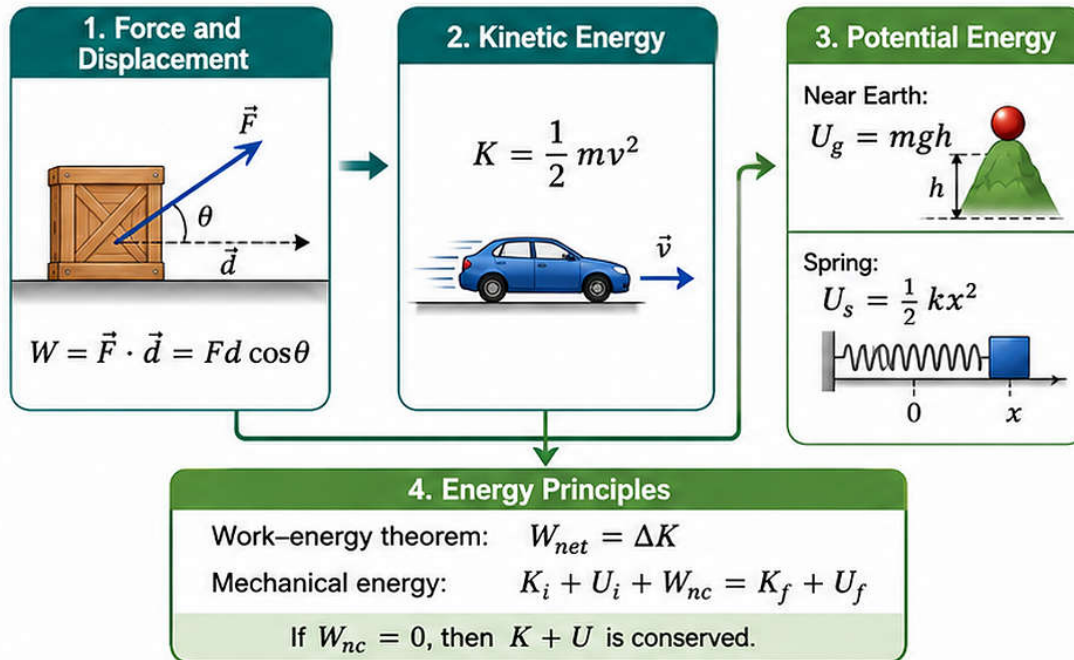


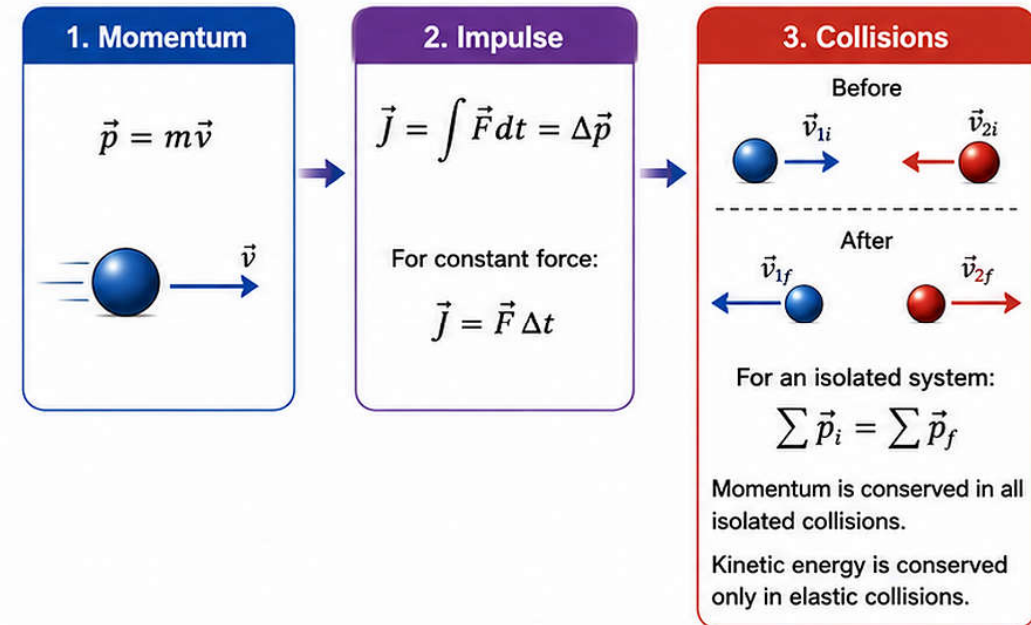
Lecture 3

Work, Energy and Momentum

WORK-ENERGY



IMPULSE-MOMENTUM



Course Progress

- ☒ Lecture 1 — Kinematics
- ☒ Lecture 2 — Newton's Laws
- ☒ **Lecture 3 — Work, Energy and Momentum**



Energy methods connect
force and **displacement**.

Momentum methods connect
force and **time**.



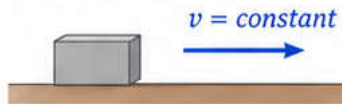
Choose the method that best
matches the information given
and the quantity required.

Newton's Laws and Free-Body Diagrams — Recap

Newton's laws connect the forces acting on a body to its motion.

Newton's laws

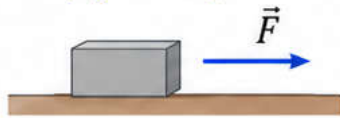
1 First law (Inertia)



$$\sum \vec{F} = 0 \Rightarrow \vec{v} = \text{constant}$$

Zero net force means zero acceleration.

2 Second law (Dynamics)



$$\sum \vec{F} = m\vec{a}$$

The vector sum of the external forces determines the acceleration.

3 Third law (Action-Reaction)

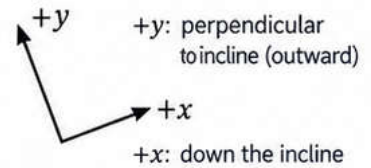
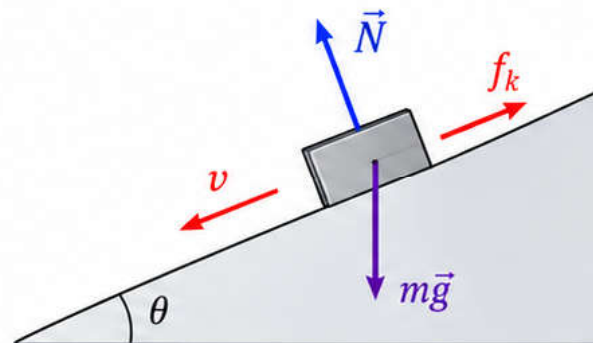


$$\vec{F}_{A \rightarrow B} = -\vec{F}_{B \rightarrow A}$$

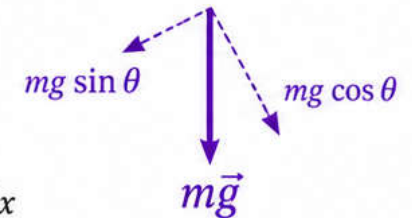
The pair acts on two different bodies and must never be placed on the same free-body diagram.

Free-body diagram

Block sliding down a rough incline



Components of weight
(not additional forces)



$$\sum F_x = mg \sin \theta - f_k = ma_x$$

$$\sum F_y = N - mg \cos \theta = 0$$

$$f_k = \mu_k N$$



Free-body diagram rule: isolate one object, draw every external force once, then project Newton's second law onto convenient axes.

Choosing a Mechanics Method — Recap

The question determines the most efficient physical principle.

Question	Primary principle	What it determines
 What is the acceleration?	Newton's second law $\sum \vec{F} = m\vec{a}$	Acceleration from the external forces at that instant. A free-body diagram is required.
 How do position and velocity change with time?	Kinematics $v = v_0 + at$ $x = x_0 + v_0t + \frac{1}{2}at^2$	Motion when the acceleration is known and constant. Kinematics describes motion; it does not identify the forces.
 What speed or position follows from a change of state?	Work and energy $W_{net} = \Delta K$ $K_i + U_i + W_{nc} = K_f + U_f$	Relates two states without calculating the complete time history. This lecture develops this method.
 What happens during a short interaction or collision?	Impulse and momentum $\vec{J} = \Delta \vec{p}$ $\sum \vec{p}_i = \sum \vec{p}_f$ (isolated system)	Velocity changes during short interactions.

Quick decision aid:

1

Need acceleration at an instant?
Use forces and Newton's second law.

2

Need to relate initial and final states?
Use energy when the relevant energy terms and non-conservative work are known.

Many problems require more than one method, but each equation answers a different question.



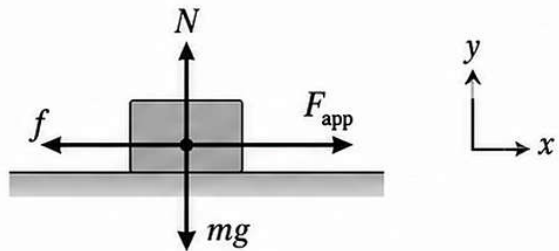
Energy methods complement Newton's laws; they do not replace the force model.

Why Do We Need Energy?

Newton's Laws

Known:

- ✓ Forces
- ✓ Mass



$$\sum \vec{F} = m\vec{a}$$

Acceleration

Motion as a function of time

Newton's
Laws

Energy
Methods

*Different methods —
Same physics*

Energy Method

Known:

- ✓ Initial state
- ✓ Final state



Work–Energy / Energy Balance

$$K_i + U_i + W_{nc} = K_f + U_f$$

No need to determine
every force separately

Relates initial and final states

If $W_{nc} = 0$, mechanical energy is conserved.

Newton's Laws

Uses forces

Finds acceleration

Describes motion step by step

Best when forces or time dependence are needed



Energy Method

Uses work and energy

Compares states

Relates initial → final

Best when only speeds or positions are needed



Key idea: Choose the method that gives the simplest correct solution.

