

PHYSICS FOR ENGINEERING STUDENTS

Electric Circuits

From Voltage to Current and Systematic Circuit Analysis

Lecture 9 — Book-style student notes

Learning focus

Current, voltage, resistance and Ohm's law

Series and parallel resistor networks

Kirchhoff's current and voltage laws

Nodal and mesh analysis

Power balance and solution verification

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Work, Energy and Momentum

Newton's laws describe motion locally: forces determine acceleration at each instant, and kinematics then follows the object step by step. Energy and momentum provide complementary global descriptions. Energy connects different states of motion, while momentum is especially effective for short interactions such as impacts and collisions. This chapter develops these methods and explains when each is the most efficient choice.

Learning objectives

- Calculate work and determine whether it is positive, negative, or zero.
- Use kinetic, gravitational, and elastic potential energy in an initial–final state analysis.
- Include friction and other non-conservative forces through their work.
- Calculate average and instantaneous power and efficiency.
- Apply impulse and momentum conservation to collisions in one, two, and three dimensions.
- Distinguish elastic, inelastic, and perfectly inelastic collisions.

1. From Newton's laws to energy methods

A correct dynamics solution begins with a system and a free-body diagram. Newton's second law, $\Sigma F = ma$, is the natural tool when forces or acceleration are required. Kinematic equations then connect acceleration with velocity, time, and displacement. Newton's third law identifies equal and opposite interaction forces acting on different bodies.

However, some problems contain position-dependent forces, complicated paths, or many intermediate stages even though only the initial and final speeds are required. In such cases, following the motion instant by instant is unnecessarily laborious. Energy methods combine the effects of forces over displacement and often replace differential equations with algebra.

Method selection: Use forces for acceleration, kinematics for detailed motion, energy for relations between states, and momentum for brief interactions in which the time history of the force is unimportant.

A simple example: frictionless fall

A mass released from rest at height h loses gravitational potential energy and gains kinetic energy. Choosing the bottom as zero gravitational potential energy gives

$$mgh = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{(2gh)}$$

The mass cancels, and we obtain the final speed without calculating the fall time or acceleration history. This is the principal advantage of the initial–final state method.

2. Work and the transfer of energy

Work measures energy transferred by a force acting through a displacement. For a constant force $\mathbf{F}$ and displacement $\mathbf{d}$, work is the scalar product

$$\mathbf{W} = \mathbf{F} \cdot \mathbf{d} = Fd \cos\theta,$$

where θ is the angle between force and displacement. The SI unit is the joule: $1 \text{ J} = 1 \text{ N}\cdot\text{m}$. Only the component of force parallel to the displacement performs work. A perpendicular force changes direction but performs no work on the particle.

Positive, negative, and zero work

Situation	Angle	Work	Physical interpretation
Force along displacement	$\theta = 0^\circ$	$W = +Fd$	Energy is transferred to the object.
Force opposite displacement	$\theta = 180^\circ$	$W = -Fd$	Energy is removed from the object.
Force perpendicular	$\theta = 90^\circ$	$W = 0$	No energy transfer by this force.
Force at a general angle	$0^\circ < \theta < 180^\circ$	$W = Fd \cos\theta$	The sign follows $\cos\theta$.

Gravity does positive work on a falling body and negative work on a rising one. Kinetic friction usually performs negative work because it opposes relative sliding. In uniform circular motion, the centripetal resultant is perpendicular to the instantaneous displacement, so it changes velocity direction but not kinetic energy.

Work done by a varying force

When force varies with position, divide the path into small displacements and sum the contributions. In the continuous limit,

$$\mathbf{W} = \int \mathbf{F} \cdot \mathbf{dr}.$$

Geometrically, in one dimension the work equals the signed area under the force–position graph. This integral is the bridge between Newton’s local description and the energy description.

Work–kinetic energy theorem

The net work done by all forces equals the change in kinetic energy:

$$W_{\text{net}} = \Delta K = K_f - K_i, \quad K = \frac{1}{2}mv^2.$$

This follows from Newton’s second law and is valid whether the forces are conservative or non-conservative. Because kinetic energy depends on v^2 , doubling speed quadruples kinetic energy. Kinetic energy is a scalar and cannot be negative.

3. Mechanical energy and the initial–final state method

Mechanical energy is the sum of kinetic energy and relevant potential energies. Near Earth's surface, gravitational potential energy is $U_g = mgh$. For an ideal spring obeying Hooke's law, elastic potential energy is $U_s = \frac{1}{2}kx^2$, where x is deformation from equilibrium and k is the spring constant.

$$E_{\text{mech}} = K + U_g + U_s = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2.$$

Potential energy belongs to a system, not to an isolated object. Gravitational potential energy belongs to the object–Earth system; elastic potential energy belongs to the object–spring system. The zero level of potential energy is arbitrary, but the same reference must be used consistently in the initial and final states.

