

LECTURE 2

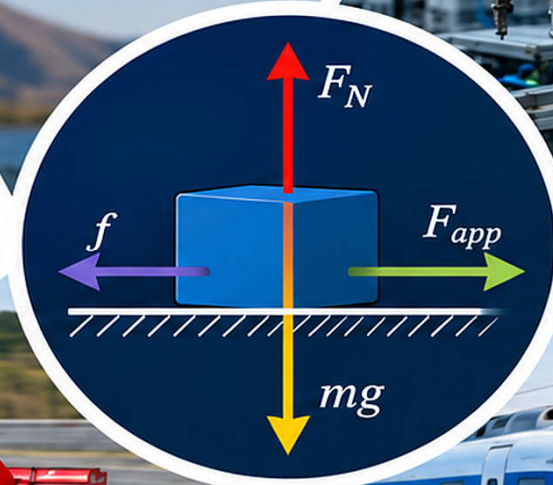
Newton's Laws of Motion

Understanding how forces govern the motion of objects in the real world.



Today you will learn:

- Newton's Three Laws
- Types of Forces
- Free Body Diagrams
- Applying Forces to Solve Problems
- Engineering Applications



PHYSICS FOR
ENGINEERS



Understand Forces.

Analyze Motion.

Solve Real-World Problems.



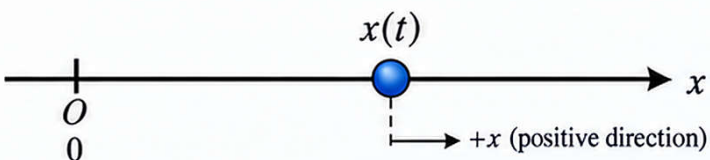
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2 Kinematics in One Picture

Kinematics describes how motion changes in time, without identifying the forces that cause it.

1. State of motion

A particle moves along the x -axis. Its state at time t is described by its position, velocity, and acceleration.



Position	Velocity	Acceleration
$x(t)$	$v(t) = \frac{dx}{dt}$	$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
time derivative	time derivative	

$x(t)$ is the signed position (m).

$v(t)$ is the signed velocity (m/s).

$a(t)$ is the signed acceleration (m/s²).

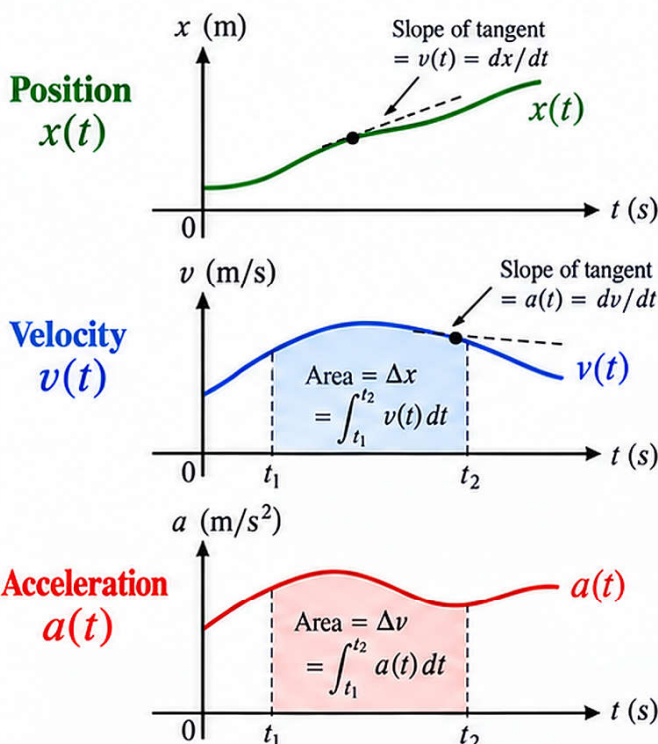
In one dimension, x , v , and a are signed quantities (positive or negative) that indicate direction.

Reverse relations (integration):

$$\Delta x = \int_{t_1}^{t_2} v(t) dt, \quad \Delta v = \int_{t_1}^{t_2} a(t) dt$$

2. Graphs carry physical meaning

The position, velocity, and acceleration graphs reveal key features of the motion.

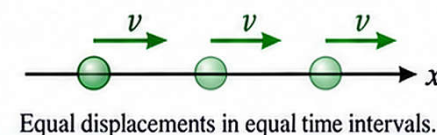


3. Common motion models

These models are idealized but widely used and illustrate how kinematics describes different types of motion.

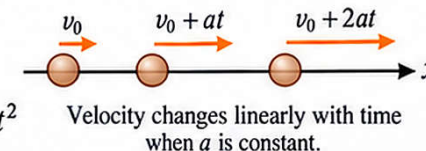
Uniform motion (constant velocity)

$a = 0$
 $v = \text{constant}$
 $x = x_0 + vt$



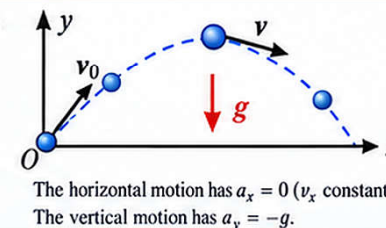
Constant acceleration

$a = \text{constant}$
 $v(t) = v_0 + at$
 $x(t) = x_0 + v_0t + \frac{1}{2}at^2$



Projectile motion (no air resistance)

Horizontal and vertical motions are independent:
 $a_x = 0, \quad a_y = -g$
 $x(t) = x_0 + v_{0x}t$
 $y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$



KEY IDEA:

Kinematics connects position, velocity, and acceleration through derivatives, integrals, and graphs.
Dynamics answers the next question: **why does the motion change?**

3

The Four Constant-Acceleration Equations

For one-dimensional motion with constant acceleration and a consistent sign convention.

Equation set

1

$$v = v_0 + at$$

does not contain
 x

2

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

does not contain
 v

3

$$v^2 = v_0^2 + 2a(x - x_0)$$

does not contain
 t

4

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

does not contain
 a

Choose the equation by the missing variable

Known / required quantities

Best equation

Need v ; know v_0, a, t

1

$$v = v_0 + at$$

Need x ; know x_0, v_0, a, t

2

$$x = x_0 + v_0 t + \frac{1}{2} at^2$$

Time is absent; know v_0, v, a, x_0

3

$$v^2 = v_0^2 + 2a(x - x_0)$$

Acceleration is absent;
know x_0, v_0, v, t

4

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

Example: choosing an equation

Given v_0, a , and Δx ; find v .

Time is not given, so use **Equation 3**.

Validity conditions

1

Acceleration is constant over the interval.

2

Motion is treated along one coordinate axis.

3

All quantities use the same positive direction and sign convention.

4

For projectile motion, apply the equations separately to x and y :
 $a_x = 0, a_y = -g$.



If acceleration varies with time, use calculus or a numerical method.

4 Velocity vs. Acceleration

1 High velocity, Zero acceleration

$$a = 0$$

Constant speed



Speed is constant.
No change in velocity.

2 Zero velocity, Large acceleration

$$v = 0, a \neq 0$$

Starting to move

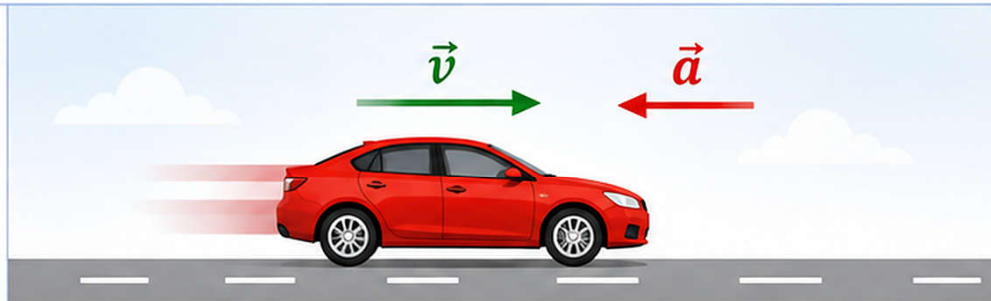


Velocity changes from
zero. Object speeds up.

