

## PHYSICS FOR ENGINEERING STUDENTS

### Laws of motion

---

*Force, tension and acceleration*

### Lecture 2 — Book-style student notes

#### Learning focus

Dynamics

Three laws of motion

Conservation laws

Frictions

*University of Szeged • Institute of Physics*

## Newton's Laws of Motion

---

Kinematics describes how an object moves; dynamics explains why that motion changes. This chapter develops a systematic method for translating a physical situation into a mathematical model. The central tools are Newton's laws, vector components, free-body diagrams and consistent sign conventions.

### Learning objectives

- Distinguish velocity from acceleration and connect dynamics to constant-acceleration kinematics.
- Identify external forces and represent them in a correct free-body diagram.
- Apply Newton's three laws to horizontal, inclined, connected and circular-motion systems.
- Model static and kinetic friction, tension and normal force.
- Check an answer using direction, units, limiting cases and physical plausibility.

### From kinematics to dynamics

Position  $x(t)$  specifies an object's location. Velocity is the time rate of change of position, and acceleration is the time rate of change of velocity. Because velocity is a vector, acceleration can arise from a change in speed, a change in direction, or both.

$$\mathbf{v} = d\mathbf{x}/dt$$

$$\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{x}/dt^2$$

For motion with constant acceleration, the familiar kinematic equations remain essential:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{v}^2 = \mathbf{v}_0^2 + 2(\mathbf{x} - \mathbf{x}_0)$$

These equations do not determine the acceleration by themselves. Dynamics supplies the missing connection: forces determine acceleration. Once acceleration has been found from Newton's second law, kinematics predicts velocity and position.

**Core question:** Kinematics asks “What does the motion do?” Dynamics asks “What interaction causes the motion to change?”

## Velocity is not acceleration

An object may have a large velocity and zero acceleration when it moves in a straight line at constant speed. It may also have zero instantaneous velocity but non-zero acceleration—for example, at the highest point of a vertical throw. During braking, velocity and acceleration point in opposite directions. In uniform circular motion, speed is constant but velocity changes direction continuously, so the acceleration is not zero.

### 1. Force as a vector quantity

A force is a push or pull arising from an interaction. It may change the translational motion of an object, rotate it, or deform it. Force is a vector: both magnitude and direction are required. In one-dimensional problems, direction can be represented by a sign; in two or three dimensions, components must be used.

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

The force unit follows directly from Newton's second law. A net force of one newton gives a one-kilogram body an acceleration of one metre per second squared.

### Net force and components

Several forces can act simultaneously. Their vector sum is the net external force. Internal forces between parts of the chosen system cancel in pairs when the complete system is analysed, but external forces do not.

$$\mathbf{F}_{\text{net}} = \Sigma \mathbf{F}_{\text{external}}$$

$$\Sigma \mathbf{F}_x = m\mathbf{a}_x$$

$$\Sigma \mathbf{F}_y = m\mathbf{a}_y$$

The x- and y-equations are independent component equations. A body may be in equilibrium vertically while accelerating horizontally. This is why the normal force on a horizontally pushed block can balance its weight even though the block accelerates along the surface.

## Common external forces

| Force         | Typical model                 | Direction                                                     |
|---------------|-------------------------------|---------------------------------------------------------------|
| Weight        | $W = mg$                      | Vertically downward near Earth                                |
| Normal force  | $N$ (constraint force)        | Perpendicular to the surface                                  |
| Friction      | $f_s$ or $f_k$                | Parallel to the surface; opposes relative or impending motion |
| Tension       | $T$                           | Along a taut string, pulling away from the body               |
| Applied force | $F_{app}$                     | As specified by the interaction                               |
| Drag          | Often depends on $v$ or $v^2$ | Opposite motion relative to the fluid                         |

## Balanced and unbalanced forces

Balanced forces mean zero net force, not necessarily zero velocity. A book resting on a table and a car travelling in a straight line at constant speed are both examples of zero acceleration. Unbalanced forces produce acceleration in the direction of the vector net force.

$$\Sigma \mathbf{F} = \mathbf{0} \Rightarrow \mathbf{a} = \mathbf{0} \Rightarrow \mathbf{v} = \text{constant}$$

**Important:** “No net force” means no change in velocity. It does not mean that the object must be at rest.

## 2. Newton's three laws

### First law: the law of inertia

In an inertial reference frame, an object remains at rest or continues with constant velocity unless a non-zero net external force acts on it. Inertia is not a force; it is the tendency of matter to resist changes in velocity. Mass quantifies translational inertia: a larger mass requires a larger net force to produce the same acceleration.