Conservative forces

A force is conservative if its work between two points is independent of path and can be represented by a potential energy. Gravity and the ideal spring force are standard examples. When only conservative forces act within the chosen system, mechanical energy is conserved:

$$E_{\text{mech},i} = E_{\text{mech},f},$$

$$K_i + U_{g,i} + U_{s,i} = K_f + U_{g,f} + U_{s,f}$$

Kinetic and potential energy may change substantially, but their sum remains constant. Conservation does not mean that each energy form remains unchanged; it means that energy is transformed rather than created or destroyed.

Five-step energy strategy

- Define the system and separate it conceptually from the surroundings.
- Choose initial and final states that simplify the energy terms.
- Select one consistent reference level for gravitational potential energy.
- List every energy form in each state and include non-conservative work if necessary.
- Solve symbolically, insert units, and test whether the result is physically plausible.

4. Worked example: falling block and vertical spring

A block of mass m is released from rest a height h above an initially uncompressed vertical spring of constant k . It falls, contacts the spring, and compresses it. We seek the maximum compression x . Air resistance and friction are neglected, and the spring is ideal.

Choice of system, states, and reference

Choose the block–spring–Earth system. Take the level of the uncompressed spring as $U^g = 0$. Initially, the block is at rest at height h and the spring is uncompressed, so $K_i = 0$, $U_{g,i}^g = mgh$, and $U_{s,i}^g = 0$. At maximum compression the block again has zero instantaneous speed, but it lies x below the reference level. Therefore $K_f = 0$, $U_{g,f}^g = -mgx$, and $U_{s,f}^g = \frac{1}{2}kx^2$.

$$E_i = mgh, \quad E_f = -mgx + \frac{1}{2}kx^2.$$

Only gravity and the ideal spring force act, so mechanical energy is conserved:

$$mgh = -mgx + \frac{1}{2}kx^2.$$

Solving the quadratic equation

Rearranging gives

$$\frac{1}{2}kx^2 - mgx - mgh = 0.$$

The quadratic formula produces two mathematical roots. Compression must be positive, so the physical solution is

$$x = [mg + \sqrt{(mg)^2 + 2kmgh}] / k.$$

This result contains all governing parameters. Increasing m , g , or h increases compression; increasing k decreases it. In the limit $k \rightarrow \infty$, the spring is extremely stiff and $x \rightarrow 0$. In the limit $m \rightarrow 0$ or $h \rightarrow 0$, compression also tends toward zero.

Numerical illustration

For $m = 2.0$ kg, $h = 1.5$ m, $k = 800$ N/m, and $g = 9.81$ m/s², substitution gives approximately $x = 0.133$ m = 13.3 cm. The units are metres, as required. The same equation can be obtained from the work–kinetic energy theorem: gravity performs $mg(h + x)$ of positive work, while the spring performs $-\frac{1}{2}kx^2$ of work, and the net change in kinetic energy is zero.

Physical lesson: At maximum compression the speed is zero, but the spring force and weight need not balance. Zero velocity at an instant does not imply zero acceleration.

5. Common conservation-of-energy applications

Pendulum released from rest

A pendulum bob released from a vertical height difference h above its lowest point converts gravitational potential energy into kinetic energy. Neglecting resistance, $mgh = \frac{1}{2}mv^2$ and therefore $v = \sqrt{2gh}$ at the bottom. The string tension does no work because it is perpendicular to the instantaneous displacement.

Motion on a frictionless track

For a body released from rest at height h_1 and later observed at height h_2 ,

$$mgh_1 = mgh_2 + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2g(h_1 - h_2)}.$$

Only the difference in height matters, not the detailed shape or length of the track. This is a characteristic property of conservative gravity.

Projectile without air resistance

At initial height h_0 and speed v_0 , and later at height y , conservation gives

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv^2 + mgy,$$

$$v = \sqrt{v_0^2 + 2g(h_0 - y)}.$$

Energy gives the speed but not the direction of velocity. Kinematics or momentum components may still be needed when direction, time, or trajectory is required.

Loop-the-loop condition

At the top of a vertical loop of radius R , the minimum speed that maintains contact occurs when the normal force becomes zero. Radially, $mg = mv_{\text{top}}^2/R$, so $v_{\text{top}}^2 = gR$. If the body starts from rest at height h above the bottom, energy conservation gives $mgh = mg(2R) + \frac{1}{2}mgR$, hence

$$h_{\text{min}} = 5R/2.$$

This example demonstrates method combination: Newton's second law supplies the contact condition, while energy relates the top of the loop to the starting state.