3 High velocity, Negative acceleration

$$a < 0$$

Braking / slowing down



Velocity decreases.
Object slows down.

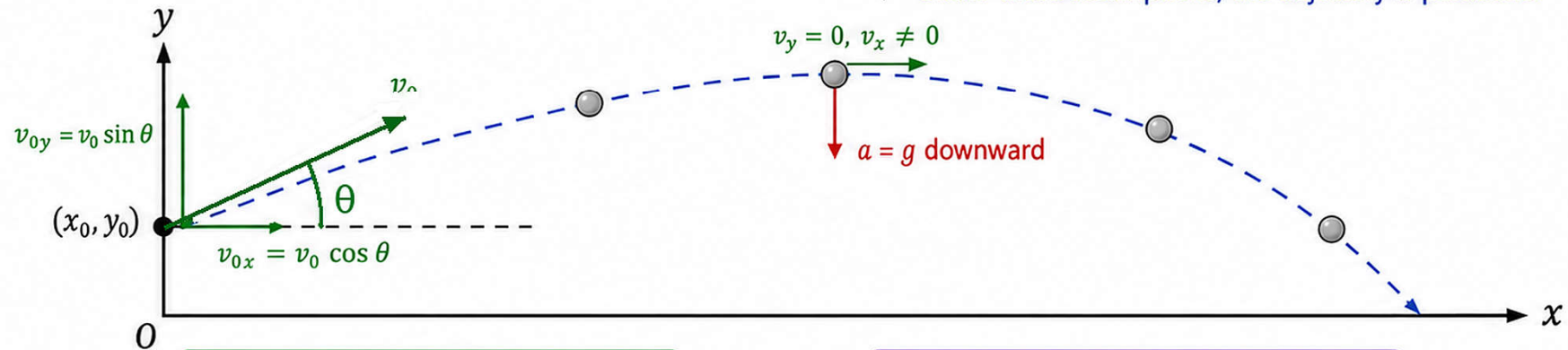


Velocity describes motion. Acceleration describes change in motion. They are related, but different!

5 Projectile Motion

Projectile motion = combination of horizontal and vertical motion.

Assume uniform gravity and negligible air resistance.
Under these assumptions, the trajectory is parabolic.



Horizontal Motion

$$\begin{aligned} a_x &= 0 \\ v_x &= v_0 \cos \theta = \text{constant} \\ x &= x_0 + (v_0 \cos \theta)t \end{aligned}$$

+

Vertical Motion

$$\begin{aligned} a_y &= -g \\ v_y &= v_0 \sin \theta - gt \\ y &= y_0 + (v_0 \sin \theta)t - \frac{1}{2}gt^2 \end{aligned}$$



Parabolic Trajectory

$$y - y_0 = (x - x_0)\tan \theta - \frac{g(x - x_0)^2}{2v_0^2 \cos^2 \theta}$$



With +y upward: $a_y = -g \approx -9.81 \text{ m/s}^2$.

Transition to Today's Lecture

Last Lecture: Kinematics

We described how motion changes.



Position

$$x(t)$$

Where is the object?



Velocity

$$\vec{v}(t) = d\vec{x}/dt$$

How fast and in which direction?



Acceleration

$$\vec{a}(t) = d\vec{v}/dt$$

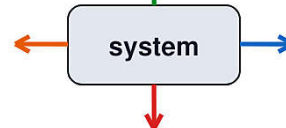
How does velocity change?

Kinematics determines $\vec{a}(t)$, but does not identify the physical interactions that produce it.



What causes acceleration?

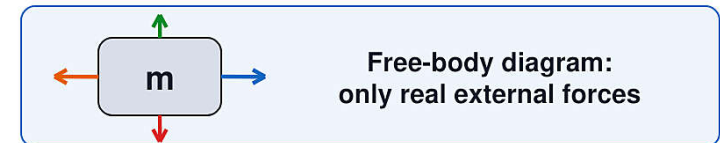
A physical model is required



Today's Lecture: Dynamics

We explain why motion changes.

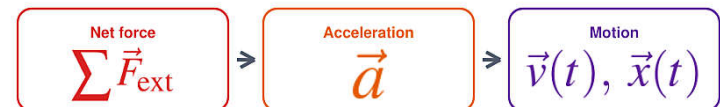
1. Identify the system and all external forces



2. Add the forces vectorially

$$\sum \vec{F}_{\text{ext}} = m\vec{a}$$

3. Use acceleration to predict the motion



Newton's Second Law provides the link

$$\sum \vec{F}_{\text{ext}} = m\vec{a}$$

Valid for a constant-mass system observed from an inertial reference frame.

KINEMATICS describes the motion. **DYNAMICS** explains it from the net external force.

Why Forces Matter?

Everything that moves, stops, speeds up, or changes direction does so because of **forces**.



Big Idea

Forces are the causes of motion change.

Forces are everywhere in engineering and in daily life.



Start and Accelerate

Engines push the car forward. Without force, it won't speed up.



Stop

Brakes apply a force opposite to the motion.



Stay in the Air

Rotors push air downward, air pushes drone upward.



Change Direction

Robotic arms apply forces to move and position parts.



Follow a Curve

Forces provide the "push" needed to turn.



Stay in Orbit

Gravity provides the centripetal force to keep satellites in orbit.



Takeaway

Forces explain motion.

Understanding forces is the **foundation of engineering mechanics**.



LECTURE 2

Newton's Laws of Motion

What is a Force?



A force is a **push** or a **pull** that can change an object's motion or shape.



Forces cause changes in motion.



Forces can also deform objects.



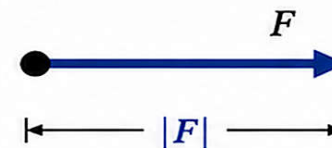
Forces are vector quantities.

A FORCE IS A VECTOR QUANTITY

It has two characteristics:

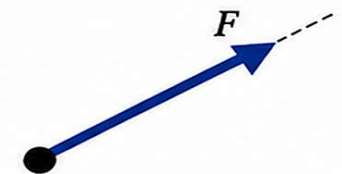
Magnitude

How strong the force is.



Direction

The line along which it acts.



EXAMPLES IN ENGINEERING

Pushing a Cart



Applied force moves the cart.

Lifting with a Crane



Tension force lifts the load.

Braking a Car



Friction force slows the car.

Drone in Flight



Thrust up balances weight down.

Pressing a Button



Force deforms the button and closes the switch.

UNITS

The SI unit of force is the **newton (N)**.

1 newton is the force that gives a mass of 1 kg an acceleration of 1 m/s².

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

Common multiples:

$$1 \text{ kN} = 10^3 \text{ N}$$

$$1 \text{ MN} = 10^6 \text{ N}$$

$$1 \text{ mN} = 10^{-3} \text{ N}$$



Newton's First Law (Law of Inertia)

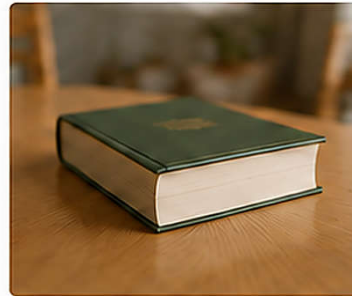


An object remains at rest or moves in a straight line at constant speed unless acted upon by a net external force.