2. Positive, Negative and Zero Work

Definition

Work done by a constant force during a displacement is:

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

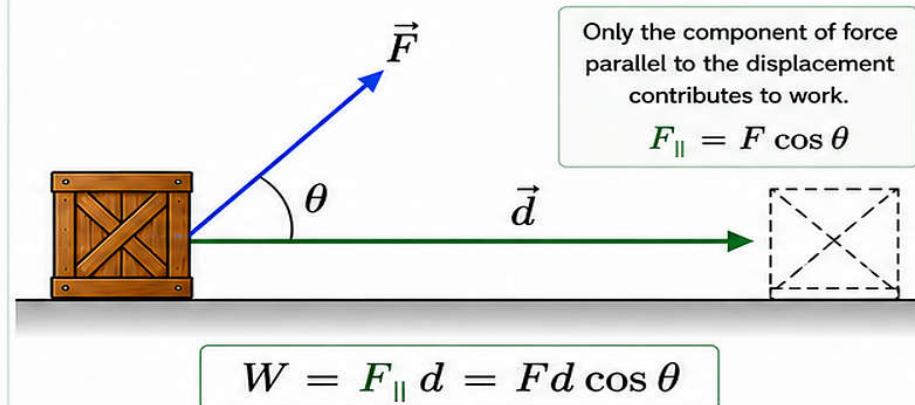
$\vec{F}$: force (N)

$\vec{d}$: displacement (m)

θ : angle between force and displacement

W : work (joule, J)

Geometric interpretation

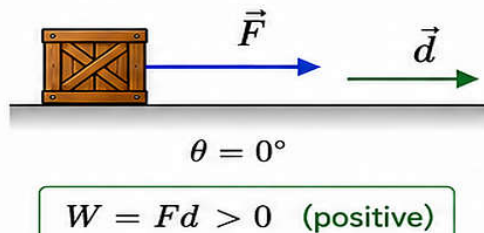


Key points

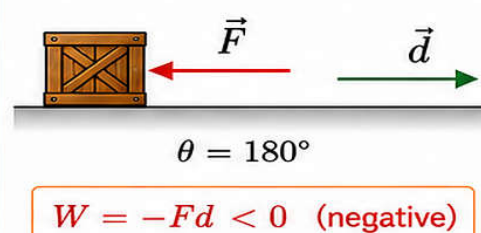
- + Work is a scalar.
- Unit: joule (J) = N·m
- 💡 Positive work transfers energy to the object.
- Negative work transfers energy from the object.
- 0 Zero work occurs when $d = 0$ or when $\vec{F} \perp \vec{d}$.

Examples

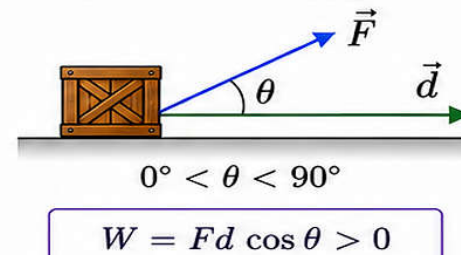
1. Force in the direction of displacement



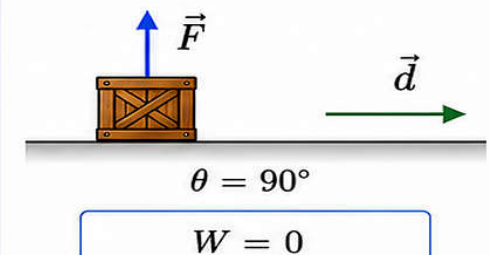
2. Force opposite to displacement



3. Force at an acute angle



4. Force perpendicular to displacement



Physical meaning:

Work is a transfer of energy caused by a force acting through a displacement.

Force



Work



Energy transfer

3. Energy and the Initial–Final State Method

What is Energy?

Energy is a scalar quantity that measures a system's capacity to do work or produce physical change.

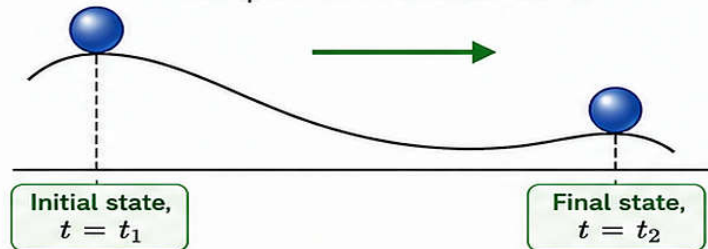
Energy can be stored, transferred, and transformed.



The total energy of an isolated system is conserved.

Initial–Final State Method

Instead of following the motion step by step, compare two selected states.



We write an energy balance between these states.

General Energy Balance

$$K_i + U_i + W_{nc} = K_f + U_f$$

If $W_{nc} = 0$:

$$K_i + U_i = K_f + U_f$$

Mechanical energy is conserved only when no non-conservative work changes it.

- No need to determine the force at every point.
- Directly relates initial and final states.
- Useful for complex paths.

Kinetic Energy, K

Energy associated with motion.

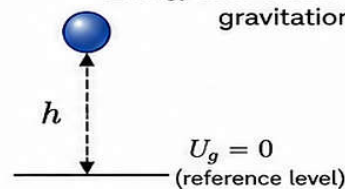


$$K = \frac{1}{2}mv^2$$

Common Forms of Mechanical Energy

Gravitational Potential Energy, U_g

Energy associated with position in a gravitational field.



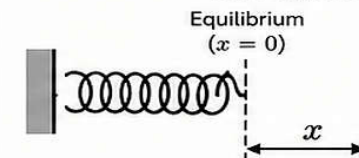
h is measured vertically from the chosen reference level where $U_g = 0$.

$$U_g = mgh$$

(near Earth's surface)

Elastic Potential Energy, U_s

Energy stored in an ideal spring displaced from equilibrium.



$$U_s = \frac{1}{2}kx^2$$

x is the extension ($x > 0$) or compression ($x < 0$) measured from equilibrium.



Important:

Always define the system and the reference levels, then include all relevant energy transfers.



Define System



Choose Reference Levels



Identify Energy & Work Terms



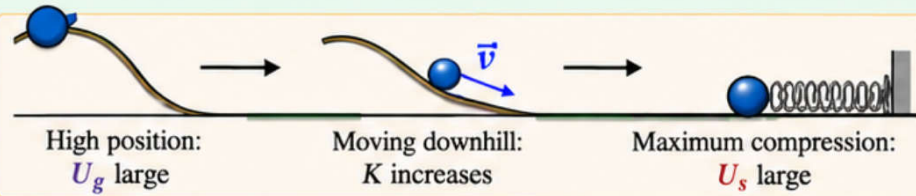
Write the Energy Balance

4. Energy Transformations

E

Closed system (cart + track + spring + Earth): different pathways, the same conserved total energy.

A. Without Dissipative Interactions



Initial state

During motion

Final state

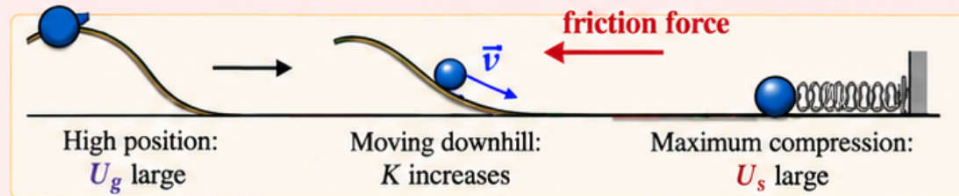


$$U_g \longrightarrow K \longrightarrow U_s$$

Mechanical energy remains constant.

$$W_{nc} = 0 \implies E_{\text{mech}} = \text{constant}$$

B. With Friction



Initial state

During motion

Final state



$$U_g \longrightarrow K + E_{\text{int}} \longrightarrow U_s + E_{\text{int}}$$

Mechanical energy decreases as internal energy increases.

$$E_{\text{mech}} + E_{\text{int}} = \text{constant}$$

Energy forms: U_g K U_s E_{int}

Total energy is conserved. Friction changes the final distribution, not the total amount.

6. How to Set Up an Energy Problem



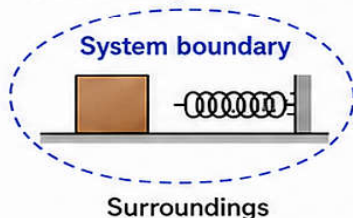
Strategy: Define the system, choose initial and final states, identify energy and work terms, then write the energy balance.

$$E_{mech,i} + W_{nc} = E_{mech,f}$$

1. Define the System

Choose the object or collection of objects to analyze.

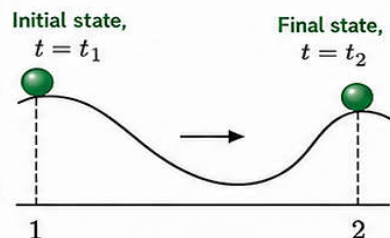
Everything outside the system is the **surroundings**.



2. Choose Initial and Final States

Select two states that simplify the problem.

Label known and unknown quantities.



3. Identify Energy and Work Terms

Include only terms relevant to the chosen system:

- Kinetic energy, K
- Gravitational potential energy, U_g
- Elastic potential energy, U_s
- Non-conservative work, W_{nc}

Choose consistent reference levels for potential energies.

4. Write Each State

Write the mechanical energy at each state:

$$E_{mech,i} = K_i + U_{g,i} + U_{s,i}$$

$$E_{mech,f} = K_f + U_{g,f} + U_{s,f}$$

Determine W_{nc} , including its sign.

5. Apply the Energy Balance

General:

$$E_{mech,i} + W_{nc} = E_{mech,f}$$

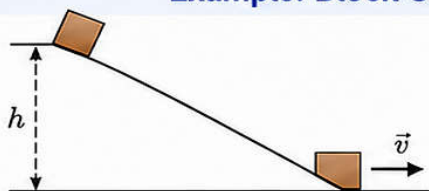
Special case:

If $W_{nc} = 0$:

$$E_{mech,i} = E_{mech,f}$$

Zero non-conservative work is the condition for mechanical-energy conservation.

Example: Block Sliding Down a Frictionless Ramp



Initial (top)

$$K_i = 0$$

$$U_{g,i} = mgh$$

Final (bottom)

$$K_f = \frac{1}{2}mv^2$$

$$U_{g,f} = 0$$

Since $W_{nc} = 0$:

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$



Good Practice

- Draw a clear sketch.
- Define the system and reference levels.
- Choose states that simplify the algebra.
- Keep the sign of W_{nc} consistent.
- Check units and limiting cases.
- Ask whether the result is physically reasonable.



Key message: Energy methods relate states directly, but the system and energy transfers must be defined carefully.

7. Falling Object and Spring – Problem Setup

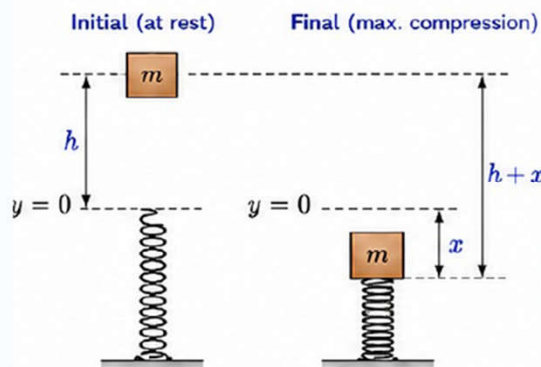


A block of mass m is released from rest a vertical distance h above an initially uncompressed vertical spring of constant k . Find the maximum compression x .

Why this example?

