

PHYSICS FOR ENGINEERING STUDENTS

Kinematics

From position to prediction

Lecture 1 — Book-style student notes

Learning focus

Reference frames and coordinates

Position, displacement and velocity

Acceleration and graphical interpretation

Constant-acceleration problem solving

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Kinematics: Describing and Predicting Motion

Kinematics is the branch of mechanics that describes motion without first specifying the interaction that caused it. Its central task is to determine how position, velocity and acceleration vary with time. A reliable prediction combines an initial state with a valid evolution law. The method is widely applicable because the same language describes vehicles, projectiles, machines, particles and many other moving systems.

Learning objectives

- Define position, displacement, velocity and acceleration relative to a stated reference frame.
- Choose coordinates that match the geometry and maintain a consistent sign convention.
- Interpret slopes and areas on position-time, velocity-time and acceleration-time graphs.
- Apply the constant-acceleration equations only when their assumptions are satisfied.
- Separate horizontal and vertical projectile motion and solve relative-motion problems.
- Check a solution using units, signs, limiting cases and an independent representation.

The connected description of motion

Position tells where an object is, velocity tells how position changes, and acceleration tells how velocity changes. Differentiation moves from position toward acceleration; integration reconstructs velocity and position from acceleration when the initial conditions are known. The three descriptions are complementary, not competing. A graph, equation, verbal statement and motion diagram should all describe the same physical motion.

$$x(t) \rightarrow v(t) = dx/dt \rightarrow a(t) = dv/dt = d^2x/dt^2$$

Core question: What information is known about the motion, which quantity is required, and which representation connects them with the fewest unsupported assumptions?

1. Predicting motion from an initial state and an evolution law

A prediction requires two kinds of information. Initial conditions specify the state at a chosen time, usually x_0 and v_0 at $t=0$. The evolution law specifies how the state changes. When acceleration is constant, velocity changes linearly with time and position changes quadratically. The equations are therefore not a list of unrelated formulas; they are different consequences of one physical assumption.

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

For $x_0=0$ m, $v_0=2$ m s⁻¹ and $a=+2$ m s⁻², the object has $v=8$ m s⁻¹ and $x=15$ m at $t=3$ s. A motion diagram at one-second intervals shows displacements of 3 m, 5 m and 7 m. The increasing gaps are the visual signature of positive acceleration. Substitution should follow, not replace, this qualitative prediction.

The equations are valid only over an interval in which acceleration is constant. If acceleration changes with time, one must use the actual function $a(t)$, integrate it, or divide the motion into intervals with appropriate models. A single constant calculated from endpoints may reproduce the total change but need not describe the intermediate motion.

Reading the terms as physical contributions

In $x=x_0+v_0t+\frac{1}{2}at^2$, the first term locates the chosen origin-relative starting point. The second is the displacement that would occur if the initial velocity remained unchanged. The third is the additional displacement created by constant acceleration. Reading the equation term by term makes signs easier to predict: a negative acceleration contribution can reduce, cancel or eventually overcome a positive initial-motion contribution.

The relation $v^2=v_0^2+2a\Delta x$ eliminates time, but it does not remove the assumptions of one-dimensional constant acceleration. Squaring also hides the sign of velocity, so the physical direction must be recovered from the motion description or another equation. A square root often produces two algebraic signs, but the event and direction determine which one is admissible.

Key idea: Initial conditions locate the starting state; the evolution law determines how that state develops. Both are required for prediction.

2. Kinematics describes how motion changes

Kinematics and dynamics answer different questions. Kinematics asks how the motion evolves: where is the object, how quickly and in which direction is it moving, and how is that velocity changing? Dynamics asks why the motion changes by connecting acceleration to forces, mass and interactions. Keeping the questions separate allows motion to be described before its cause is identified.

If $x(t)$ is known, differentiation gives $v(t)$ and $a(t)$. If $a(t)$ is known together with v_0 and x_0 , integration reconstructs the motion. The integration variable is a temporary symbol, so the limits state the physical interval explicitly. Initial conditions appear because integration determines changes, not absolute starting values.

$$v(t) = v_0 + \int_0^t a(\tau) d\tau$$

$$x(t) = x_0 + \int_0^t v(\tau) d\tau$$

The same measured positions can be summarized by a table, plotted as x versus t , fitted with a function, or represented by a motion diagram. Agreement among these representations is a powerful error check. If an equation predicts increasing position while the plotted points decrease, the model, sign convention or data transfer must be reconsidered.

Differentiation emphasizes local behavior and can amplify noise in measured position data. Integration accumulates information and may smooth short-scale fluctuations, but it also carries any offset through the entire interval. In experimental work, derivatives and integrals therefore depend on sampling interval, filtering and numerical method as well as on the underlying motion. The mathematical definitions remain exact, while their estimates from finite data have practical limitations.