6. Non-conservative forces, power, and efficiency

Mechanical-energy change

Friction, drag, motors, and external applied forces can transfer energy across the system boundary or transform mechanical energy into thermal, sound, or deformation energy. The work of non-conservative forces equals the change in mechanical energy:

$$W_{\text{nc}} = \Delta E_{\text{mech}} = (K + U_g + U_s)_f - (K + U_g + U_s)_i.$$

For a block sliding a distance d on a horizontal surface with kinetic friction $f_k = \mu_k mg$, friction performs $W_{\text{fr}} = -\mu_k mgd$. If its initial speed is v_0 ,

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -\mu_k mgd,$$

$$v = \sqrt{(v_0^2 - 2\mu_k gd)},$$

provided the expression under the square root is non-negative. Otherwise the body stops before travelling the assumed distance. The “lost” mechanical energy has not disappeared; it has mainly become internal thermal energy of the surfaces and surroundings.

Power

Power measures the rate of work or energy transfer. Average and instantaneous power are

$$P_{\text{avg}} = W/\Delta t = \Delta E/\Delta t, \quad P = dW/dt = dE/dt.$$

For translational motion, instantaneous power is the scalar product

$$P = \mathbf{F} \cdot \mathbf{v}.$$

Thus the same driving force requires more power at higher speed. A force perpendicular to velocity transfers no instantaneous power. The watt is a joule per second: 1 W = 1 J/s.

Efficiency

$$\eta = P_{\text{useful}}/P_{\text{in}} \times 100\% = E_{\text{useful}}/E_{\text{in}} \times 100\%.$$

Real devices have $\eta < 100\%$ because some input energy becomes heat, sound, vibration, or other unwanted forms. A motor with 800 W input and 75% efficiency delivers 600 W of useful output power.

Worked power examples

Example 1: A student lifts a 20 kg box vertically through 3.0 m in 4.0 s at approximately constant speed. The useful work done on the box is the increase in gravitational potential energy, $W = mgh = (20 \text{ kg})(9.81 \text{ m/s}^2)(3.0 \text{ m}) = 588.6 \text{ J}$. The average useful power is

$$P_{\text{avg}} = W/\Delta t = 588.6 \text{ J}/4.0 \text{ s} = 147 \text{ W}.$$

Because the speed is constant, the lifting force is approximately mg . The same result follows from $P = Fv$: the average vertical speed is $3.0 \text{ m}/4.0 \text{ s} = 0.75 \text{ m/s}$, and $P = (196.2 \text{ N})(0.75 \text{ m/s}) = 147 \text{ W}$. The agreement demonstrates that power may be calculated from either energy per time or force times velocity.

Example 2: A car provides a driving force of 1500 N while travelling at a steady 30 m/s. The mechanical power delivered at the wheels is $P = Fv = 45,000 \text{ W} = 45 \text{ kW}$. If the drivetrain efficiency is 80%, the required input power is $P_{\text{in}} = P_{\text{out}}/\eta = 45 \text{ kW}/0.80 = 56.25 \text{ kW}$. The remaining 11.25 kW is dissipated in the drivetrain and surroundings. At steady speed the net force is zero, but the engine power is not zero: the driving force balances resistance while energy is continually dissipated.

7. Linear momentum and impulse

Linear momentum describes translational motion and is defined by

$$\mathbf{p} = m\mathbf{v}.$$

Unlike energy, momentum is a vector. Its direction is the direction of velocity, and separate components may be positive, negative, or zero according to the chosen coordinate axes. The SI unit $\text{kg}\cdot\text{m/s}$ is equivalent to $\text{N}\cdot\text{s}$.

Impulse–momentum theorem

Newton's second law can be written as $F_{\text{net}} = dp/dt$. Integrating over an interaction from t_i to t_f gives

$$\mathbf{J}_{\text{net}} = \int \mathbf{F}_{\text{net}} dt = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i.$$

For a constant or average force, $J = F_{\text{avg}}\Delta t$. The area under a force–time graph is impulse. For a fixed momentum change, increasing the contact time reduces average force:

$$F_{\text{avg}} = \Delta p / \Delta t.$$

Catching and striking a ball

When a player catches a ball and brings it to rest, moving the hand backward increases stopping time and reduces the average force. When a bat reverses the ball's direction, the momentum change is actually larger than for simply stopping it. If the presentation compares equal momentum changes, the shorter collision time still produces the larger force. In real deflection problems, both the momentum change and contact duration must therefore be evaluated carefully.