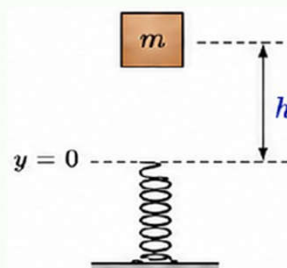
- Gravitational and elastic potential energies are involved.
- The initial–final state method avoids intermediate speeds.
- The block descends a total distance $h + x$.

1. Physical Situation



The block falls through h , contacts the spring, then compresses it by x .

2. Initial State (i)

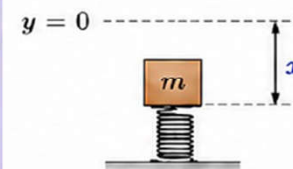


Choose $U_g = 0$ at the uncompressed spring top.

$$\begin{aligned} K_i &= 0 \\ U_{g,i} &= mgh \\ U_{s,i} &= 0 \\ \hline E_i &= mgh \end{aligned}$$

3. Final State (f)

At maximum compression, the block is momentarily at rest.



$$\begin{aligned} K_f &= 0 \\ U_{g,f} &= -mgx \\ U_{s,f} &= \frac{1}{2} kx^2 \\ \hline E_f &= -mgx + \frac{1}{2} kx^2 \end{aligned}$$

System



Block + spring + Earth

Known



m, k, h, g

Find



Maximum compression x

Assumptions



- Negligible air resistance
- Ideal massless spring
- Hooke's law applies
- Spring initially uncompressed



Key idea

The gravitational–energy decrease over the total descent $h + x$ becomes elastic potential energy:

$$mg(h + x) = \frac{1}{2} kx^2$$

9. Falling Object and Spring – Solution



With the system defined as block + spring + Earth and negligible air resistance, $W_{nc} = 0$.

Initial energy:

$$E_{mech,i} = mgh$$

Final energy:

$$E_{mech,f} = -mgx + \frac{1}{2}kx^2$$

Energy balance:

$$mgh = -mgx + \frac{1}{2}kx^2$$

1. Rearrange the Equation

$$mgh = -mgx + \frac{1}{2}kx^2$$



$$\frac{1}{2}kx^2 - mgx - mgh = 0$$

This is a quadratic equation in x .

We require the physically meaningful root $x \geq 0$.

2. Solve for the Compression x

General form: $ax^2 + bx + c = 0$

$$a = \frac{1}{2}k, \quad b = -mg, \quad c = -mgh$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2kmgh}}{k}$$

The minus sign gives $x \leq 0$ and is not the physical compression.

3. Physical Solution

We take the positive root:

$$x = \frac{mg + \sqrt{(mg)^2 + 2kmgh}}{k}$$

The negative root gives $x \leq 0$ and is not the physical compression.

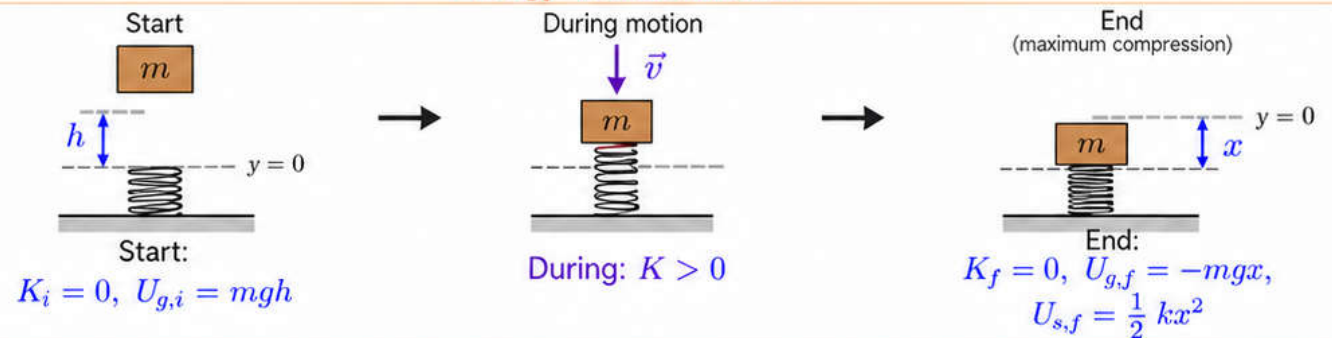
Therefore, x is the maximum compression of the spring.

Interpretation



- Gravitational potential energy *decreases* during the total descent $h + x$.
- Elastic potential energy *increases* as the spring is compressed.
- Kinetic energy is zero initially and at maximum compression, but *positive in between*.
- Total mechanical energy remains constant because $W_{nc} = 0$.

Energy Transformation



10. Falling Object and Spring — Discussion



Big idea:

During the total descent $h + x$, gravitational potential energy is transformed into kinetic energy and finally into elastic potential energy.

Key takeaways:

- Initial and final states can determine the maximum compression.
- Intermediate motion need not be calculated.
- The result must pass dimensional and limiting-case checks.

1. What Affects Maximum Compression?

$$x = \frac{mg + \sqrt{(mg)^2 + 2kmgh}}{k}$$

Mass m	larger $m \rightarrow$ larger x
Gravity g	larger $g \rightarrow$ larger x
Drop height h	larger $h \rightarrow$ larger x
Spring constant k	larger $k \rightarrow$ smaller x



These trends follow from the exact solution.

2. Limiting and Dimensional Checks

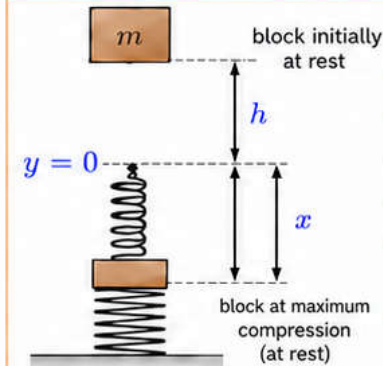
- **Very stiff spring**, $k \rightarrow \infty$: $x \rightarrow 0$
- **Vanishing mass**, $m \rightarrow 0$: $x \rightarrow 0$
- **No initial gap**, $h \rightarrow 0$: $x \rightarrow \frac{2mg}{k}$
- **Large drop height**: $x \approx \sqrt{\frac{2mgh}{k}}$

Thus x grows approximately as $\sqrt{h}$ when h is large.

$$x_{eq} = \frac{mg}{k}; \text{ for } h = 0, \quad x_{max} = 2x_{eq}.$$

Dimensional check: the numerator has units of N; division by k (N/m) gives x in metres.

3. Assumptions and Model



- System: block + spring + Earth
- Negligible air resistance
- Ideal massless spring
- Hooke's law applies
- Spring initially uncompressed
- Uniform gravitational field

4. Alternative Viewpoint: Work–Energy

Work by gravity over total downward displacement $h + x$:

$$W_g = mg(h + x)$$

Work by the spring during compression:

$$W_s = -\frac{1}{2} kx^2$$

Since $K_i = K_f = 0$: $W_g + W_s = \Delta K = 0$

$$mg(h + x) - \frac{1}{2} kx^2 = 0$$

This is equivalent to the energy-conservation result.

5. Physical Interpretation

1. If h doubles, x increases, but not generally by a factor of $\sqrt{2}$; that is only the large- h approximation.
2. A softer spring produces a larger maximum compression.
3. Dissipation such as air resistance reduces the maximum compression because energy becomes thermal/internal energy.



Overall lesson: A correct model, consistent reference levels, and dimensional and limiting-case checks are essential.

11. Four Problems for the Energy-State Method



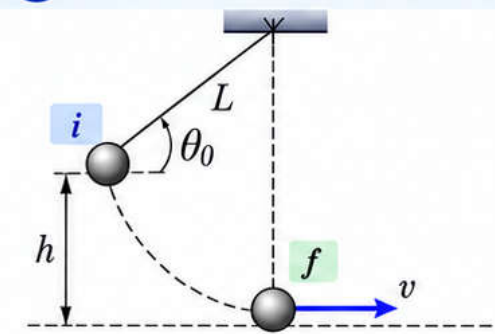
Same method – different motion

For each problem, identify a useful initial state i and final state f . The detailed energy balance and solution are developed on the following slides.



Assume no air resistance or dissipative friction unless stated otherwise.

1 Pendulum Released from Rest

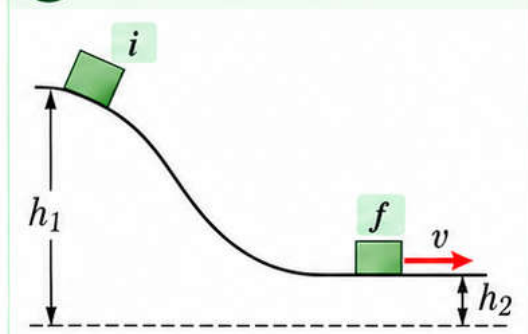


Problem:
The bob is released from rest. Determine its speed and the string tension at the lowest point.

Given: m, L, θ_0, g

Find: v_{bottom} and T_{bottom}

2 Block on a Frictionless Track

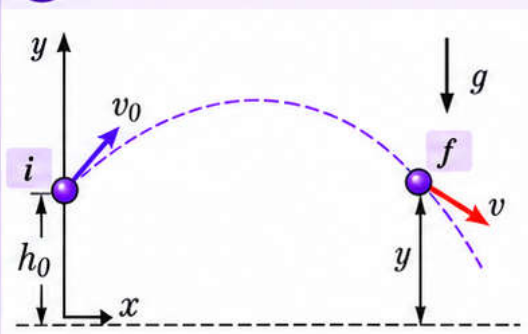


Problem:
A block slides without friction. Determine its speed at the selected height h_2 .

Given: m, h_1, h_2, g

Find: v at state f

3 Projectile without Air Resistance

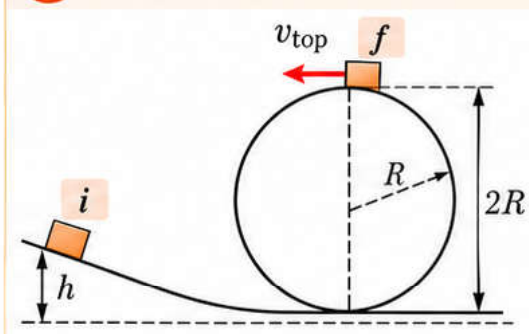


Problem:
A projectile is launched from height h_0 . Determine its speed when it reaches height y .

Given: v_0, h_0, y, g

Find: speed magnitude v at state f

4 Loop-the-Loop



Problem:
Determine the minimum release height required for the body to complete the loop without losing contact.

Given: R, g ; released from rest

Find: minimum h

Common setup used in the next four worked examples



The energy equation and algebra are introduced only on the detailed solution slides.