- **Inertia** is the tendency of an object to resist changes in its state of motion.
- A **net external force** is required to start, stop, speed up, slow down, or change direction.

EXAMPLES

1. At Rest Stays at Rest



A book on a table remains at rest while the net external force is zero.

2. In Motion Stays in Motion



A car maintains constant velocity in a straight line while the net external force is zero.

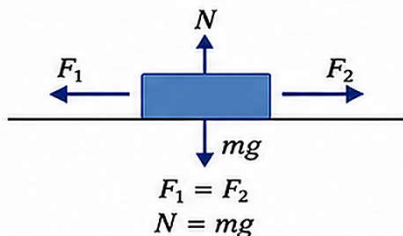
3. Inertia in Daily Life



When the bus stops suddenly, your body tends to keep moving forward relative to the bus.

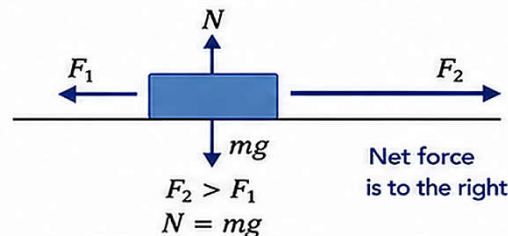
FREE-BODY VIEW

No Net Force ($\Sigma F = 0$)



Object remains at rest or moves with constant velocity.

Net Force ($\Sigma F \neq 0$)



Object accelerates in the direction of the net force.

KEY POINTS

- ✓ This law is also called the **Law of Inertia**.
- ✓ It does not mean “objects want to be at rest”—it applies to **both rest and uniform motion**.
- ✓ Only an **unbalanced (net) external force** can change motion.
- ✓ It describes motion in an **inertial reference frame** when the net external force is zero.

Newton's Second Law (Law of Acceleration)

$$\sum \vec{F}_{\text{ext}} = \vec{F}_{\text{net}} = m\vec{a}$$

for constant mass

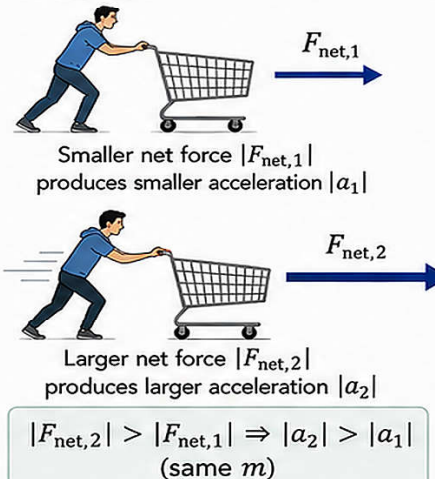
$\vec{F}_{\text{net}}$ = net external force (N)
 m = mass (kg)
 $\vec{a}$ = acceleration (m/s²)

- The **magnitude of the acceleration** of an object is directly proportional to the **magnitude of the net force** acting on it.
- The **magnitude of the acceleration** is inversely proportional to the **mass of the object**.
- The **acceleration vector** is in the **same direction** as the **net-force vector**.

WHAT THIS MEANS

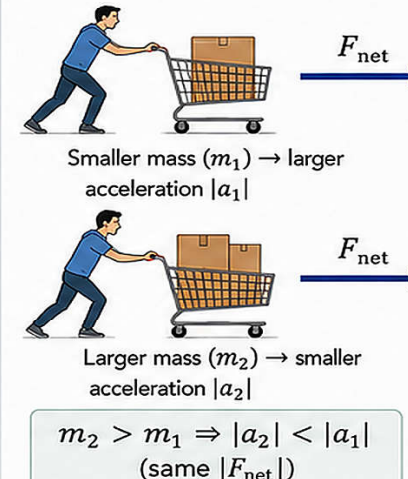
1. More Force → More Acceleration

Same cart mass; friction is neglected. A larger net force produces a larger acceleration.



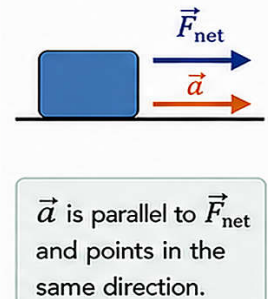
2. More Mass → Less Acceleration

Same net force acts on both carts; friction is neglected.



3. Direction

Both the acceleration vector and the net-force vector are in the same direction.



EXAMPLES IN ENGINEERING

Accelerating a Car



The road's forward static-friction force produces the car's acceleration.

Rocket Launch



Net upward force gives $a = (T - mg)/m$ when drag is neglected.

Robotic Arm



For the same load, a larger net actuator force produces greater acceleration.

KEY POINTS



This law **quantifies** how forces cause changes in motion.



Mass acts as **inertia**: more mass means more resistance to acceleration.



Doubling the net force **doubles** the acceleration (if mass is constant).



Doubling the mass **halves** the acceleration (if net force is constant).

Newton's Third Law (Law of Action–Reaction)



For every action, there is an **equal** and **opposite** reaction.

$$\vec{F}_{A \rightarrow B} = -\vec{F}_{B \rightarrow A}$$

The two forces are **simultaneous** and act on different objects, so they do not cancel on either object's free-body diagram.

- If object A exerts a force on object B, object B exerts an equal magnitude force on object A in the opposite direction.
- These forces act on different objects.
- The forces occur at the same time.

KEY POINTS

- ✓ Newton's Third Law states that forces come in pairs.
- ✓ The action and reaction forces have equal magnitude.
- ✓ The forces act in opposite directions.
- ✓ The forces act on different objects.
- ✓ These forces occur at the same time.
- ✓ Action and reaction do not cancel because they act on different objects.

EXAMPLES

Rocket Launch



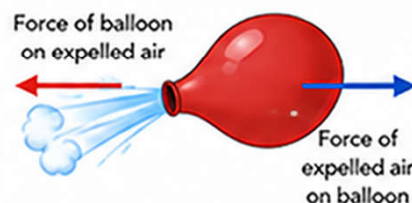
The rocket pushes exhaust downward; the exhaust pushes the rocket upward with an equal and opposite force.

Walking



The person pushes the ground backward; the ground pushes the person forward with an equal and opposite force.

Balloon



The balloon pushes air backward; the expelled air pushes the balloon forward.

REAL-WORLD APPLICATIONS

Swimming



The swimmer pushes water backward; water pushes the swimmer forward.

Gun Recoil



The gun pushes the bullet forward; the bullet pushes the gun backward with equal force.

Boat Propeller



The propeller pushes water backward; water pushes the propeller and boat forward.

Space Travel



The rocket pushes exhaust backward; the exhaust pushes the rocket forward.

Newton's Laws Together



Newton's three laws work together to describe and predict the motion of objects.

- The **First Law** describes motion when the net external force is zero.
- The **Second Law** tells us how a net force changes motion.
- The **Third Law** tells us that forces always come in pairs.

SUMMARY OF THE THREE LAWS

Law	What It Describes	Key Idea
First Law	Motion when the net external force is zero.	Velocity remains constant when $\sum \vec{F}_{ext} = 0$.
Second Law	How net force changes motion.	$\sum \vec{F}_{ext} = m\vec{a}$ (constant mass).
Third Law	Relationship between forces.	$\vec{F}_{A \rightarrow B} = -\vec{F}_{B \rightarrow A}$; the forces act on different objects.

EXAMPLE: PUSHING A BOX ACROSS THE FLOOR

1 Before the Push

Static forces are balanced; the net external force is zero.



First Law:

The box remains at rest.

2 During the Push

The push exceeds friction, so the net force is rightward and the box accelerates.



Second Law:

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

3 When the Push Stops

After the push ends, kinetic friction is the net force. The moving box slows down.