Physical meaning before calculation

Before differentiating or integrating, identify what each symbol represents and the interval over which the relationship applies. A mathematically valid manipulation can still answer the wrong physical question if the reference frame, event or component has been changed silently.

3. Reference frames and relative motion

Position and velocity are not properties of an object alone. They are measured relative to a reference frame that provides an origin, coordinate axes and a clock. A passenger walking forward at 2 m s^{-1} inside a train moving at 20 m s^{-1} has velocity 2 m s^{-1} relative to the train and 22 m s^{-1} relative to the ground, provided all motion is collinear, and the positive directions agree.

$$\mathbf{x}_{P/G} = \mathbf{x}_{P/T} + \mathbf{x}_{T/G}$$

$$\mathbf{v}_{P/G} = \mathbf{v}_{P/T} + \mathbf{v}_{T/G}$$

The notation A/B means “A measured relative to B.” Reversing the order changes the sign. Relative velocity is a vector relation, so components must be added geometrically in two or three dimensions. For frames moving at constant relative velocity, acceleration is the same in classical mechanics because differentiating the constant frame velocity gives zero.

A reference frame is not the same as a coordinate system. The frame identifies the physical observer and clock; coordinates assign numerical labels to positions within that frame. One laboratory frame can use Cartesian, polar or another coordinate system. Changing coordinates changes the numerical components, not the physical position or motion.

Relative velocity as a change of viewpoint

Suppose a boat moves relative to water while the water moves relative to the shore. The shore observer sees the vector sum of the boat-through-water velocity and the water-through-shore velocity. Aiming directly across the river does not generally produce a path perpendicular to the bank. Conversely,

reaching a point directly opposite requires an upstream velocity component that cancels the current. The geometry should be drawn before component equations are written.

The addition rule is associative, so an intermediate frame can be introduced when it simplifies the data. Nevertheless, every term must form a consistent chain: A relative to B plus B relative to C gives A relative to C. Adding A/B to C/B is not a valid frame chain even if the units match. Subscript logic is therefore a physical check analogous to dimensional analysis.

Problem-solving rule: Attach every position and velocity to a named frame. Write the frame relation symbolically before inserting numbers or directions.

4. Choosing coordinates that match the geometry

A good coordinate system follows the symmetry of the motion. Cartesian coordinates are natural for straight paths, rooms and many laboratory systems. Polar coordinates describe radial distance and direction and are useful for rotation, radar and orbital motion. A body-fixed frame follows a vehicle or robot and can simplify component descriptions, although the frame may be non-inertial if it accelerates or rotates.

$$\mathbf{r} = x \hat{i} + y \hat{j}$$

$$x = r \cos\theta, \quad y = r \sin\theta$$

The axes and positive directions should be chosen once and shown on the sketch. Convenient choices reduce algebra: take the x-axis along an incline, the vertical axis upward for a projectile, or one axis along the known relative velocity. A negative answer then communicates direction relative to the chosen positive axis; it is not automatically an error.

Coordinates also help expose constraints. Motion confined to a straight rail has one varying coordinate even though it occurs in a three-dimensional room. A projectile in a vertical plane needs two coordinates. Reducing the description to independent coordinates removes unnecessary unknowns without discarding physical information.

Coordinate choice and component signs

A vector does not become negative; individual components become negative relative to chosen axis directions. The same velocity vector can have different components after the axes rotate. This is why the diagram and coordinate definition must appear before numerical components. Components can be added algebraically only when they refer to the same basis directions.

A moving or rotating coordinate system may simplify geometry but complicate the evolution equations. At the introductory level, most problems are best solved in a fixed inertial laboratory or

ground frame. A body-fixed frame is useful for describing what an onboard observer measures, but acceleration transformations require care if that frame turns or changes speed.

5. Position, displacement and distance

Position $x(t)$ is a signed coordinate assigned at a specified time in a specified frame. Its sign states location relative to the origin, not the direction of motion. A car at $x=-3$ m is three metres on the negative side of the origin; it may be moving either left or right. A complete position statement therefore includes frame, origin, axis, positive direction and time.

$$\mathbf{r}(t) = x(t)\hat{i}$$

Displacement compares two positions. In one dimension it is final position minus initial position; as a vector it points directly from the initial location to the final location. It depends only on the endpoints. If an object travels from $x=+4$ m to $x=-1$ m, its displacement is -5 m, regardless of the route followed between the points.

$$\Delta x = x_f - x_i$$

Distance travelled accumulates the length of the actual path and is never negative. If an object moves from 0 m to 5 m and then back to 2 m, its displacement is +2 m while its distance is 8 m. Confusing these quantities leads directly to confusion between average velocity and average speed.