12. Pendulum Released from Rest – Problem Setup

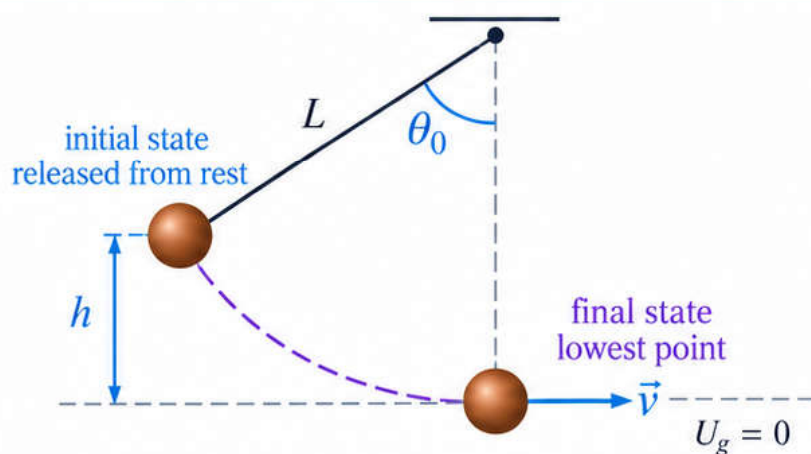
Problem

A pendulum bob of mass $m = 0.50$ kg and length $L = 1.20$ m is released from rest at $\theta_0 = 60^\circ$ from the vertical. Find its speed and the string tension at the lowest point.

Why this example?

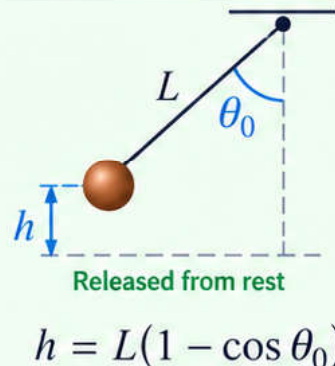
Energy conservation gives the speed. Radial dynamics is then required to determine the tension.

1. Physical Situation



The bob descends through h and reaches the lowest point with maximum speed.

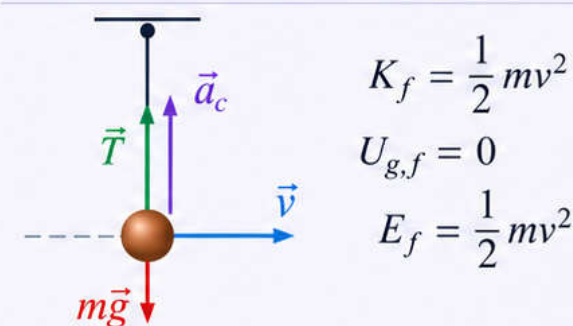
2. Initial State (I)



$$\begin{aligned} K_i &= 0 \\ U_{g,i} &= mgh \\ E_i &= mgh \end{aligned}$$

Reference: $U_g = 0$ at the lowest point

3. Final State and Required Quantities



Quantities to determine

Speed at the lowest point
 $v = ?$

String tension at the lowest point
 $T = ?$

At the bottom, the radial direction points upward toward the pivot.

System

bob + Earth

Known

m, L, θ_0, g

Find

v at bottom
 T at bottom

Assumptions

massless string, no air resistance,
frictionless pivot

Key idea

Use energy first, then apply the radial force equation.

13. Pendulum Released from Rest – Solution

System: bob + Earth

No air resistance; massless string;
frictionless pivot

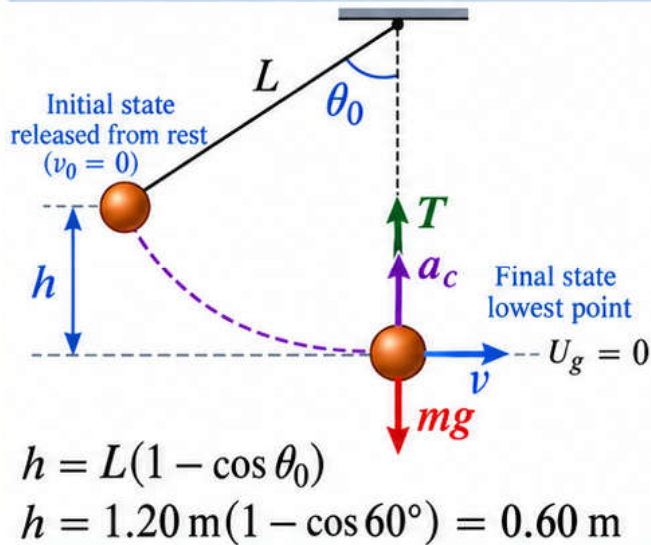
Energy balance

$$E_{\text{mech},i} = E_{\text{mech},f}$$

Radial dynamics at bottom

$$\sum F_r = ma_c \quad a_c = \frac{v^2}{L}$$

1. Geometry and Force Directions



2. Energy Conservation → Speed

For the bob + Earth system

$$E_i = E_f$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2gL(1 - \cos \theta_0)}$$

$$v = \sqrt{2(9.81)(0.60)} = 3.43 \text{ m s}^{-1}$$

Mass cancels from the equation.

Energy gives the speed, but not the string tension.

3. Radial Dynamics → Tension

At the lowest point, inward is upward

$$\sum F_r = ma_c = m \frac{v^2}{L}$$

$$T - mg = m \frac{v^2}{L}$$

$$T = mg + m \frac{v^2}{L}$$

$$T = mg[3 - 2 \cos \theta_0]$$

$$T = (0.50)(9.81)[3 - 2 \cos 60^\circ] = 9.81 \text{ N}$$

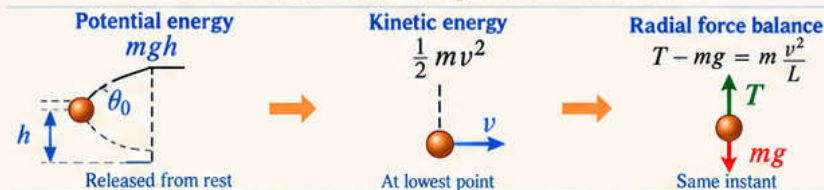
+ radial direction
↑
toward pivot

Dynamics gives the tension after v is known.

Interpretation

- Mass cancels from the speed equation.
- $T > mg$ because inward acceleration is upward.
- Energy alone cannot determine T .
- Larger θ_0 gives larger v and larger T .

How the Methods Complement Each Other



Energy describes the state change; dynamics determines the instantaneous force.

Final Results

$$h = 0.60 \text{ m}$$

$$v = 3.43 \text{ m s}^{-1}$$

$$T = 9.81 \text{ N} = 2.00 mg$$

Use $g = 9.81 \text{ m s}^{-2}$

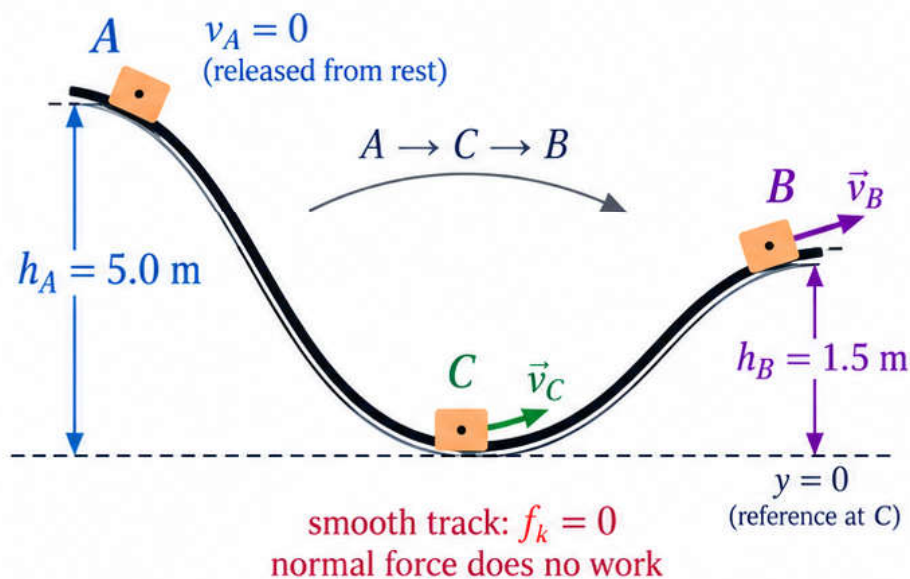
14. Block on a Frictionless Track – Problem Setup

A 2.0 kg block starts from rest at A, 5.0 m above the lowest point. Determine its speed at the lowest point C and at B, where B is 1.5 m above C.

Why this example?

Conservation of mechanical energy connects speed to vertical height; the shape of the track does not affect the result.

Physical Situation



1. Initial State A

Position (height):

$$y_A = h_A = 5.0 \text{ m}$$

Speed:

$$v_A = 0 \quad (\text{released from rest})$$

Kinetic energy:

$$K_A = 0$$

Gravitational potential energy:

$$U_{g,A} = mgh_A$$

Total mechanical energy:

$$E_A = mgh_A$$

2. Lowest State C

Position (height):

$$y_C = 0 \quad (\text{reference level})$$

Speed:

$$v_C = ? \quad (\text{maximum speed})$$

Kinetic energy:

$$K_C = \frac{1}{2} mv_C^2$$

Gravitational potential energy:

$$U_{g,C} = 0$$

Total mechanical energy:

$$E_C = \frac{1}{2} mv_C^2$$

3. Higher State B

Position (height):

$$y_B = h_B = 1.5 \text{ m}$$

Speed:

$$v_B = ?$$

Kinetic energy:

$$K_B = \frac{1}{2} mv_B^2$$

Gravitational potential energy:

$$U_{g,B} = mgh_B$$

Total mechanical energy:

$$E_B = \frac{1}{2} mv_B^2 + mgh_B$$

System

block + Earth

Known

m, h_A, h_B, g

Find

v_C and v_B

Assumptions

released from rest, frictionless track, negligible air resistance, uniform g

Key idea

Same total mechanical energy at A, C, and B.

15. Block on a Frictionless Track – Solution and Discussion

System: block + Earth

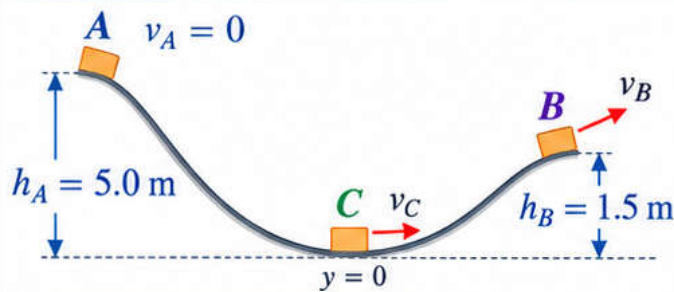
No non-conservative work: $W_{nc} = 0$

Mechanical energy is conserved:

$$K_i + U_{g,i} = K_f + U_{g,f} \quad E_A = E_C = E_B$$

Choose $U_g = 0$ at C
($y = 0$ at the lowest point).