The first law also defines the class of reference frames in which the standard form of Newtonian mechanics is valid. It corrects the common misconception that a continuing motion needs a continuing forward force. In an ideal frictionless environment, no force is required to maintain constant velocity; a force is required to change it.

## Second law: the quantitative law of motion

$$\Sigma \mathbf{F} = m\mathbf{a} \quad \text{valid for constant mass}$$

Acceleration is directly proportional to the net force and inversely proportional to mass. Doubling the net force doubles acceleration if mass is unchanged. Doubling the mass halves acceleration if the same net force is applied. The acceleration vector points in the direction of the net force, not necessarily in the direction of velocity.

$$\mathbf{a} = \Sigma \mathbf{F}/m$$

The equation must be applied to a clearly specified system. Writing “ $\mathbf{F} = m\mathbf{a}$ ” before identifying the body often produces mistakes, because forces acting on different bodies become mixed.

## Third law: interaction pairs

If body A exerts a force on body B, then body B simultaneously exerts an equal-magnitude, opposite-direction force on body A:

$$\mathbf{F}_{A \rightarrow B} = -\mathbf{F}_{B \rightarrow A}$$

The two forces act on different objects, so they never cancel within the free-body diagram of one object. A person walking pushes the ground backward, while the ground pushes the person forward. A rocket pushes exhaust gases backward, and the gases push the rocket forward. The forces are equal, but the accelerations need not be equal because the masses can differ.

## How the laws work together

Consider a box initially at rest. Before the push, its forces are balanced and the first law predicts no change in motion. During the push, a non-zero net force produces acceleration according to the second law. At the same time, the box pushes back on the person according to the third law. If the push stops and friction is absent, the first law predicts that the box continues at constant velocity.

**Diagnostic test for a third-law pair:** The forces have the same interaction type, occur simultaneously, have equal magnitude and opposite direction, and act on two different bodies.

## 3. Reference frames and free-body diagrams

### Inertial and non-inertial frames

Motion is measured relative to a reference frame. Frames moving at constant velocity relative to one another are inertial: Newton's laws have the same form in all of them. A train moving in a straight line at constant speed is approximately inertial. A train that accelerates, brakes or turns is non-inertial.

In an accelerating frame, an observer may introduce an apparent or “fictitious” force to retain an equation resembling Newton's second law. For example, a passenger feels pushed backward when a bus accelerates forward. No physical agent pushes the passenger backward; the sensation reflects the passenger's inertia viewed from the accelerating bus.

**Modelling convention:** For everyday engineering problems, the Earth is usually treated as an inertial frame unless rotational effects or very high precision matter.

### What is a free-body diagram?

A free-body diagram (FBD) is a simplified drawing of one isolated object or system showing every external force acting on it. It is not a picture of the entire scene. The table, rope or neighbouring body is removed and replaced by the force it exerts on the chosen object.

### A reliable six-step procedure

- Identify the object or system to analyse.
- Isolate it and represent it by a simple box or point.
- List all external interactions: gravity, contact, rope, spring, fluid and applied forces.
- Draw one arrow for each external force, in the correct direction.
- Label every arrow clearly and choose coordinate axes that simplify the equations.
- Check that no action–reaction partner acting on another body has been included.

Arrow length may be used qualitatively to suggest magnitude, but the direction and label are essential. Redraw the FBD whenever the physical situation changes—for example, when a body begins to slide and static friction changes to kinetic friction.

### Example: pushed block on a horizontal floor

The block experiences weight  $mg$  downward, normal force  $N$  upward, an applied force  $F_{\text{app}}$  to the right and kinetic friction  $f_k$  to the left. With no vertical acceleration:

$$\Sigma \mathbf{F}_y = N - mg = \mathbf{0} \Rightarrow N = mg$$

$$\Sigma \mathbf{F}_x = \mathbf{F}_{\text{app}} - \mathbf{f}_k = m\mathbf{a}$$

This pair of equations contains all information required for the translational motion. The vertical balance does not imply that all forces are balanced; the horizontal resultant may still be non-zero.

#### 4. A systematic problem-solving method

Dynamics becomes much more reliable when every problem is approached in the same order. The objective is to translate words and geometry into force equations without skipping the physical model.