Second Law:

$\vec{a}$ is leftward because the net force is leftward.

Third Law in Action:

The person pushes the box forward; the box pushes back on the person with an equal and opposite force.



These forces form a Third-Law pair; they act on different objects and do not cancel on one free-body diagram.

ENGINEERING APPLICATIONS

Vehicle Design



Driven tires push the road backward; the road's forward static friction accelerates the vehicle.

Braking Systems



Brakes create wheel torque; tire-road friction provides the external force that slows the vehicle.

Robotics



Actuator forces and torques determine link acceleration; contact forces occur in Third-Law pairs.

Aerospace



The engine pushes exhaust backward; exhaust pushes the rocket forward. The net force determines acceleration.

Reference Frames, Inertia, and Newton's First Law

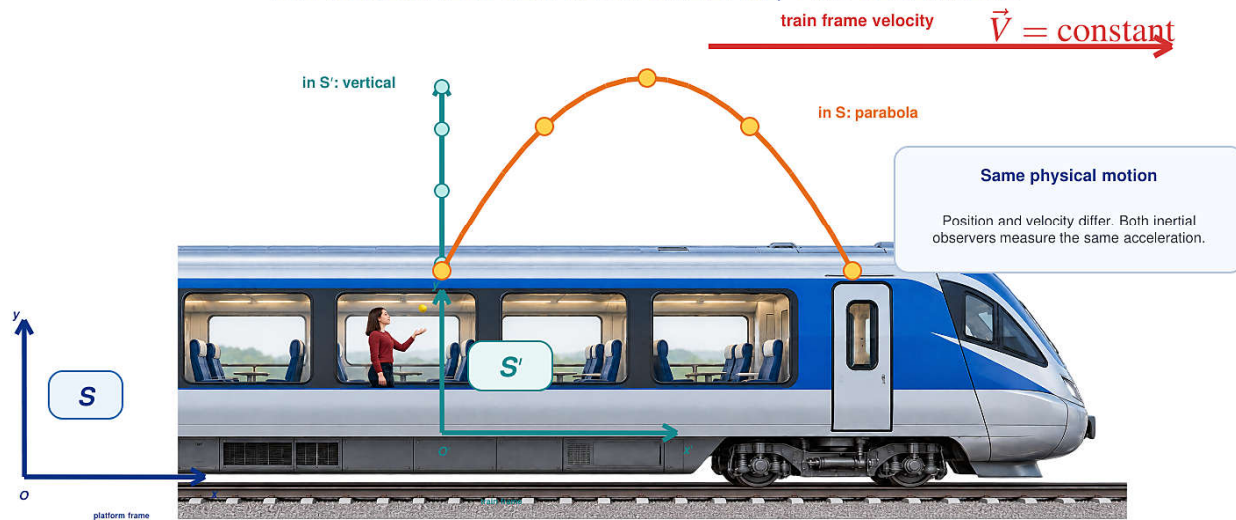
REFERENCE FRAME

An observer, spatial coordinates and a clock used to assign position and time to an event. A reference frame may accelerate or rotate.

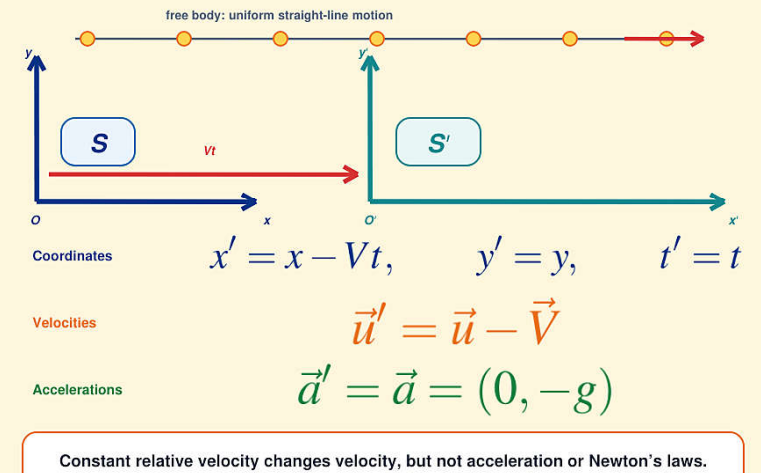
INERTIAL FRAME

A reference frame in which Newton's first law is valid: a free body keeps constant velocity without adding an inertial force.

PHYSICAL INTERPRETATION: ONE BALL, TWO OBSERVERS



GALILEAN TRANSFORMATION BETWEEN S AND S'



NEWTON'S FIRST LAW PROVIDES THE TEST

INERTIAL FRAME

The frame is at rest or moves with constant velocity relative to an inertial frame.

$$\sum \vec{F} = 0 \implies \vec{a} = 0, \vec{v} = \text{constant}$$

NON-INERTIAL FRAME

The frame accelerates or rotates. Apparent force terms enter the equation.

$$\sum \vec{F}_{\text{real}} + \vec{F}_{\text{in}} = m\vec{a}'$$

$$\vec{F}_{\text{in}} = -m\vec{A}_{S'/S}$$

A coordinate system labels events. An inertial frame is a physical class of reference frames defined by Newton's first law.

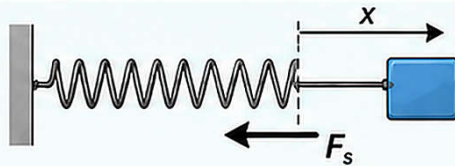
Types of Forces

Common examples — not an exhaustive list



1. APPLIED FORCE

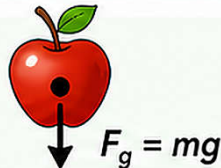
A contact force exerted on an object by a person or another object.



2. ELASTIC FORCE

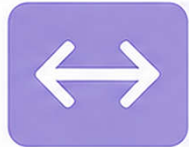
A restoring force produced when an elastic object is stretched or compressed; it opposes the deformation.

$$F_s = -kx$$



3. GRAVITATIONAL FORCE

An attractive force between masses; near Earth, an object's weight points toward Earth's center.



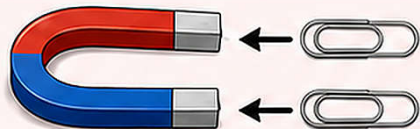
4. FRICTION FORCE

A contact force that opposes relative motion or the tendency to move between surfaces.



5. AIR RESISTANCE (DRAG)

A fluid force that acts opposite an object's velocity relative to the air.



6. MAGNETIC FORCE

A force between magnets, moving charges, or magnetic materials; it may attract or repel.

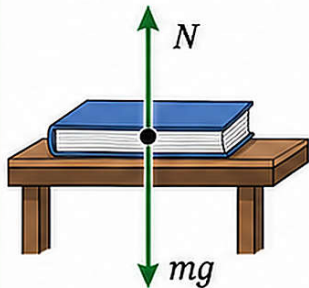
Balanced vs. Unbalanced Forces

- Forces can be **balanced** or **unbalanced**.
- The net force determines whether an object's velocity remains constant or changes.

Balanced Forces

- The vector sum of all forces is zero: $\Sigma \vec{F} = 0$.
- There is no acceleration; velocity remains constant.

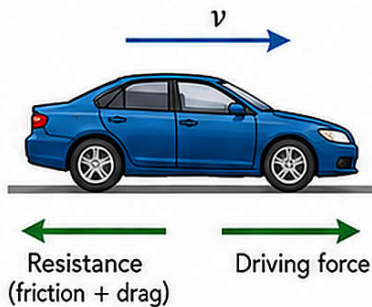
1. Book on a Table



$$N = mg$$

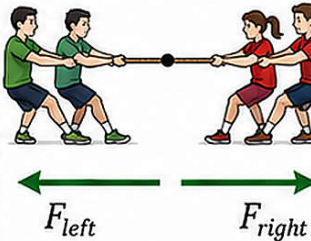
$$\Sigma F_y = 0$$

2. Car at Constant Velocity



$$\Sigma F_x = 0, a = 0$$

3. Tug-of-War (Equilibrium)



$$F_{left} = F_{right}$$

$$\text{Net force} = 0$$

Unbalanced Forces

- The vector sum of all forces is nonzero: $\Sigma \vec{F} \neq 0$.
- The object accelerates in the direction of the net force.