A. INITIAL STATE A — ENERGY ACCOUNTING



$$\begin{aligned} K_A &= 0 \\ U_{g,A} &= mgh_A \\ E_A &= mgh_A \end{aligned}$$

Released from rest.

$$h_A = 5.0 \text{ m}, \quad v_A = 0$$

C. LOWEST POINT C — SOLVE FOR THE MAXIMUM SPEED

$$\begin{aligned} E_A &= E_C \\ mgh_A &= \frac{1}{2} mv_C^2 \\ gh_A &= \frac{1}{2} v_C^2 \quad (m \text{ cancels}) \\ v_C &= \sqrt{2gh_A} \\ v_C &= \sqrt{2(9.81)(5.0)} \\ v_C &= 9.90 \text{ m s}^{-1} \end{aligned}$$

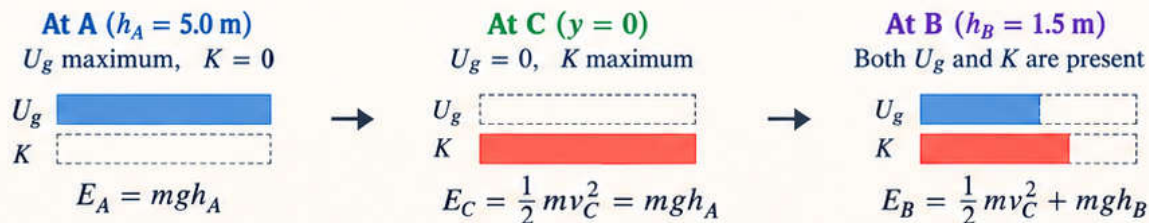
Maximum speed because C is the minimum height.

B. HIGHER STATE B — SOLVE FOR THE SPEED AT h_B

$$\begin{aligned} E_A &= E_B \\ mgh_A &= \frac{1}{2} mv_B^2 + mgh_B \\ g(h_A - h_B) &= \frac{1}{2} v_B^2 \quad (m \text{ cancels}) \\ v_B &= \sqrt{2g(h_A - h_B)} \\ v_B &= \sqrt{2(9.81)(5.0 - 1.5)} \\ v_B &= 8.29 \text{ m s}^{-1} \end{aligned}$$

Speed at B is less than at C because B is higher.

Energy Transformation



Total mechanical energy remains constant: $E_A = E_C = E_B = mgh_A$.

Analysis and Physical Interpretation

- $v_C > v_B$ because C is lower than B.
- Speed depends only on the vertical height change, not on the track shape.
- Mass cancels from both speed equations.
- Equal heights correspond to equal speeds for the same conserved mechanical energy.
- The normal force changes direction but does no work because it is perpendicular to the displacement.

Numerical comparison: $9.90 \text{ m s}^{-1} > 8.29 \text{ m s}^{-1}$.

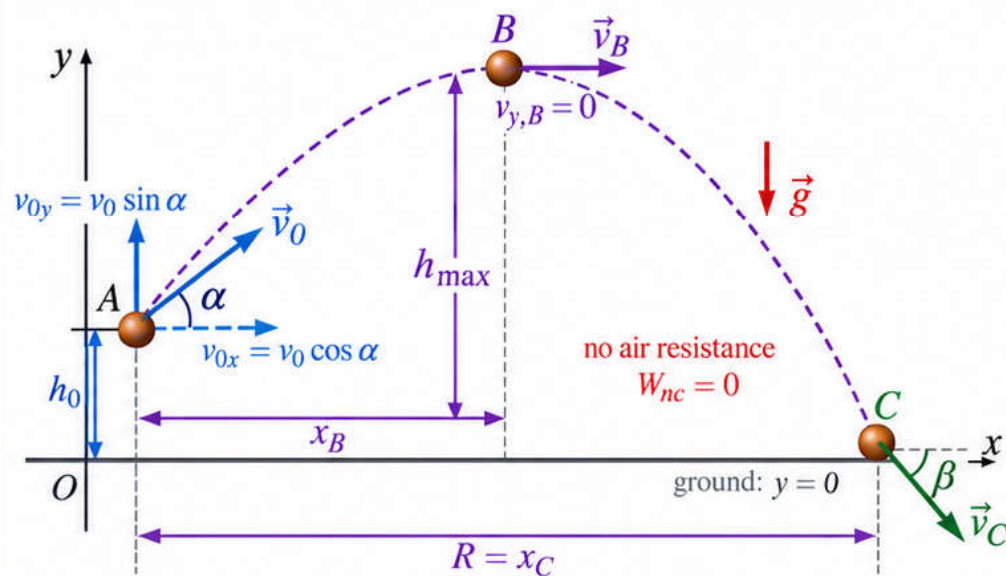
16. Projectile Motion – Problem Setup

A 0.20 kg ball is launched from height $h_0 = 1.50$ m with speed $v_0 = 14.0$ m s⁻¹ at $\alpha = 40^\circ$. Neglect air resistance. Determine the speed and position at the apex, and the speed, direction, and horizontal position at impact.

Why this example?

Energy determines speed and height. Velocity components are required to determine direction.

1. Physical Situation



2. Initial State A (Launch)

$$x_A = 0 \quad y_A = h_0$$

$$v_A = v_0$$

$$v_{0x} = v_0 \cos \alpha$$

$$v_{0y} = v_0 \sin \alpha$$

$$E_A = \frac{1}{2} m v_0^2 + m g h_0$$

3. Apex State B

$$x_B = ?$$

$$y_B = h_{\max} = ?$$

$$v_A = v_0$$

$$v_{y,B} = 0$$

$$v_B = v_{0x} = ?$$

$$E_B = \frac{1}{2} m v_B^2 + m g h_{\max}$$

4. Impact State C

$$x_C = R = ?$$

$$y_C = 0$$

$$v_{x,C} = v_{0x}$$

$$v_{y,C} < 0$$

$$v_C = ?$$

$$\beta = ?$$

$$E_C = \frac{1}{2} m v_C^2$$

System

ball + Earth

Known

m, v_0, α, h_0, g

Find

$v_B, h_{\max}, x_B, v_C, \beta, R$

Assumptions

uniform gravity, no air resistance,
point-particle model

Method

energy gives speeds and heights;
kinematics gives times, horizontal
distances, and an independent check.

17. Projectile Motion – Solution by Energy Conservation

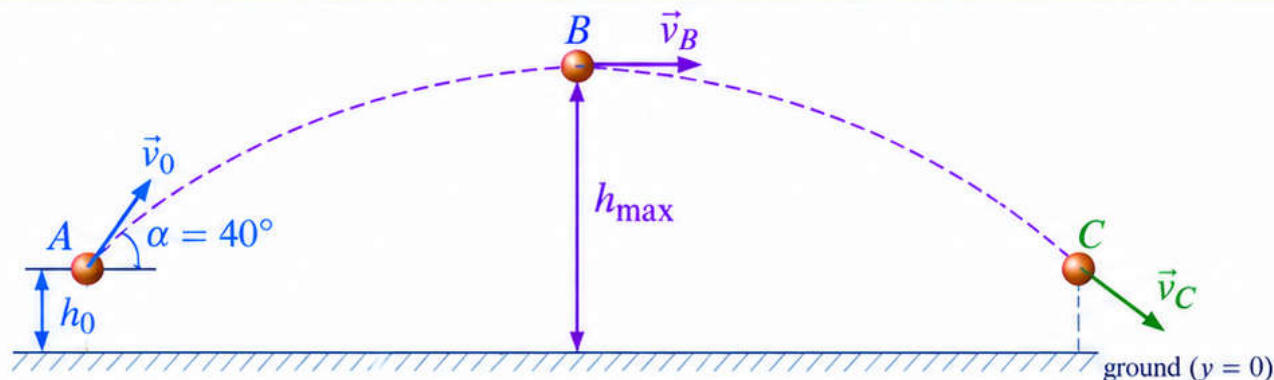
System: ball + Earth

$W_{nc} = 0$

$E_A = E_B = E_C$

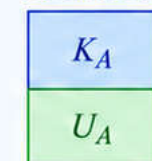
$K + U_g = \text{constant}$

Reference $U_g = 0$ at ground.



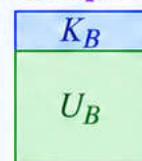
Energy at the three states (qualitative)

A: Launch



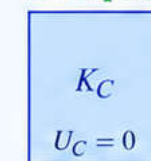
$E_A = K_A + U_A$

B: Apex



$E_B = K_B + U_B$

C: Impact



$E_C = K_C$

$E_A = E_B = E_C$ (total mechanical energy is constant)

1. State A – Launch

$$K_A = \frac{1}{2} m v_0^2$$

$$U_A = m g h_0$$

$$E_A = \frac{1}{2} m v_0^2 + m g h_0$$

$$m = 0.20 \text{ kg} \quad h_0 = 1.50 \text{ m}$$

$$v_0 = 14.0 \text{ m s}^{-1} \quad \alpha = 40^\circ$$

$$g = 9.81 \text{ m s}^{-2}$$

2. State B – Apex

At the apex $v_{y,B} = 0$.

With no horizontal external force,

$$v_B = v_0 \cos \alpha = 10.72 \text{ m s}^{-1}.$$

Energy balance: $\frac{1}{2} m v_0^2 + m g h_0 = \frac{1}{2} m v_B^2 + m g h_{\max}$

Cancel m and rearrange:
$$h_{\max} = h_0 + \frac{v_0^2 - v_B^2}{2g} = h_0 + \frac{v_0^2 \sin^2 \alpha}{2g}$$

$$h_{\max} = 1.50 + \frac{(14.0)^2 \sin^2 40^\circ}{2(9.81)} = 5.63 \text{ m}.$$

$$v_B = 10.72 \text{ m s}^{-1}$$

$$h_{\max} = 5.63 \text{ m}$$

3. State C – Impact

At impact $y_C = 0$.

Energy balance:

$$\frac{1}{2} m v_0^2 + m g h_0 = \frac{1}{2} m v_C^2$$

Cancel m :

$$v_C = \sqrt{v_0^2 + 2g h_0}$$

$$= \sqrt{[(14.0)^2 + 2(9.81)(1.50)]}$$

$$= 15.01 \text{ m s}^{-1}.$$

$$v_C = 15.01 \text{ m s}^{-1}$$

What the energy method provides:

- Speed at a specified height.
- Maximum height after the apex condition $v_y = 0$ fixes v_B .
- Results independent of mass and trajectory shape.

What it does not provide:

- Flight time.
- Horizontal position or range.
- Direction of the impact velocity.