##### Step 1: identify the system and assumptions

State which body or combination of bodies is analysed. Record relevant assumptions: rigid surface, negligible air resistance, massless inextensible string, frictionless pulley, constant gravitational acceleration, or particle approximation. Assumptions determine which force models are valid.

##### Step 2: draw the FBD

Draw only forces acting on the chosen system. Do not draw velocity or acceleration as forces. If several bodies move differently, draw a separate FBD for each unless they are deliberately treated as one combined system.

##### Step 3: choose axes and signs

Choose axes along expected motion or along a surface. On an incline, axes parallel and perpendicular to the plane avoid unnecessary trigonometry. Declare the positive directions before writing equations and keep them unchanged.

##### Step 4: apply Newton's laws component by component

$$\Sigma \mathbf{F}_x = m\mathbf{a}_x \qquad \Sigma \mathbf{F}_y = m\mathbf{a}_y$$

If a component of acceleration is zero, its force sum is zero. Use third-law pairs to relate interactions between bodies, but never place both members of the pair on one FBD.

##### Step 5: solve and verify

Solve the simultaneous equations symbolically when possible, then substitute numerical values with units. A negative answer is not automatically wrong: it means the actual vector direction is opposite the chosen positive direction.

##### Worked example: crate on a rough floor

A 20 kg crate is pushed to the right by 50 N. Kinetic friction is 12 N to the left. Taking right as positive:

$$F_{\text{net},x} = 50 \text{ N} - 12 \text{ N} = 38 \text{ N}$$

$$a = F_{\text{net},x}/m = 38 \text{ N}/(20 \text{ kg}) = 1.9 \text{ m/s}^2$$

The positive result confirms that the crate accelerates to the right. Vertically,  $N = mg$  because there is no vertical acceleration.

## Five checks

- Units: force divided by mass must give  $\text{m/s}^2$ .
- Direction: does the sign agree with the net-force direction?
- Magnitude: is the acceleration plausible for the forces and mass?
- Limiting case: does the formula behave correctly if friction becomes zero or mass becomes very large?
- Consistency: does the result satisfy every original equation?

## 5. Friction and horizontal motion

Friction is a contact force parallel to the interface. It opposes relative sliding or the tendency to slide. Its direction must be inferred from the possible relative motion, not simply from the direction in which the object happens to face.

### Static friction

Static friction acts while the surfaces do not slide relative to one another. It adjusts from zero up to a maximum value. Therefore, static friction is generally an inequality, not automatically  $\mu_s N$ :

$$0 \leq \mathbf{f}_s \leq \mathbf{f}_{s,\max} = \mu_s \mathbf{N}$$

Only at impending motion does  $\mathbf{f}_s$  reach  $\mu_s \mathbf{N}$ . If a 10 N horizontal push is applied and the body remains at rest, static friction is 10 N in the opposite direction, provided 10 N is below its maximum value.

### Kinetic friction

Once sliding begins, the kinetic-friction magnitude is approximately proportional to the normal force:

$$\mathbf{f}_k = \mu_k \mathbf{N}$$

Typically  $\mu_s > \mu_k$ , which explains why starting an object moving often requires more force than keeping it sliding. Both coefficients are dimensionless and depend on the pair of surfaces and their condition.

### General horizontal-block result

For a block of mass  $m$  on a horizontal surface, pushed horizontally by  $F_{\text{app}}$  and moving to the right,  $N = mg$  and  $\mathbf{f}_k = \mu_k mg$ . Newton's second law gives:

$$\mathbf{F}_{\text{app}} - \mu_k mg = m\mathbf{a}$$

$$\mathbf{a} = \mathbf{F}_{\text{app}}/m - \mu_k \mathbf{g}$$

The expression shows two competing effects: the applied force per unit mass accelerates the block, while friction supplies the opposing acceleration  $\mu_k g$ . If  $F_{\text{app}} = \mu_k mg$ , the acceleration is zero and a moving block maintains constant speed. If the calculated acceleration is negative, the velocity decreases while the block still moves to the right.

**A subtle but important correction**

The simplified statement “friction never helps motion” can be misleading. Friction always opposes relative or impending sliding at the contact, but it can propel an object. Static friction from the road accelerates a driven car forward, and static friction from the ground pushes a walking person forward. The correct rule concerns relative motion at the interface.