Endpoint and path quantities

Displacement is an endpoint quantity; distance is a path quantity. They are equal in magnitude only for one-directional motion along a straight path without reversal.

Successive displacements add with sign or vector direction. If Δx_1 moves from A to B and Δx_2 from B to C, their sum is the direct displacement from A to C. Distances also add, but as non-negative path lengths. This difference explains why a complete round trip has zero net displacement while its distance, travel time and energy use are generally nonzero.

6. Average and instantaneous velocity

Average velocity describes net displacement over a time interval. It is a vector quantity and inherits the sign of displacement in one dimension. Average speed is total distance divided by elapsed time and is non-negative. These quantities are equal in magnitude only when the motion has no reversal and follows the relevant straight path.

$$v_{avg} = \Delta x / \Delta t$$

Instantaneous velocity is the limiting value of the average velocity as the time interval becomes very small. Geometrically, it is the slope of the tangent to the position-time graph. A positive slope means

positive velocity, a negative slope means negative velocity, and a horizontal tangent means zero instantaneous velocity. The height of the graph gives position, not speed.

$$v(t) = \lim(\Delta t \rightarrow 0) \Delta x / \Delta t = dx/dt$$

A straight position-time graph has constant slope and therefore constant velocity. A curve that becomes progressively steeper represents increasing speed in the sign direction of the slope. Curvature indicates acceleration because the slope itself is changing. Reading a graph requires separating the value of the plotted quantity from the slope of the graph.

Secant and tangent slopes

Average velocity is the slope of a secant line connecting two points on $x(t)$. Instantaneous velocity is the slope of the tangent at one point. As the two endpoints of the secant approach one another, the secant slope approaches the tangent slope when the function is smooth. This graphical limiting process gives the derivative a direct physical meaning and explains why a shorter measurement interval gives a more local velocity estimate.

Shorter intervals are not always experimentally better. When position readings have finite resolution, dividing a small displacement by a very small time can make relative uncertainty large. Practical velocity measurement balances temporal locality against measurement noise. The mathematical limit defines the ideal quantity; the instrument and sampling method determine how well it can be estimated.

Graph-reading habit: State the axes and units, identify whether the question asks for a coordinate, slope or area, and interpret the sign before calculating a number.

7. Acceleration and the relationship between v and a

Average acceleration is the change in velocity divided by the elapsed time. Instantaneous acceleration is the derivative of velocity and the second derivative of position. Since velocity is a vector, acceleration can arise from a change in speed, direction or both. An object can have zero velocity at an instant and non-zero acceleration, as at the top of a vertical trajectory.

$$a_{avg} = \Delta v / \Delta t$$

$$a(t) = dv/dt = d^2x/dt^2$$

Velocity and acceleration point in the same direction when the object speeds up, and in opposite directions when it slows down. This rule is more reliable than associating positive acceleration with speeding up. A body moving in the negative direction with negative acceleration increases its speed; a body moving negatively with positive acceleration decreases its speed.

On a velocity-time graph, acceleration is the slope. A horizontal segment indicates constant velocity and zero acceleration. A positive slope gives positive acceleration; a negative slope gives negative acceleration. At a sharp corner the idealized acceleration changes abruptly. Real systems round such corners over a finite response interval.

Zero acceleration means constant velocity, not necessarily rest. Zero velocity describes one instant and does not determine acceleration. At a turning point in one-dimensional motion, velocity passes through zero while acceleration usually remains nonzero. These distinctions prevent the common mistake of inferring one kinematic quantity directly from another without examining its rate of change.

In two dimensions, velocity can maintain constant magnitude while changing direction. Uniform circular motion is the standard example: speed is constant, but the velocity vector rotates and acceleration points inward. Thus the one-dimensional sign table is a component tool, while the general rule compares the full directions of velocity and acceleration.

Sign table for one-dimensional motion

- $v > 0$ and $a > 0$: motion toward $+x$, speeding up.
- $v > 0$ and $a < 0$: motion toward $+x$, slowing down.
- $v < 0$ and $a < 0$: motion toward $-x$, speeding up.
- $v < 0$ and $a > 0$: motion toward $-x$, slowing down.

8. Area under the velocity-time graph

The signed area under a velocity-time graph equals displacement. This follows from integrating velocity over time. Area above the time axis contributes positive displacement; area below contributes negative displacement. The total distance is obtained by adding the magnitudes of the separate areas, not by allowing positive and negative regions to cancel.

$$\Delta x = \int_{t_1}^{t_2} v(t) dt$$

For piecewise-linear graphs, displacement can be found from rectangles, triangles and trapezoids. Each width has units of time and each height has units of velocity, so the area has units of length. This unit check distinguishes the area interpretation from the slope, whose units are acceleration.