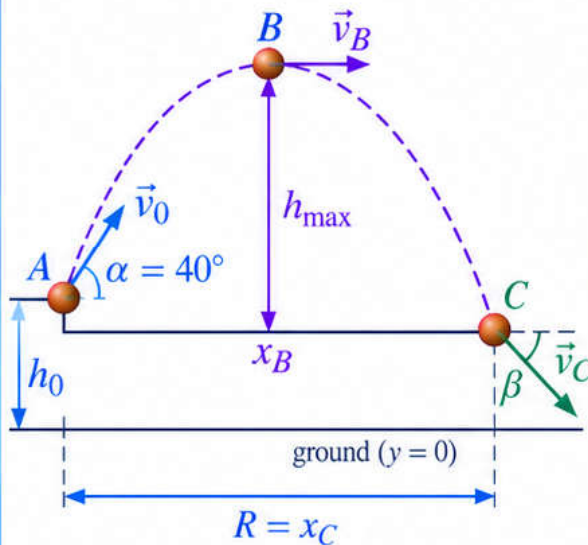
} These require component kinematics.

18. Projectile Motion – Kinematic Solution and Method Comparison

Given: $h_0 = 1.50 \text{ m}$, $v_0 = 14.0 \text{ m s}^{-1}$, $\alpha = 40^\circ$, $g = 9.81 \text{ m s}^{-2}$

Initial components: $v_{0x} = v_0 \cos \alpha = 10.72 \text{ m s}^{-1}$ $v_{0y} = v_0 \sin \alpha = 9.00 \text{ m s}^{-1}$

Projectile Diagram



1. A TO B: APEX

Apex condition $v_y(t_B) = 0$:

$$t_B = \frac{v_{0y}}{g} = \frac{9.00}{9.81} = 0.917 \text{ s.}$$

Horizontal position:

$$x_B = v_{0x} t_B = (10.72)(0.917) = 9.84 \text{ m.}$$

Maximum height:

$$\begin{aligned} h_{\max} &= h_0 + v_{0y} t_B - \frac{1}{2} g t_B^2 \\ &= 1.50 + (9.00)(0.917) - \frac{1}{2} (9.81)(0.917)^2 \\ &= 5.63 \text{ m.} \end{aligned}$$

Apex speed:

$$v_B = v_x = v_{0x} = 10.72 \text{ m s}^{-1}.$$

2. A TO C: TIME AND RANGE

At impact $y(t_C) = 0$:

$$0 = h_0 + v_{0y} t_C - \frac{1}{2} g t_C^2.$$

Positive root:

$$\begin{aligned} t_C &= \frac{v_{0y} + \sqrt{v_{0y}^2 + 2gh_0}}{g} \\ &= \frac{9.00 + \sqrt{(9.00)^2 + 2(9.81)(1.50)}}{9.81} \\ &= 1.99 \text{ s.} \end{aligned}$$

Range (horizontal distance):

$$\begin{aligned} R = x_C &= v_{0x} t_C \\ &= (10.72)(1.99) = 21.3 \text{ m.} \end{aligned}$$

3. IMPACT VELOCITY AND DIRECTION

Velocity components at impact:

$$\begin{aligned} v_{x,C} &= v_{0x} = 10.72 \text{ m s}^{-1} \\ v_{y,C} &= v_{0y} - g t_C = 9.00 - (9.81)(1.99) \\ &= -10.51 \text{ m s}^{-1}. \end{aligned}$$

Impact speed:

$$\begin{aligned} v_C &= \sqrt{v_{x,C}^2 + v_{y,C}^2} \\ &= \sqrt{(10.72)^2 + (-10.51)^2} \\ &= 15.01 \text{ m s}^{-1}. \end{aligned}$$

Direction (below horizontal):

$$\begin{aligned} \beta &= \tan^{-1} \left(\frac{|v_{y,C}|}{v_{x,C}} \right) \\ &= \tan^{-1} \left(\frac{10.51}{10.72} \right) = 44.4^\circ \\ &\text{below horizontal.} \end{aligned}$$

Method Comparison: Kinematics

- Gives time, horizontal position, range, and velocity direction.
- Requires component equations and a chosen coordinate system.
- Also gives $h_{\max}$ and speed.

Method Comparison: Energy Conservation

- Gives speed at a specified height without calculating time.
- Efficient for v_C and $h_{\max}$.
- Does not determine time, horizontal distance, or direction by itself.

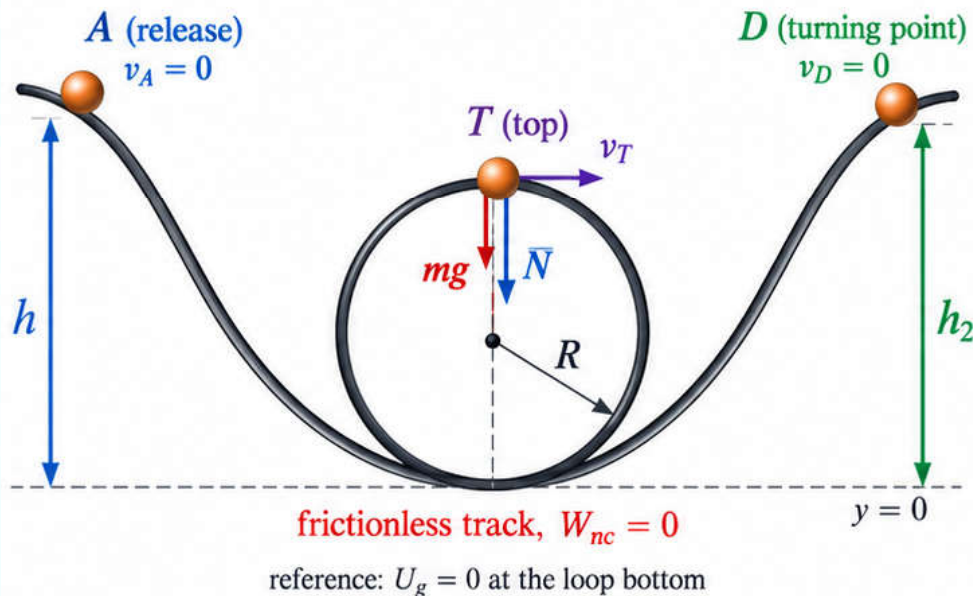
Consistency Check

$h_{\max} = 5.63 \text{ m}$ and $v_C = 15.01 \text{ m s}^{-1}$ agree with the energy method.

18. Loop-the-Loop – Problem Setup

A point mass m is released from rest at height h on a frictionless track containing a vertical loop of radius $R = 1.20$ m. Determine the minimum release height required to maintain contact throughout the loop, and the maximum height h_2 reached on the opposite incline.

Physical Situation



1. Initial State A

Released from rest at height h

$$y_A = h$$

$$v_A = 0$$

$$K_A = 0$$

$$U_A = mgh$$

$$E_A = mgh$$

2. Critical Top State T

Top of the loop (minimum contact)

$$y_T = 2R$$

$$v_T = ?$$

$$K_T = \frac{1}{2}mv_T^2$$

$$U_T = 2mgR$$

$$E_T = \frac{1}{2}mv_T^2 + 2mgR$$

Radial equation at top:

$$mg + N = \frac{mv_T^2}{R}$$

Contact condition: $N \geq 0$

Threshold for minimum release height: $N = 0$

At $N = 0$ gravity alone supplies the required centripetal acceleration. Any smaller speed causes loss of contact.

3. Opposite Turning State D

Turning point on the right incline

$$y_D = h_2$$

$$v_D = 0$$

$$K_D = 0$$

$$U_D = mgh_2$$

$$E_D = mgh_2$$

System

point mass + Earth

Known

m, R, g

Find

h_{min} and h_2

Assumptions

point mass, frictionless track, negligible air resistance, uniform g

Key condition

At T , $N \geq 0$; at the limiting case $N = 0$.

Key idea

Energy conservation determines speeds and turning heights, but the radial contact condition at T is also required.

19. Loop-the-Loop — Solution and Discussion

System: point mass + Earth
 $W_{nc} = 0$ (frictionless track)

$$E_A = E_T = E_D$$

Reference: $U_g = 0$ at loop bottom ($y = 0$)

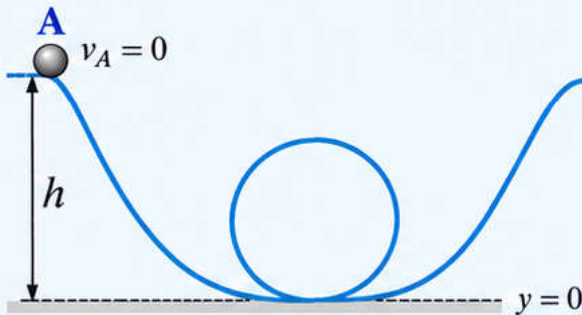
Top contact condition:
 $N \geq 0$

1. STATE A: RELEASE (initial state)

$$y_A = h$$

$$v_A = 0$$

$$E_A = mgh$$



2. STATE T: CRITICAL TOP (minimum contact)

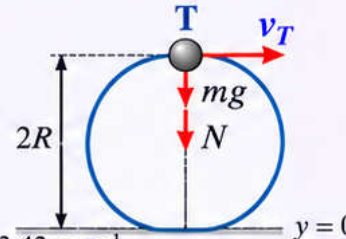
(a) Radial dynamics at the top

$$mg + N = m \frac{v_T^2}{R}$$

For minimum release height: $N = 0$

$$mg = m \frac{v_{T,min}^2}{R}$$

$$v_{T,min} = \sqrt{gR} = \sqrt{(9.81)(1.20)} = 3.43 \text{ m s}^{-1}$$



$N = 0$ is the limiting contact case. Gravity alone supplies the required centripetal acceleration. If $v_T < \sqrt{gR}$, the mass loses contact before or at the top.

(b) Energy from A to T

$$E_A = E_T \quad mgh_{min} = \frac{1}{2}mv_{T,min}^2 + mg(2R)$$

$$gh_{min} = \frac{1}{2}gR + 2gR = \frac{5}{2}gR$$

$$h_{min} = \frac{5}{2}R = \frac{5(1.20)}{2} = 3.00 \text{ m} = 2.5R$$

Merely reaching the top ($h = 2R$) gives $v_T = 0$ and cannot maintain contact.

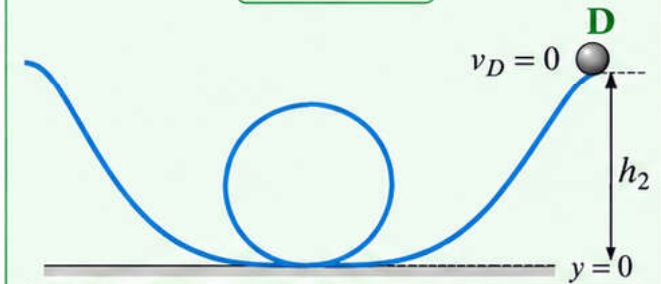
3. STATE D: OPPOSITE TURNING POINT

At D, $v_D = 0$ and $y_D = h_2$.