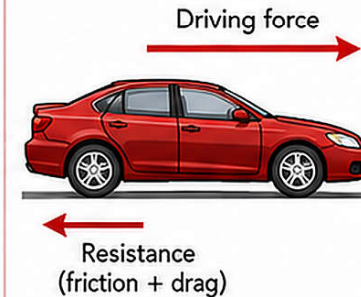
1. Kicking a Ball



During contact:

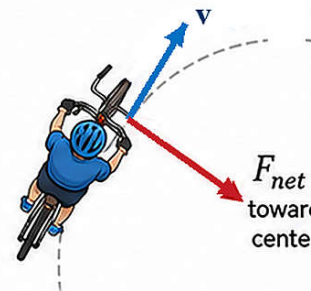
$$\Sigma F_x > 0, a \rightarrow$$

2. Accelerating Car



$$a \rightarrow$$

3. Changing Direction



Net force is perpendicular to velocity.
 a is toward the center.



Key Idea

Balanced forces: $\Sigma \vec{F} = 0 \rightarrow$ constant velocity.

Unbalanced forces: $\Sigma \vec{F} \neq 0 \rightarrow$ acceleration.

Free-Body Diagram (FBD)

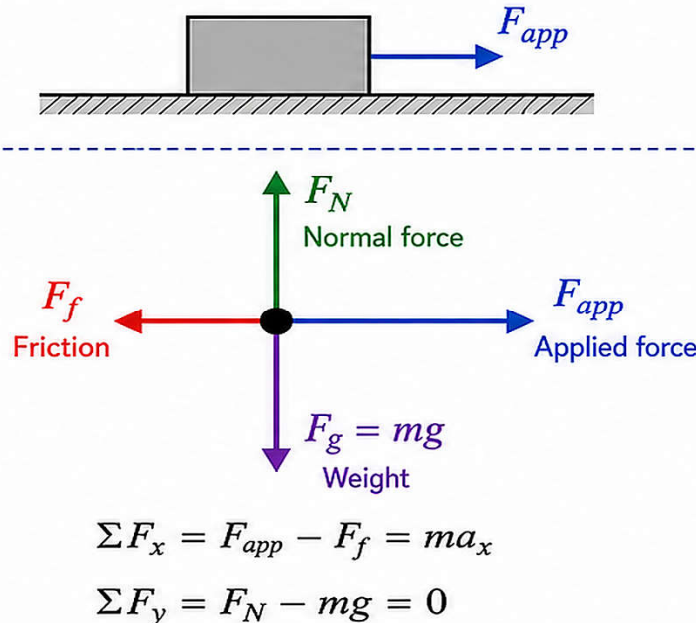
Isolate the object. Then draw only the external forces acting on it.

What Is a Free-Body Diagram?

A free-body diagram is a simplified representation of an isolated object showing all **external** forces acting on that object.

- 1 Choose the object
- ↓
- 2 Isolate it
- ↓
- 3 Draw the external forces

Example: Block Pushed Across a Rough Horizontal Surface



Rules for Drawing an FBD

1. Choose and isolate one object.
2. Replace it with a simple box or dot.
3. Draw only forces acting on that object.
4. Draw every force from the object.
5. Show the correct direction and label.
6. Do not draw velocity or acceleration as forces.

Key Check

Ask: Which object exerts each force, and on which object does it act?

Then apply Newton's second law: $\Sigma \vec{F} = m\vec{a}$.

How to Draw an FBD



A free-body diagram shows all **external** forces acting on one chosen object. Follow these steps.

1 Identify the Object

Choose the single object or system to analyse.



2 Isolate the Object

Replace it with a simple box or dot. Remove the surroundings from the diagram.



3 Identify External Forces

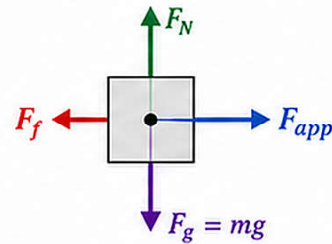
Ask which other objects interact with it.

- Earth → weight
- Floor → normal force and friction
- Person → applied force

Include only forces that actually act in the situation.

4 Draw the Force Vectors

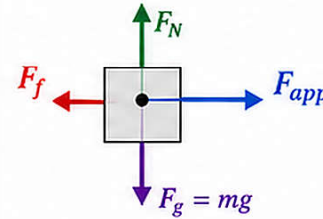
Draw each vector from the object in its physical direction.



All arrows start at the same point. Friction points **opposite** to the relative motion or its tendency.

5 Label and Scale

Label every force clearly. When the diagram is drawn to scale, arrow length represents force magnitude.



6 Check and Apply Newton's Law

Have any forces been missed?

Are the directions and relative magnitudes sensible?

$$\sum F_x = F_{app} - F_f = ma_x$$

$$\sum F_y = F_N - mg = 0$$

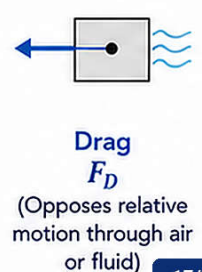
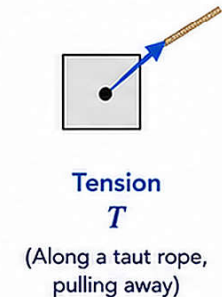
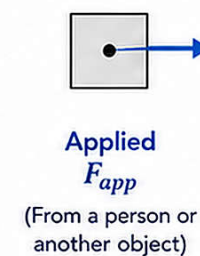
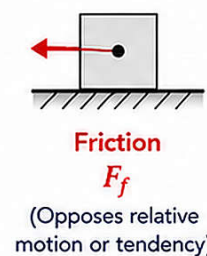
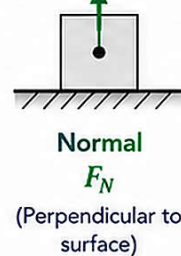
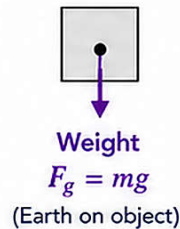
Use the FBD with Newton's laws to solve for unknowns.



Important

- Draw only **external** forces on the chosen object.
- Do not draw velocity or acceleration as forces.
- Friction opposes relative motion or its tendency.
- Redraw the FBD if the interactions change.

Common Forces



Solving Problems with Newton's Laws

1 Identify the system → 2 Draw the FBD → 3 Choose axes → 4 Apply $\Sigma F = ma$ → 5 Solve and interpret

1. Problem and Given

A block of mass m slides to the right on a horizontal table. A horizontal force F_{app} is applied to the right. The coefficient of kinetic friction is μ_k . Find the acceleration a_x of the block.

Given / Assumptions:

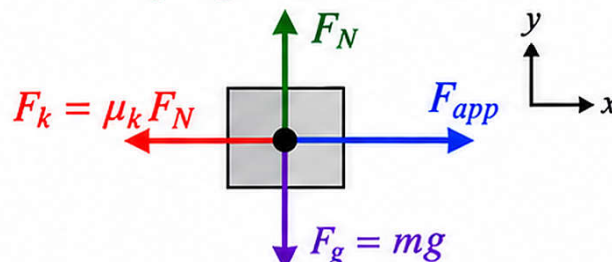
- Mass: m
- Applied force: F_{app} to the right
- Coefficient of kinetic friction: μ_k
- Horizontal surface
- Air resistance is neglected
- Take $+x$ to the right and $+y$ upward.