**Common error:** Do not write  $f_s = \mu_s N$  unless the surface is exactly at the threshold of slipping. Otherwise determine the required static friction from equilibrium or dynamics and verify that it does not exceed  $\mu_s N$ .

**6. Inclined planes**

On an incline of angle  $\theta$ , choose  $x$  parallel to the slope and  $y$  perpendicular to it. Resolve the weight into these axes rather than resolving the normal force. For a block on the plane:

$$W_{\parallel} = mg \sin\theta \quad W_{\perp} = mg \cos\theta$$

The parallel component tends to move the block down the slope. The perpendicular component presses the block into the surface. If there is no acceleration perpendicular to the plane:

$$N - mg \cos\theta = 0 \Rightarrow N = mg \cos\theta$$

**Block sliding down a rough incline**

Suppose the block slides down and kinetic friction acts up the slope. Taking down the incline as positive:

$$f_k = \mu_k N = \mu_k mg \cos\theta$$

$$mg \sin\theta - \mu_k mg \cos\theta = ma$$

$$a = g(\sin\theta - \mu_k \cos\theta)$$

The mass cancels: within this ideal model, all masses have the same acceleration on a given incline with the same coefficient of kinetic friction. The result is positive if gravity's downslope component exceeds friction. It is zero when  $\tan\theta = \mu_k$ , corresponding to constant velocity while sliding.

**Frictionless limit**

$$\mu_k = 0 \Rightarrow a = g \sin\theta$$

This provides a useful limiting-case check. At  $\theta = 0^\circ$ , acceleration is zero; at  $\theta = 90^\circ$ , the model approaches free fall with  $a = g$ .

### Static block on an incline

If the block is at rest, static friction acts up the slope with the magnitude required for equilibrium:

$$f_s = mg \sin\theta$$

Rest is possible only if this required value does not exceed  $\mu_s N$ :

$$mg \sin\theta \leq \mu_s mg \cos\theta \Rightarrow \tan\theta \leq \mu_s$$

The critical angle  $\theta_c$  at which slipping begins therefore satisfies  $\tan\theta_c = \mu_s$ . This method distinguishes the equilibrium calculation from the limiting friction condition.

### Interpretation of a negative result

If the coordinate direction is down the incline and the equation yields a  $< 0$ , the acceleration is up the incline. This can occur while the block is still moving down, in which case it slows. It does not automatically mean the block is moving up.

## 7. Tension and pulley systems

Tension is the pulling force transmitted by a taut rope, string or cable. It acts along the string and pulls away from the object. An ideal string is massless and inextensible; an ideal pulley is massless and frictionless. Under these assumptions, the tension has the same magnitude throughout one continuous string, and connected bodies have related accelerations.

### Single hanging mass

For a mass  $m$  at rest, the upward tension balances weight:

$$T - mg = 0 \Rightarrow T = mg$$

If downward is positive and the mass accelerates downward with magnitude  $a$ :

$$mg - T = ma \Rightarrow T = m(g - a)$$

Thus, tension is less than weight during downward acceleration and greater than weight during upward acceleration.

### Atwood machine

Two masses  $m_1$  and  $m_2$  are connected over an ideal fixed pulley. Let  $m_2 > m_1$ , so  $m_2$  moves down and  $m_1$  moves up with the same acceleration magnitude  $a$ . Separate FBDs give:

$$T - m_1g = m_1a$$

$$m_2g - T = m_2a$$

Adding eliminates the internal tension and gives the system acceleration:

$$a = (m_2 - m_1)g / (m_1 + m_2)$$

Substitution into either body equation gives:

$$T = m_1(g + a) = m_2(g - a) = 2m_1m_2g/(m_1 + m_2)$$

### Block on a frictionless table connected to a hanging mass

Let  $m_1$  lie on a frictionless table and  $m_2$  hang vertically. The string constraint gives equal acceleration magnitudes. For  $m_1$ ,  $T = m_1a$ . For  $m_2$ ,  $m_2g - T = m_2a$ . Therefore:

$$a = m_2g/(m_1 + m_2) \quad T = m_1m_2g/(m_1 + m_2)$$

With  $m_1 = 4.0 \text{ kg}$ ,  $m_2 = 6.0 \text{ kg}$  and  $g = 9.8 \text{ m/s}^2$ :

$$a = 5.88 \text{ m/s}^2 \quad T = 23.5 \text{ N}$$

**Modelling warning:** Equal tension is not universal. It can fail if the rope has significant mass, the pulley has rotational inertia, or axle friction is present.