Energy from A to D:

$$E_A = E_D \quad mgh = mgh_2$$

$$h_2 = h$$



For the minimum-release case:

$$h_2 = h_{min} = 3.00 \text{ m.}$$

With no dissipation, the point mass rises back to its original release height after completing the loop.

Physical Interpretation

- The contact condition is most restrictive at the top because the required normal force is smallest there.
- $N = 0$ is the limiting contact boundary. Gravity alone supplies the centripetal acceleration.
- For $h > 5R/2$, $N_T > 0$ and the mass maintains contact with margin.

Energy Transformation (minimum-release case)

State	Kinetic Energy K	Gravitational Potential Energy U_g	Total Energy E
A (release)	0	$mgh_{min} = \frac{5}{2}mgR$	$E_A = \frac{5}{2}mgR$
T (top, threshold)	$\frac{1}{2}mgR$	$2mgR$	$E_T = \frac{5}{2}mgR$
D (turning point)	0	$mgh_2 = \frac{5}{2}mgR$	$E_D = \frac{5}{2}mgR$

$$E_A = E_T = E_D.$$

13. Power: The Rate of Doing Work and Transferring Energy

P

Core idea

Power measures how rapidly work is done or energy is transferred.

Sign of power

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

$P > 0$: energy enters the object

$P < 0$: energy leaves the object

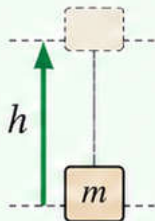
$P = 0$: this force transfers no energy at that instant

1. Average power

Average power describes energy transfer over a finite time interval.

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$1 \text{ W} = 1 \text{ J s}^{-1}$$



$$W_{\text{lifter}} = mgh$$

$$P_{\text{avg, lifter}} = \frac{mgh}{\Delta t}$$

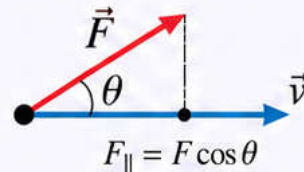
Same energy transfer in less time means greater average power.

2. Instantaneous power

Instantaneous power is the rate of work at one moment.

$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

$$P = Fv \cos \theta$$



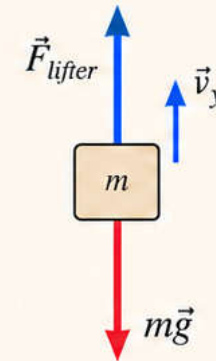
Only the component of force parallel to the velocity transfers energy.

$$\theta < 90^\circ: \\ P > 0$$

$$\theta = 90^\circ: \\ P = 0$$

$$\theta > 90^\circ: \\ P < 0$$

3. Power and gravitational energy



$$U_g = mgy$$

$$\frac{dU_g}{dt} = mgv_y$$

$$P_g = \vec{F}_g \cdot \vec{v} = -mgv_y = -\frac{dU_g}{dt}$$

For constant upward speed:

$$P_{\text{lifter}} = +mgv_y$$

Gravity does negative work during upward motion. The lifter increases the gravitational potential energy.

Interpretation

- Power is a **rate**, not an amount of **energy**.
- Average power applies to a time interval. Instantaneous power applies to one moment.
- Net power changes kinetic energy: $P_{\text{net}} = \frac{dK}{dt}$.
- A force perpendicular to the velocity has zero instantaneous power.

Quick practice

- 1) A 20 kg box is lifted 3.0 m in 4.0 s at constant speed. Find the average power.
- 2) A car exerts a 1500 N driving force at 30 m s⁻¹. Find the power at the wheels.

147 W

45 kW

15. Momentum



Key takeaway:

- Momentum describes translational motion and is especially useful for interactions and collisions.
- It depends on both mass and velocity.



Big idea: Momentum helps us understand and predict the motion of objects, especially during short interactions such as collisions.

1. Definition

The linear momentum $\vec{p}$ of a particle of mass m moving with velocity $\vec{v}$ is

$$\vec{p} = m\vec{v}$$



SI unit:

kg·m/s (equivalently N·s)

Important notes

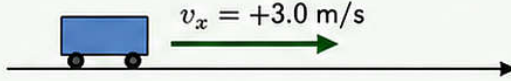
- Momentum has the same direction as velocity because mass is a positive scalar.
- If $\vec{v} = \vec{0}$, then $\vec{p} = \vec{0}$.

2. Properties of momentum

- 1 Momentum is a vector quantity with magnitude and direction.
- 2 Momenta add according to vector addition.
- 3 Momentum depends on mass and velocity.
- 4 The component of momentum along a chosen axis can be positive, negative, or zero.
- 5 Momentum depends on the reference frame.

Example (1D)

A 2.0 kg cart moves to the right at 3.0 m/s. Define right as the $+x$ direction.



$$p_x = mv_x = (2.0 \text{ kg})(3.0 \text{ m/s}) = +6.0 \text{ kg}\cdot\text{m/s}.$$

3. Intuition

A. Same speed: $m_2 > m_1$ and $v_2 = v_1$



Therefore, $|p_2| > |p_1|$.

B. Same mass: $m_2 = m_1$ and $v_2 > v_1$



Therefore, $|p_2| > |p_1|$.

At fixed speed, more mass means greater momentum magnitude; at fixed mass, greater speed means greater momentum magnitude.



Take-home message: Momentum is a fundamental property of motion:

$$\vec{p} = m\vec{v}$$

16. Conservation of Momentum



Key takeaway: If the net external impulse on a system is zero over the chosen time interval, its total momentum is conserved.

1. Statement of the law

$$\vec{J}_{ext} = \Delta \vec{P} = \vec{P}_f - \vec{P}_i$$

If $\vec{J}_{ext} = \vec{0}$, then $\vec{P}_f = \vec{P}_i$.

$$\sum_k \vec{p}_{k,i} = \sum_k \vec{p}_{k,f}$$

- $\vec{P} = \sum_k \vec{p}_k$ is the total momentum of the system.
- $\vec{p}_k = m_k \vec{v}_k$, where m_k and $\vec{v}_k$ are the mass and velocity of particle k .

2. System and condition

Strictly isolated system:

$$\sum \vec{F}_{ext} = \vec{0} \text{ at all times.}$$

For momentum conservation over t_i to t_f , the weaker sufficient condition is zero net external impulse:

$$\vec{J}_{ext} = \int_{t_i}^{t_f} \sum \vec{F}_{ext} dt = \vec{0}$$

External forces may act yet give zero net impulse over the interval.

Examples:

- Short collision with negligible external impulse.
- Explosion of an initially closed system.
- Carts on a nearly frictionless track.



Internal interaction forces exchange momentum between parts of the system and cancel in the total momentum balance under Newton's third law.

3. Mathematical form (two particles and N particles)

For two particles with masses m_1, m_2 and velocities $\vec{v}_1, \vec{v}_2$:

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_1 \vec{v}_{1,f} + m_2 \vec{v}_{2,f}$$

In components:

$$\begin{cases} m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f} \\ m_1 v_{1y,i} + m_2 v_{2y,i} = m_1 v_{1y,f} + m_2 v_{2y,f} \\ m_1 v_{1z,i} + m_2 v_{2z,i} = m_1 v_{1z,f} + m_2 v_{2z,f} \end{cases}$$

For a system of N particles:

$$\sum_{k=1}^N m_k \vec{v}_{k,i} = \sum_{k=1}^N m_k \vec{v}_{k,f}$$

Conservation holds independently in x , y , and z when the corresponding component of net external impulse is zero.



Take-home message:

With zero net external impulse, total momentum is unchanged; internal interactions only redistribute momentum among the objects.

17. Book Example: Catch or Deflect the Ball?

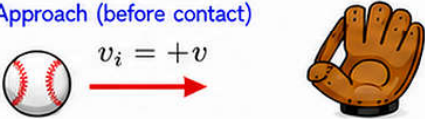


Key takeaway: Average force is set by both impulse and contact time: $|\vec{F}_{avg}| = \frac{|\Delta\vec{p}|}{\Delta t}$. Catching reduces force by increasing contact time; reversing the ball also increases its momentum change.

1. The situation

- Take rightward as positive.
 - A ball of mass m approaches a glove or bat with initial velocity $v_i = +v$ to the right.
- Two possible outcomes:
- Catch** (stop): ball stops, $v_f = 0$.
 - Deflect** (rebound): ball reverses direction and leaves to the left, $v_f = -u$, where $u > 0$.

Approach (before contact)



The ball approaches the glove or bat.

2. Momentum change of the BALL

Catch (stop)

$$p_i = +mv, \quad p_f = 0$$
$$\Delta p_{catch} = p_f - p_i = -mv$$
$$|\Delta p_{catch}| = mv$$

Deflect (rebound)

$$p_i = +mv, \quad p_f = -mu$$
$$\Delta p_{deflect} = p_f - p_i = -mu - mv = -m(u+v)$$
$$|\Delta p_{deflect}| = m(u+v)$$

If $u = v$, then $|\Delta p_{deflect}| = 2mv$

The rebound produces a larger momentum change than the catch; they are NOT the same.

3. Impulse, force, and contact time

Impulse and force on the BALL:

$$\vec{J}_{ball} = \Delta\vec{p}_{ball} \quad \vec{F}_{avg, on ball} = \frac{\Delta\vec{p}_{ball}}{\Delta t}$$

By **Newton's third law**, the impulse and average force on the hand or bat have equal magnitude and opposite direction:

$$\vec{J}_{hand/bat} = -\vec{J}_{ball}, \quad \vec{F}_{avg, on hand/bat} = -\vec{F}_{avg, on ball}$$

Magnitude relation:

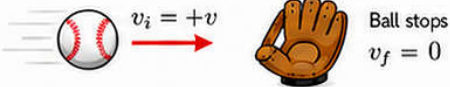
$$|\vec{F}_{avg}| = \frac{|\Delta\vec{p}|}{\Delta t}$$

- Average force depends on BOTH momentum change and contact time.
- For the same $|\Delta p|$, a longer contact time gives a smaller average force.

When comparing catch and rebound, the rebound generally has both a larger $|\Delta p|$ and often a shorter Δt , so its average force is typically much larger.

4. Catch vs. deflect: what happens to the ball?

Catch (stop the ball)



- Ball stops.
- $|\Delta p| = mv$.
- Glove moving backward with the ball can increase Δt and reduce average force.

Deflect (rebound)



- Ball reverses direction and leaves to the left.
- $|\Delta p| = m(u+v)$, or $2mv$ if $u = v$.
- Often shorter Δt and larger $|\Delta p|$, producing a larger average force.