2. Free-Body Diagram

Context (block on table)



Free-body diagram (isolated block)



F_N : normal force (up)
 F_g : weight (down)
 F_{app} : applied force (right)
 F_k : kinetic friction (left)

3. Equations and Solution

- 1 Vertical direction ($a_y = 0$):

$$\Sigma F_y = F_N - mg = 0 \Rightarrow F_N = mg$$
- 2 Kinetic friction:

$$F_k = \mu_k F_N = \mu_k mg$$
- 3 Horizontal direction:

$$\Sigma F_x = F_{app} - F_k = ma_x$$
- 4 Acceleration:

$$a_x = \frac{F_{app} - F_k}{m} = \frac{F_{app} - \mu_k mg}{m}$$



Interpretation

If $F_{app} > \mu_k mg$, then $a_x > 0$:
 the block speeds up while moving right.



Special Cases

- If $F_{app} = \mu_k mg$, then $a_x = 0$: constant velocity.
- If $F_{app} < \mu_k mg$, then $a_x < 0$: the block slows while moving right.
- This kinetic-friction model applies only while the block is sliding.



Units Check

$[N]/[kg] = m/s^2$
 μ_k is dimensionless.

Example 2 – Inclined Plane



A block of mass m is on an incline of angle θ above the horizontal.
The coefficient of kinetic friction between the block and the surface is μ_k .
Find the acceleration of the block down the incline.

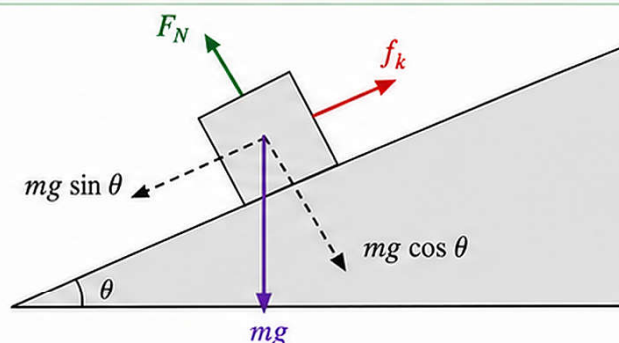
1. Given & Assumptions

- Mass: m
- Incline angle: θ
- Coefficient of kinetic friction: μ_k
- Block slides down the incline
- Find: acceleration a

Assumptions

- Kinetic friction acts up the incline.
- No other forces (air resistance neglected).
- The incline is fixed and rigid.

2. Free Body Diagram (FBD)



mg	= weight (vertical downward)
F_N	= normal force (perpendicular to surface, away from surface)
f_k	= kinetic friction (parallel to surface, up the incline)
$mg \sin \theta$	= component of weight (parallel down the incline)
$mg \cos \theta$	= component of weight (perpendicular into the surface)

Note: Do not add mg and its components simultaneously.

3. Solution

- 1 Choose axes: Take down the incline as positive.
- 2 Normal force (perpendicular to incline):
$$F_N = mg \cos \theta$$
- 3 Kinetic friction (up the incline):
$$f_k = \mu_k F_N = \mu_k mg \cos \theta$$
- 4 Apply Newton's Second Law along the incline:

$$\begin{aligned}\sum F_{\parallel} &= ma \\ mg \sin \theta - f_k &= ma \\ mg \sin \theta - \mu_k mg \cos \theta &= ma\end{aligned}$$

- 5 Solve for acceleration:

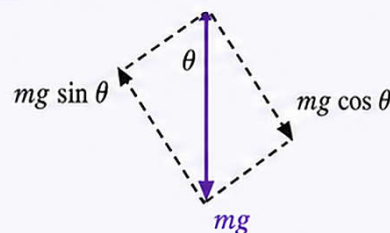
$$a = g[\sin \theta - \mu_k \cos \theta]$$

(down the incline)

Geometry & Components

The weight mg is resolved into two components:

- Parallel to the incline (down the incline): $mg \sin \theta$
- Perpendicular to the incline (into the surface): $mg \cos \theta$



Interpretation

- If $\sin \theta > \mu_k \cos \theta$, then $a > 0$: the block accelerates down the incline.
- If $\sin \theta = \mu_k \cos \theta$, then $a = 0$: the block moves down at constant speed.
- If $\sin \theta < \mu_k \cos \theta$, then $a < 0$: the acceleration points up the incline, so the downward-moving block slows down.

Note: After the block stops, static friction must be considered.

Friction



Friction is a contact force tangent to the **interface**. It **opposes** relative sliding—or the tendency to slide—between two surfaces.

Static:

$$0 \leq F_s \leq \mu_s F_N$$

Kinetic:

$$F_k \approx \mu_k F_N$$

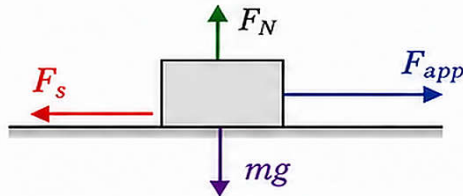
F_s, F_k : friction forces (N)
 μ_s, μ_k : dimensionless coefficients
 F_N : normal force (N)

Types of Friction

1. Static Friction

Acts when there is no slipping. Adjusts to the required value up to a maximum:

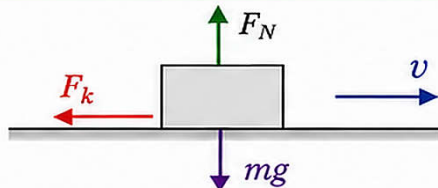
$$0 \leq F_s \leq F_{s,\max} = \mu_s F_N$$



2. Kinetic Friction

Acts when the surfaces slide relative to each other.

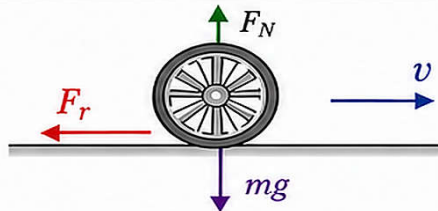
$$F_k \approx \mu_k F_N$$



3. Rolling Resistance

Results mainly from deformation and energy loss during rolling. Often modeled as $F_r = C_{rr} F_N$.

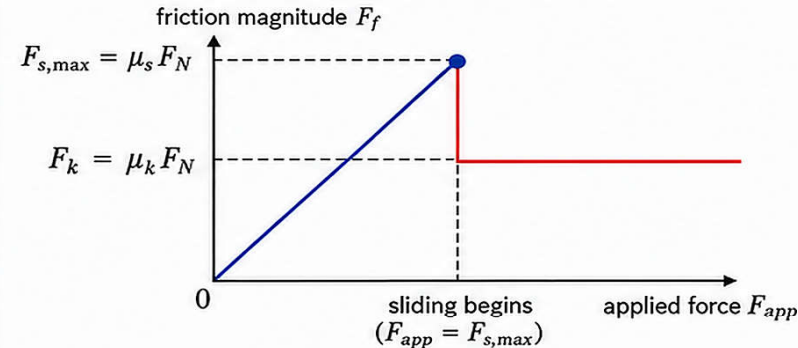
Rolling resistance is not the same as sliding friction.



Key Points

- Friction acts tangent to the contact surface.
- Its direction opposes relative motion or impending relative motion—not necessarily the object's overall motion.
- Static friction is not always $\mu_s F_N$; that is only its maximum value.
- Typically $\mu_s > \mu_k$ for the same pair of surfaces.
- Friction can enable motion: walking, rolling without slipping, and vehicle acceleration.