This principle underlies airbags, seat belts, helmets, padding, and crumple zones. These devices do not necessarily reduce the required momentum change, but they lengthen the interval over which it occurs and distribute the force more safely.

8. Conservation of momentum

For a system of particles, internal forces occur in equal and opposite pairs and cancel in the total momentum balance. If the net external impulse is zero, total momentum remains constant:

$$\Sigma \mathbf{p}_i = \Sigma \mathbf{p}_f.$$

For two particles,

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}.$$

Conservation holds separately in every coordinate direction. A system may conserve x-momentum while an external impulse changes y-momentum; consequently, isolation must be assessed component by component. During a brief collision, gravity and friction may be negligible compared with the large contact forces, so momentum can be approximately conserved even though the system is not perfectly isolated over longer times.

Applications

Application	Momentum principle	Engineering purpose
Vehicle safety	Increase stopping time for a given Δp	Reduce average force on occupants.
Rocket propulsion	Exhaust gains backward momentum	Rocket gains forward momentum.
Firearm recoil	Bullet and gun acquire opposite momenta	Larger gun mass or longer recoil time reduces speed or force.
Pile driver	Large falling momentum produces a large impulse	Drive a pile into the ground efficiently.

9. Collisions in one dimension

In any collision of an isolated system, total momentum is conserved. Kinetic energy is conserved only in an elastic collision. In an inelastic collision, some kinetic energy becomes internal energy, sound, heat, or permanent deformation. In a perfectly inelastic collision the bodies stick together.

Collision type	Momentum	Kinetic energy	Restitution
Elastic	Conserved	Conserved	$e = 1$
Inelastic	Conserved	Not conserved	$0 < e < 1$
Perfectly inelastic	Conserved	Not conserved	$e = 0$

Perfectly inelastic collision

If masses m_1 and m_2 stick together and move with common velocity v_f , conservation of momentum gives

$$v_f = (m_1 v_{1i} + m_2 v_{2i}) / (m_1 + m_2).$$

Velocity signs are essential. Kinetic energy after the collision is less than or equal to its initial value; the difference is transformed into other energy forms.

Worked example: perfectly inelastic collision

A 0.20 kg puck moving at +5.0 m/s strikes a stationary 0.30 kg puck, and the pucks stick together. Take the original direction as positive. Initial momentum is $(0.20 \text{ kg})(5.0 \text{ m/s}) = 1.0 \text{ kg}\cdot\text{m/s}$. Conservation of momentum gives

$$v_f = 1.0 \text{ kg}\cdot\text{m/s} / (0.20 \text{ kg} + 0.30 \text{ kg}) = 2.0 \text{ m/s}.$$

The initial kinetic energy is $K_i = \frac{1}{2}(0.20)(5.0)^2 = 2.5 \text{ J}$. The final kinetic energy is $K_f = \frac{1}{2}(0.50)(2.0)^2 = 1.0 \text{ J}$. Thus 1.5 J of kinetic energy is transformed into deformation, sound, and internal energy. Momentum is conserved, but kinetic energy is not. This numerical comparison is the clearest way to verify the collision classification.

Elastic collision in one dimension

For an elastic collision, use both momentum and kinetic-energy conservation:

$$m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f},$$

$$\frac{1}{2}m_1 \mathbf{v}_{1i}^2 + \frac{1}{2}m_2 \mathbf{v}_{2i}^2 = \frac{1}{2}m_1 \mathbf{v}_{1f}^2 + \frac{1}{2}m_2 \mathbf{v}_{2f}^2.$$

Solving gives

$$\mathbf{v}_{1f} = [(m_1 - m_2)/(m_1 + m_2)]\mathbf{v}_{1i} + [2m_2/(m_1 + m_2)]\mathbf{v}_{2i},$$

$$\mathbf{v}_{2f} = [2m_1/(m_1 + m_2)]\mathbf{v}_{1i} + [(m_2 - m_1)/(m_1 + m_2)]\mathbf{v}_{2i}.$$

Equal masses exchange velocities in a one-dimensional elastic collision. A light object striking a much heavier stationary object rebounds approximately with its original speed. These limiting cases are valuable checks.