5. Comparison summary

Aspect	Catch (stop)	Deflect (rebound)
What happens	Ball stops.	Ball reverses direction.
Momentum change $ \Delta p $	mv .	$m(u+v)$, or $2mv$ if $u = v$.
Typical contact time Δt	Can be longer (glove moves backward).	Often shorter.
Average force magnitude $ \vec{F}_{avg} $	Smaller.	Typically much larger.
Why	Smaller $ \Delta p $ and/or longer Δt .	Larger $ \Delta p $ and often shorter Δt .



Take-home message: Do not compare forces from contact time alone—compare both $|\Delta p|$ and Δt .



This is our bridge to collisions.

16. Two-Dimensional Collisions and Momentum Projections

One vector law — independent scalar projections

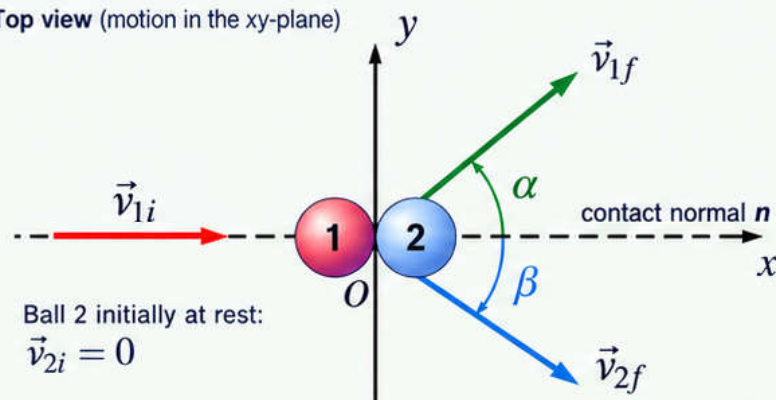
For an isolated two-body system, total linear momentum is conserved.

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Choose any orthogonal axes; apply the same sign convention before and after the collision.

1. Collision geometry — special case

Top view (motion in the xy-plane)



For smooth spheres, the contact impulse acts along the line of centres.

- 1 Choose axes and signs.
- 2 Resolve each velocity.
- 3 Conserve momentum in each direction.

2. Momentum projections

General planar equations

Conserve momentum along each orthogonal axis:

$$x: m_1 v_{1x,i} + m_2 v_{2x,i} = m_1 v_{1x,f} + m_2 v_{2x,f}$$

$$y: m_1 v_{1y,i} + m_2 v_{2y,i} = m_1 v_{1y,f} + m_2 v_{2y,f}$$

For the diagram shown ($\vec{v}_{2i} = 0$)

$$m_1 v_{1i} = m_1 v_{1f} \cos \alpha + m_2 v_{2f} \cos \beta$$

$$0 = m_1 v_{1f} \sin \alpha - m_2 v_{2f} \sin \beta$$

The initial y-momentum is zero; the final y-components are equal and opposite in total momentum.

3. What closes the problem?

A. Elastic collision

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Kinetic energy supplies one additional scalar equation.

B. Inelastic collision

Momentum is conserved, but kinetic energy is not. A restitution law or another physical constraint is required.

$$e = - \frac{v_{2f,n} - v_{1f,n}}{v_{2i,n} - v_{1i,n}}$$

where v_{1n} and v_{2n} are velocity components along the contact normal $\mathbf{n}$ (line of centres).

C. Planar motion

$$p_{z,i} = p_{z,f} = 0$$

Only the x- and y-equations are independent here.

Key message: Momentum conservation is one vector equation, equivalent to independent scalar equations along the chosen axes.

Special case

Equal masses + elastic collision + target initially at rest $\Rightarrow \vec{v}_{1f} \perp \vec{v}_{2f}$

18. Engineering Applications



Key takeaway: Momentum balance and impulse–momentum design are central to many engineering systems. Engineers define a system, account for external impulse, and control how momentum changes to achieve safer or more effective outcomes.

Momentum concepts in real engineering problems

1 Vehicle crash safety

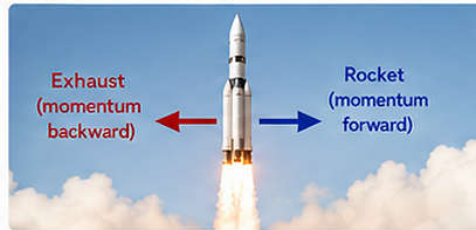


- Each body has its own Δp . Occupants must be brought from their initial velocity to rest.
- For a given occupant Δp , $|F_{avg}| = \frac{|\Delta p|}{\Delta t}$.
- Crumple zones increase the vehicle's stopping time and distance.
- Seat belts and airbags increase an occupant's stopping time, distribute forces over a larger area, and reduce peak injury loads.



Goal: Increase stopping time and manage load paths to reduce peak forces on occupants.

2 Rocket propulsion



- System: rocket + expelled propellant.
- With negligible external impulse over a short interval, their total momentum is conserved.
- Exhaust gains backward momentum; the rocket gains forward momentum.
- For steady ideal thrust (optional): $T \approx \dot{m} u_e$, where $\dot{m}$ is propellant mass-flow rate (kg/s) and u_e is exhaust speed relative to the rocket (m/s).



Thrust arises from momentum carried away by exhaust; increasing $\dot{m}$ or u_e increases T .

3 Recoil of firearms



- Bullet momentum forward is accompanied by gun momentum backward for the gun–bullet system, neglecting external impulse during firing.
- For a given recoil momentum, a larger gun mass produces a smaller recoil velocity: $|v_g| = \frac{|p_g|}{m_g}$.
- Recoil pads and mechanisms increase the stopping time and reduce the average/peak force felt by the shooter.



Design manages recoil velocity and the forces transmitted to the shooter.

4 Pile driving and impact hammers



- Before impact, the falling hammer has momentum $p = mv$ and kinetic energy $K = \frac{1}{2}mv^2$.
- During impact, the hammer's momentum change delivers impulse $J = \Delta p$ to the pile.
- Penetration also depends on transferred energy and soil resistance; momentum and impulse are not interchangeable.
- Designers control hammer mass, impact speed, contact time, and energy transfer.



Large hammer momentum; controlled impulse and energy transfer.



Engineers use momentum in two related ways:

1. **Momentum balance:** define the system and account for external impulse.
2. **Impulse management:** control Δp and Δt to shape forces.



Design insight:

Force depends not only on how much momentum changes, but also on **how quickly** and through **what load path** the change occurs.



Take-home message: Good engineering controls momentum transfer, contact time, energy dissipation, and load paths.



Next: Lecture summary

20. Summary – Key Takeaways



Big picture: Energy is a scalar accounting tool for changes and transfers; momentum is a vector quantity that describes translational motion and interactions.

1. Work and Energy

- **Work by a force:** $W = \int \vec{F} \cdot d\vec{r}$; for constant $\vec{F}$ and straight displacement, $W = Fd \cos \theta$
- **Kinetic energy:** $K = \frac{1}{2}mv^2$
- **Net-work theorem:** $W_{net} = \Delta K$
- **Mechanical energy:** $E_{mech} = K + U_g + U_s$
- **Gravitational potential (near Earth):**
 $U_g = mgy$ (relative to a chosen zero level)
- **Elastic potential:** $U_s = \frac{1}{2}kx^2$
- **Mechanical-energy relation for the chosen system:**
 $W_{nc,ext} = \Delta E_{mech}$
 - If $W_{nc,ext} = 0$, then $E_{mech,i} = E_{mech,f}$
- Total energy remains conserved even if mechanical energy is converted to internal (thermal) energy.

Key energy terms



Kinetic
 $K = \frac{1}{2}mv^2$



Elastic potential
 $U_s = \frac{1}{2}kx^2$



Gravitational potential
 $U_g = mgy$

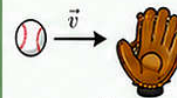
2. Momentum and Impulse

- **Momentum:** $\vec{p} = m\vec{v}$
- **Impulse:** $\vec{J} = \int_{t_i}^{t_f} \vec{F} dt$
- **Net impulse:** $\vec{J}_{net} = \Delta \vec{p}$
- **For a system:** $\vec{J}_{ext} = \Delta \vec{P}_{system}$
 - If $\vec{J}_{ext} = \vec{0}$, then $\vec{P}_i = \vec{P}_f$
- **Average-force magnitude:** $|\vec{F}_{avg}| = \frac{|\Delta \vec{p}|}{\Delta t}$
- Catching can reduce force by increasing Δt ; rebound generally has larger $|\Delta \vec{p}|$ and often shorter Δt .

Catch vs. Deflect/Rebound

Incoming ball speed = v

Catch (ball stops)



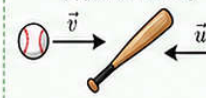
$$\vec{v}_f = \vec{0}$$

$$\Delta \vec{p}_{catch} = m\vec{0} - m\vec{v}$$

$$= -m\vec{v}$$

$$|\Delta \vec{p}_{catch}| = mv$$

Deflect / Rebound (opposite direction)



$$\vec{v}_f = -\vec{u}$$

$$\Delta \vec{p}_{rebound} = m(-\vec{u}) - m\vec{v}$$

$$= -m(\vec{v} + \vec{u})$$

$$|\Delta \vec{p}_{rebound}| = m(v + u)$$

If $u = v$, then $|\Delta \vec{p}| = 2mv$

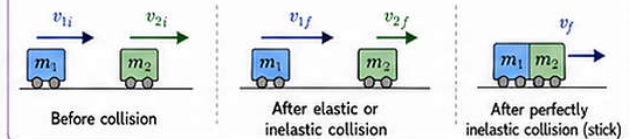
They do NOT have the same momentum change.

3. Collisions

- Types: elastic, inelastic, perfectly inelastic.
- **1D elastic collision formulas (masses m_1, m_2):**

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$
- Momentum is conserved in any collision only when net external impulse on the system is negligible.
- Kinetic energy is conserved only in an elastic collision.
- In an inelastic collision, kinetic energy decreases and is converted into deformation, internal energy, heat, or sound; it is not lost.
- **Perfectly inelastic:** objects stick and share a common final velocity.



Core connections

- Work transfers energy.
- Impulse changes momentum.
- Zero external nonconservative work $\Rightarrow$ mechanical energy conserved.
- Zero net external impulse $\Rightarrow$ system momentum conserved.
- Elastic collision under negligible external impulse $\Rightarrow$ both momentum and kinetic energy conserved.



Design insight

Engineers manage energy transfer, momentum change, contact time, dissipation, and system boundaries.



Why it matters

From car safety to rockets, sports, and impacts—understanding energy and momentum helps us analyze, predict, and design better systems.



Take-home message:

Energy and momentum provide complementary ways to analyze motion and interactions. Choose the system carefully, apply the correct conservation condition, and keep vector directions and energy transformations explicit.



Next: Q&A / Discussion