Static vs. Kinetic Friction



Before sliding:
 $F_s = F_{app}$, up to $F_{s,\max}$.

During sliding
 (ideal Coulomb model, constant F_N):
 F_k is approximately constant.



Remember

Friction resists relative slipping at the interface. It may oppose motion, prevent slipping, or provide the force that accelerates an object.



Typical Values

Coefficients depend strongly on material, surface condition, lubrication, temperature, and speed. Use tabulated values only as approximate guides.



Units

Forces: newtons (N)
 Coefficients μ_s , μ_k , and C_{rr} : dimensionless

Tension



Tension is the pulling force transmitted through a taut string, rope, or cable.
It acts along the string and pulls away from the object on which it acts.

T : tension force (N)

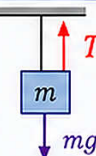
Key Properties

- A string can **pull**, but it cannot **push**.
- The tension force on an object is directed **along** the attached string, **away** from the object.
- For one ideal massless string passing over ideal frictionless, massless pulleys, the tension has the **same magnitude** throughout.
- If the string has mass, or a pulley has rotational inertia or axle friction, tensions in different segments **may differ**.
- An inextensible string **constrains** the motions of connected objects.

How Tension Appears

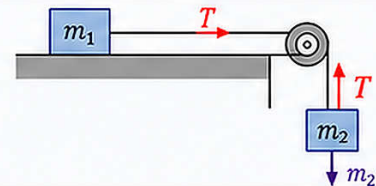
1. Hanging Mass

A mass m hangs from a straight vertical string attached to a ceiling.



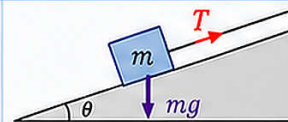
2. Block and Hanging Mass

A block m_1 on a table is connected by a string over a fixed pulley to a hanging mass m_2 .



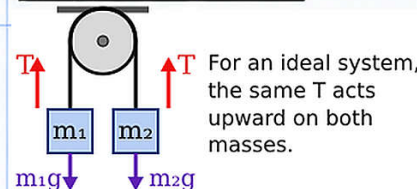
3. Block on an Incline

A block m rests on an incline and is attached to a string parallel to the incline.



4. Atwood Machine

Two masses m_1 and m_2 are connected by a string over a fixed pulley.

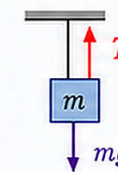


Finding Tension

1. Hanging Mass at Rest

$$\sum F_y = 0$$

$$T - mg = 0 \Rightarrow T = mg$$

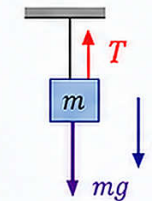


2. Mass Accelerating Downward

Choose downward as positive.

$$mg - T = ma$$

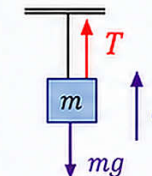
$$T = m(g - a)$$



3. Mass Accelerating Upward

$$T - mg = ma$$

$$T = m(g + a)$$



Ideal-System Assumptions



- Massless, inextensible string
- Massless, frictionless pulley
- No slipping between string and pulley

Units



Tension is a force.

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

Remember



- Draw tension on the selected object, **along** the string and **away** from that object.
- Do not confuse the force of the rope on a mass with the force of the rope on a pulley.

Worked Example



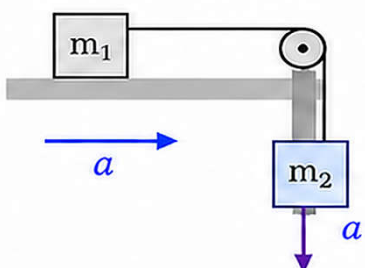
Problem:

A block of mass $m_1 = 4.0$ kg rests on a frictionless horizontal table and is connected by a light, inextensible string over an ideal pulley to a hanging mass $m_2 = 6.0$ kg. **Find the acceleration of the system and the tension in the string.**

Given:

- $m_1 = 4.0$ kg
- $m_2 = 6.0$ kg
- $g = 9.8$ m/s²
- Frictionless table; massless string and pulley

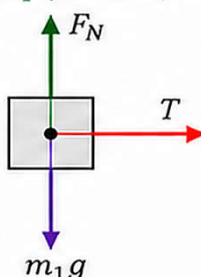
1. System Summary



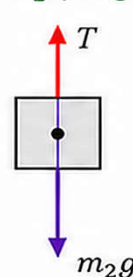
- m_1 moves toward the pulley; m_2 moves downward.
- Both masses have acceleration magnitude 5.88 m/s².
- The ideal string has the same tension, 23.5 N, throughout.

2. Free-Body Diagrams

For m_1 (on table):



For m_2 (hanging):



3. Choose Directions and Apply Newton's Second Law

Choose right as positive for m_1 and downward as positive for m_2 .

For m_1 : $\sum F_x = m_1 a$

$$T = m_1 a \quad (1)$$

For m_2 : $\sum F_y = m_2 a$

$$m_2 g - T = m_2 a \quad (2)$$

The inextensible string gives both masses the same acceleration a .

4. Solve for Acceleration

From (1): $T = m_1 a$

Substitute into (2):

$$m_2 g - m_1 a = m_2 a$$

$$m_2 g = (m_1 + m_2) a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$a = \frac{(6.0 \text{ kg})(9.8 \text{ m/s}^2)}{4.0 \text{ kg} + 6.0 \text{ kg}}$$

$$a = 5.88 \text{ m/s}^2 \quad (m_2 \text{ downward, } m_1 \text{ right})$$

5. Solve for Tension

$$T = m_1 a$$

$$T = (4.0 \text{ kg})(5.88 \text{ m/s}^2)$$

$$T = 23.52 \text{ N} \approx 23.5 \text{ N}$$

$$T = 23.5 \text{ N}$$



Key Takeaways

- Draw a separate FBD for each mass.
- Use one consistent sign convention.
- Solve the coupled equations simultaneously.



Units and Common Mistakes

- Acceleration: m/s²
- Tension: N
- Do not compare m_1 and m_2 as if this were an Atwood machine with both masses hanging.
- Do not draw the pulley or table in an FBD.

Pulley System



A pulley changes the direction of a force or allows multiple masses to be connected by a light, inextensible string.

For a light string over frictionless pulleys, the tension is the same throughout the string.

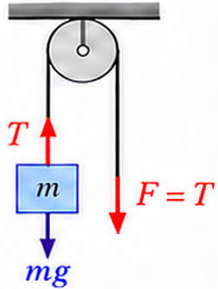
Assumptions

- The string is light (massless) and inextensible.
- Pulleys are frictionless.
- The rope remains taut; axle friction and pulley mass are neglected.

Notation

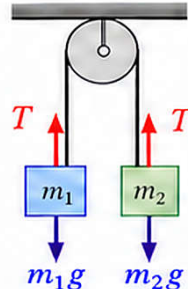
m_1, m_2 : masses (kg)
 T : tension in the string (N)
 a : magnitude of acceleration (m/s^2)
 (Same a for both masses)

1. Single Fixed Pulley (Changes Direction)



- The pulley only changes the direction of the force.
- $T = mg$ if the mass is at rest.

2. Atwood Machine (Two Masses)



If $m_2 > m_1$:

- m_2 moves down
- m_1 moves up
- Same tension T in both sides
- Same acceleration magnitude a for both masses

3. General Atwood Machine Equations

Take downward as positive for m_2 and upward as positive for m_1 (if $m_2 > m_1$).