Worked example: elastic collision

Let $m_1 = 2.0$ kg move at $v_{1i} = +3.0$ m/s and let $m_2 = 1.0$ kg move at $v_{2i} = -1.0$ m/s. Substitution into the elastic-collision formulas gives

$$v_{1f} = [(2-1)/(2+1)](3) + [2(1)/(2+1)](-1) = 0.33 \text{ m/s},$$

$$v_{2f} = [2(2)/(2+1)](3) + [(1-2)/(2+1)](-1) = 4.33 \text{ m/s}.$$

Check momentum: initially, $p_i = (2)(3) + (1)(-1) = 5$ kg·m/s. Finally, $p_f \approx (2)(0.33) + (1)(4.33) = 4.99$ kg·m/s, with the small difference caused by rounding. Check kinetic energy: $K_i = \frac{1}{2}(2)(3^2) + \frac{1}{2}(1)(1^2) = 9.5$ J, while $K_f \approx \frac{1}{2}(2)(0.33^2) + \frac{1}{2}(1)(4.33^2) = 9.48$ J, again equal within rounding. Both conservation laws are satisfied.

Coefficient of restitution

$$e = \text{relative speed of separation} / \text{relative speed of approach}.$$

Restitution supplies an additional relation when a collision is neither perfectly elastic nor perfectly inelastic. Sign conventions must be handled through velocity components, while e is normally reported as a non-negative scalar between zero and one.

10. Collisions in two and three dimensions

Momentum conservation is vectorial. Choose axes, resolve every initial and final velocity, and conserve each component:

$$\Sigma p_{x,i} = \Sigma p_{x,f}, \quad \Sigma p_{y,i} = \Sigma p_{y,f}, \quad \Sigma p_{z,i} = \Sigma p_{z,f}.$$

For an elastic collision, kinetic-energy conservation supplies one additional scalar equation. In a general two-dimensional collision of two bodies, x- and y-momentum plus kinetic energy provide three equations. Four unknown final components remain unless an angle, symmetry condition, or other constraint is specified.

Recommended procedure

- Draw the before-and-after velocity diagram and choose convenient axes.
- Resolve velocities into signed Cartesian components.
- Write momentum conservation in every independent direction.
- Add kinetic-energy conservation only if the collision is explicitly elastic.
- Solve for components, then reconstruct speeds and directions.
- Check total momentum, kinetic energy when applicable, signs, and limiting cases.

In three dimensions, add the z-component. Coordinate choice can simplify the algebra—for example, align the x-axis with one initial velocity or choose the initial motion plane as the x–y plane.

11. Summary and problem-solving checklist

Question	Most direct tool
What acceleration is produced by known forces?	Newton's second law and an FBD.
How are speed and position related over a path?	Kinematics or energy, depending on known data.
What connects two states without tracking every instant?	Work–energy and energy conservation.
What happens during a very short interaction?	Impulse and momentum.
What is conserved in an isolated collision?	Total vector momentum.
When is kinetic energy also conserved?	Only in an elastic collision.

Essential equations

$$W = Fd \cos\theta, \quad W_{\text{net}} = \Delta K, \quad E_{\text{mech}} = K + U_g + U_s,$$

$$W_{\text{net}} = \Delta E_{\text{mech}}, \quad \mathbf{P} = d\mathbf{E}/dt = \mathbf{F} \cdot \mathbf{v},$$

$$\mathbf{p} = m\mathbf{v}, \quad \mathbf{J} = \int \mathbf{F} \, dt = \Delta \mathbf{p}, \quad \Sigma \mathbf{p}_i = \Sigma \mathbf{p}_f.$$

Short review problems

- A 10 N force acts through 5.0 m at 60° to the displacement. Calculate the work.
- A 3.0 kg object is released from rest 4.0 m above the ground. Find its speed immediately before impact, neglecting air resistance.
- A 1200 kg car travels at 20 m/s. Calculate its kinetic energy and the average braking force required to stop it over 50 m.
- A 500 W motor lifts a 40 kg load at constant speed. Find the maximum ideal lifting speed.
- A 0.15 kg ball changes velocity from +20 m/s to –10 m/s in 0.050 s. Find the average force.
- A 2.0 kg cart moving at 3.0 m/s sticks to a stationary 1.0 kg cart. Find their common velocity and the kinetic energy transformed.

Answers for self-checking

1) 25 J. 2) 8.86 m/s. 3) $K = 240 \text{ kJ}$ and $F_{\text{avg}} = 4.8 \text{ kN}$ opposite the motion. 4) 1.27 m/s. 5) -90 N , where the negative sign indicates the force is opposite the initial positive direction. 6) $v_f = 2.0 \text{ m/s}$; the kinetic energy decreases from 9.0 J to 6.0 J, so 3.0 J is transformed.

Final takeaway: Energy describes the capacity for change and connects states; momentum describes directed motion and governs short interactions. The best engineering solution comes from choosing the principle that removes unnecessary detail while retaining the physics needed to answer the question.