## 8. Circular motion

In circular motion, the instantaneous velocity is tangent to the path. Even at constant speed, its direction changes, so the object has an inward acceleration directed toward the centre. This is centripetal acceleration.

$$a = v^2/r = \omega^2r = 4\pi^2r/T^2$$

The required net inward force is not a new type of force. “Centripetal force” is the name for the radial resultant of real forces such as tension, friction, gravity or the normal force:

$$\Sigma F_{\text{inward}} = ma = mv^2/r$$

### Object on a string

For a mass moving in a horizontal circle, tension may provide the inward resultant. If tension is the only horizontal radial force:

$$T = mv^2/r$$

If the string breaks, the inward force disappears and the mass continues approximately along the tangent at the release point, not radially outward. The apparent outward tendency belongs to a rotating reference frame, not to the inertial-frame FBD.

### Car on a flat curve

On a level road, static friction between tyres and road supplies the centripetal force. Vertically  $N = mg$ . At the threshold of slipping:

$$mv_{\text{max}}^2/r = f_{s,\text{max}} = \mu_s mg$$

$$v_{\max} = \sqrt{(\mu_s g r)}$$

Mass cancels in this simplified model. If speed doubles at the same radius, the required inward force becomes four times larger. If radius halves at the same speed, the required force doubles.

### Direction matters

Centripetal acceleration is radial and inward; velocity is tangential. These vectors are perpendicular in uniform circular motion. Because the acceleration has no tangential component, it changes direction but not speed. If the speed also changes, a tangential acceleration component must be added, and the total acceleration is the vector sum of radial and tangential components.

$$|\mathbf{a}| = \sqrt{(\mathbf{a}_c^2 + \mathbf{a}_t^2)}$$

**Common mistake:** Do not add a separate “centripetal force” arrow if tension, friction, gravity or normal force already provides the inward resultant. Write the radial sum of the real forces.

## 9. Banked curves, common mistakes and summary

### Ideal banked curve without friction

A banked road is tilted inward by angle  $\theta$ . With no friction, the normal force has a vertical component that balances weight and a horizontal component that supplies centripetal acceleration:

$$N \cos\theta = mg$$

$$N \sin\theta = m\mathbf{v}^2/r$$

Dividing the equations eliminates  $N$  and  $m$ :

$$\tan\theta = v^2/(rg) \quad v = \sqrt{(rg \tan\theta)}$$

This is the design or ideal speed. At that speed, no lateral friction is required. In real conditions, static friction creates a range of safe speeds. Its direction depends on the tendency to slip: at excessive speed the car tends to move up the bank and friction acts down the slope; at too low a speed it tends to slide down and friction acts up the slope.

### Frequent errors and their corrections

| Mistake                                       | Correction                                                             |
|-----------------------------------------------|------------------------------------------------------------------------|
| Skipping the FBD                              | Isolate the system and draw every external force first.                |
| Mixing third-law pairs                        | Remember that the two forces act on different bodies.                  |
| Using weight instead of mass in $ma$          | Use mass $m$ ; weight $mg$ belongs in the force sum.                   |
| Assuming $f_s = \mu_s N$ in every static case | Use $f_s \leq \mu_s N$ and calculate the required value.               |
| Treating centripetal force as an extra force  | Sum the real inward forces and set the result equal to $mv^2/r$ .      |
| Changing signs midway                         | Declare positive axes once and interpret a negative result physically. |

### Compact solution checklist

- Identify the system and assumptions.
- Draw an FBD containing external forces only.
- Choose convenient axes and positive directions.
- Resolve vectors into components.
- Apply  $\Sigma F = ma$  separately along each axis.
- Solve symbolically, substitute units, and interpret signs.
- Check limiting cases and physical plausibility.

### Essential equations

$$\Sigma F = ma \quad W = mg \quad f_k = \mu_k N \quad f_s \leq \mu_s N$$

$$a_c = v^2/r \quad \Sigma F_{\text{inward}} = mv^2/r \quad \tan\theta = v^2/(rg)$$

**Final takeaway:** The equations are short; the modelling is the real work. A correct system choice, clear FBD and consistent axes turn a complicated engineering situation into a small set of solvable equations.