For m_1 (moving up):

$$T - m_1 g = m_1 a \quad (1)$$

For m_2 (moving down):

$$m_2 g - T = m_2 a \quad (2)$$

Add (1) and (2):

$$(m_2 - m_1)g = (m_1 + m_2)a \Rightarrow a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Substitute back to find the tension:

$$T = m_1(g + a) = m_2(g - a)$$

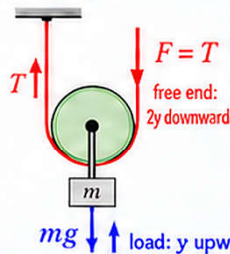
$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

4. Movable and Compound Pulley Systems

The same principles apply:

- Same tension in the same ideal string.
- Apply Newton's 2nd Law to each body.
- The inextensible string constraint relates the accelerations; connected bodies need not have the same magnitude.

Example (Movable Pulley)

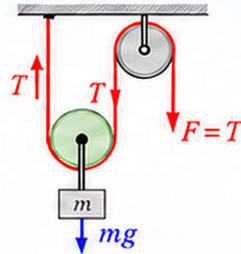


$$2T - mg = ma \quad (\text{upward positive})$$

$$|a_{\text{free}}| = 2|a_{\text{load}}|$$

$$s_{\text{free}} = 2s_{\text{load}}$$

Example (Compound Pulley)



For one ideal rope, T is the same in every segment.

Mechanical advantage equals the number of supporting rope segments.



Key Takeaway

Pulley systems use tension to relate forces and accelerations. Draw the FBD for each mass and apply Newton's 2nd Law.



Units

Tension: newton (N)
 Mass: kilogram (kg)
 Acceleration: m/s^2



Remember

- Tension acts along the string.
- A fixed ideal pulley changes only the direction of the tension force.
- A movable pulley provides force multiplication.



Pro Tip

Always start by drawing a clear FBD for each body. Label T , m , g , and the direction of acceleration.

24. Common Mistakes



Avoiding common mistakes is essential when solving dynamics problems.
Check your work and think physically!



Good Practice

- Draw a clear FBD.
- Choose and label coordinate axes.
- Check directions, signs, units, and limiting cases.
- Interpret the result physically.

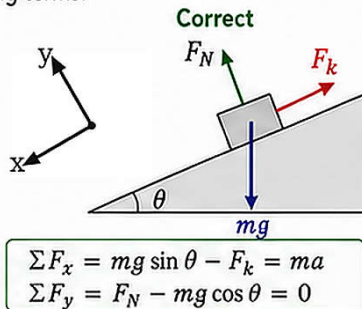
1. Skipping the Free-Body Diagram

Writing equations before identifying the forces often leads to missing terms.

Wrong

$$ma = mg - f$$

(components and directions missing)

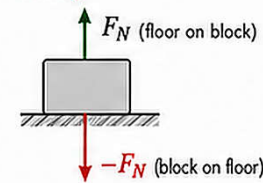


x is along the plane (down the slope); y is perpendicular to the plane.

2. Mixing Action-Reaction Pairs

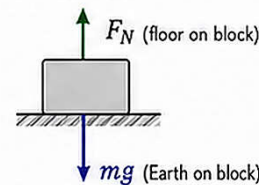
Newton's third-law forces act on different objects, so they do not belong on the same FBD.

Wrong



(Both on the block — incorrect)

Correct

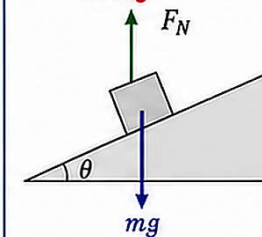


The reaction to F_N acts on the floor, not on the block.

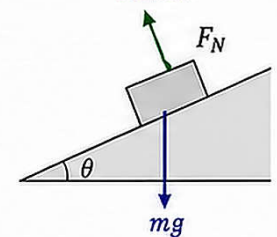
3. Wrong Normal-Force Direction

The normal force is perpendicular to the contact surface—not necessarily vertical.

Wrong



Correct



Normal means perpendicular to the surface.

4. Using Weight Instead of Mass

In Newton's second law, inertia is represented by mass m , not weight mg .

Wrong

$$F_{\text{net}} = (mg)a$$

✗ wrong units

Correct

$$F_{\text{net}} = ma$$

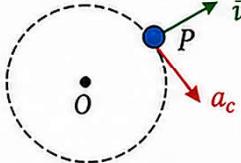
✓ $N = \text{kg} \cdot \text{m}/\text{s}^2$

Weight mg is a force; mass m measures inertia.

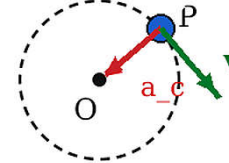
5. Misreading Circular-Motion Vectors

Velocity is tangent to the path; centripetal acceleration points toward the center.

Wrong



Correct



$\vec{v}$ is tangential; $a_c = \frac{v^2}{r}$ is radial inward.

6. Inconsistent Sign Convention

Use signed components consistently. Do not insert an extra minus sign by intuition.

Wrong

Take $+x$ right, but use a positive acceleration magnitude a for leftward acceleration and write $\Sigma F_x = -ma$ without defining it.

Correct

Define the signed component a_x .

$$\Sigma F_x = ma_x$$

If acceleration is leftward, the solution gives $a_x < 0$.

Choose positive directions once and keep them throughout.



Key Takeaway:

Reliable solutions require a correct FBD, physically correct vectors, consistent components, valid units, and a final plausibility check.

25. Summary

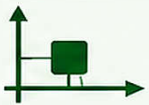


We analysed motion using Newton's laws, free-body diagrams, and models of friction, tension, inclines, and circular motion.
These tools allow us to model and solve real-world engineering problems.

The Problem-Solving Process



- 1 Define the System**
Choose one object or a clearly defined system.



- 2 Draw the FBD**
Isolate it and draw only the external forces acting on it.



- 3 Choose Axes and Apply Newton's Laws**
Define positive directions and write component equations.



- 4 Solve the Equations**
Solve for the unknown forces and accelerations.



- 5 Check and Interpret**
Check signs, directions, units, limiting cases, and physical meaning.

Key Concepts Covered



Newton's Laws

- 1st Law: $\sum \vec{F} = 0 \Rightarrow$ velocity is constant
- 2nd Law: $\sum \vec{F} = m\vec{a}$
- 3rd Law: equal and opposite forces act on different bodies



Friction

- Static: $0 \leq F_s \leq \mu_s F_N$
- Kinetic: $F_k \approx \mu_k F_N$
- Opposes relative sliding or its tendency



Tension

- Acts along a taut string and pulls away from the object
- Same magnitude throughout one ideal massless string over ideal pulleys
- May differ between segments in non-ideal systems



Circular Motion

- $a_c = \frac{v^2}{r}$, directed toward the center
- $\sum F_{\text{radial}} = m \frac{v^2}{r}$
- The centripetal force is the net inward force, not an additional force



Inclines and Banked Curves

- Resolve forces along convenient axes
- Friction changes the range of possible speeds
- Geometry and force components determine the motion

Important Equations

Newton's 2nd Law	$\sum \vec{F} = m\vec{a}$
Weight near Earth	$F_g = mg$
Static friction	$0 \leq F_s \leq \mu_s F_N$
Kinetic friction	$F_k \approx \mu_k F_N$
Centripetal acceleration	$a_c = \frac{v^2}{r} = \omega^2 r$
Net radial force	$\sum F_{\text{radial}} = m \frac{v^2}{r}$
Banked curve, no friction	$\tan \theta = \frac{v^2}{rg}$
Frictionless incline*	$a = g \sin \theta$ $F_N = mg \cos \theta$

*Block sliding freely under gravity, with no other force component along the plane.



Takeaway

Mastering these fundamentals enables you to analyse and design safe, efficient, and reliable engineering systems.



Keep Practising

Draw the diagram first, use consistent components, and always test whether the answer is physically plausible.