A single velocity-time graph simultaneously displays direction, speed and acceleration. The vertical value gives velocity, the sign gives direction, the slope gives acceleration and the accumulated area gives displacement. Marking important times and calculating interval by interval avoids sign errors when the graph crosses the axis.

Piecewise motion

Realistic motion is often modeled piecewise: acceleration, cruising, braking and rest may occupy different intervals. Position and velocity should remain continuous unless the idealized model explicitly contains an instantaneous jump. Acceleration may change between intervals. The final state of one interval becomes the initial state of the next, so a calculation should carry both time and kinematic state forward consistently.

For a trapezoidal region under $v(t)$, the area equals the average of the endpoint velocities multiplied by the interval duration. This is exactly the constant-acceleration displacement relation $\Delta x = \frac{1}{2}(v_0 + v)t$. The graphical and algebraic approaches are therefore the same model expressed in different forms. Their agreement is a valuable conceptual and numerical check.

Interpretation: Graph area is signed displacement. Split the interval at every zero crossing or change of functional form before calculating distance or net displacement.

9. Constant-acceleration models and equation selection

Constant acceleration converts the differential definitions into a small set of algebraic relations. Each equation omits one of the common variables $x - x_0$, v , v_0 , a and t . Efficient problem solving begins by listing known and required quantities, then choosing the equation that does not contain an unnecessary unknown. Selecting a familiar formula before organizing the information often creates extra algebra or an unsolvable system.

$$v = v_0 + at$$

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

The model must be checked before the equation is chosen. Constant acceleration is appropriate for ideal free fall near Earth when air resistance is neglected, for simplified vehicle acceleration over a stated interval, and for each component of projectile motion. It is not appropriate when acceleration depends strongly on speed, time or position unless the interval is divided or a more general model is used.

Solve symbolically where possible, preserve signs and units, and substitute numbers only after the equation is arranged. A negative time for a future event usually signals an inconsistent root or sign convention. A negative velocity or displacement may be entirely physical and should be interpreted rather than discarded.

Selecting and rejecting mathematical roots

Quadratic position equations can yield two times because the object may pass the same position on two different parts of its motion. A projectile can cross a given height while rising and again while falling. Both roots may be physically valid, one may precede the chosen initial time, or one may belong outside the interval over which the model applies. Root selection must be based on the event description, not on choosing the positive number automatically.

Limiting cases provide another check. Setting $a=0$ should reduce constant-acceleration relations to uniform motion. Setting $t=0$ should recover the initial state. If a derived expression fails these simple limits, an algebraic term or sign is missing. Limiting-case checks are fast because they test structure without repeating the full numerical solution.

10. A reliable kinematics workflow

A systematic workflow prevents most errors. First represent the motion with a sketch or graph and identify the event required by the question. Define the reference frame, origin, axes and clock. Translate the wording into symbols with units. Decide whether acceleration is constant and whether the problem is one-dimensional, separable into components or requires relative motion.

- List known quantities, required quantities and assumptions.
- Choose a physical model before selecting an equation.
- Solve symbolically and maintain one sign convention.
- Substitute values with units and keep sensible precision.
- Verify dimensions, sign, magnitude and limiting behavior.
- Check the result using a graph, another equation or an independent representation.

The interpretation step is part of the solution. State what a positive or negative result means in the chosen frame, whether all mathematical roots correspond to the requested event, and whether the magnitude is plausible. An answer without this interpretation may be algebraically complete but physically ambiguous.

Units and significant information

Dimensional analysis should accompany every symbolic step. A term added to a displacement must have dimensions of length; a slope on a velocity-time graph must have dimensions of acceleration. Units can identify many substitutions made into the wrong equation. They cannot confirm the sign, frame or model, so dimensional consistency is necessary but not sufficient.

Numerical precision should reflect the input information. Calculator digits do not create physical knowledge. Keep additional digits during intermediate calculation to avoid rounding drift, then report the final value at a precision supported by the data. When the problem supplies exact idealized

parameters, the purpose remains to communicate a physically interpretable result rather than a long decimal expansion.

Modelling principle: The equation is the compact end of the reasoning process. The difficult work is defining the event, frame, assumptions and valid physical model.

11. Worked example: an accelerating car

A car starts with $v_0 = 5.0 \text{ m s}^{-1}$ and accelerates uniformly at 2.5 m s^{-2} for 4.0 s . The model is one-dimensional constant acceleration with the forward direction positive. The final velocity follows directly from the velocity-time relation.

$$v = 5.0 + (2.5)(4.0) = 15.0 \text{ m s}^{-1}$$

The displacement may be calculated from $\Delta x = v_0 t + \frac{1}{2} a t^2$. Substitution gives $5.0 \times 4.0 + \frac{1}{2} \times 2.5 \times (4.0)^2 = 40 \text{ m}$. The same result follows from average velocity because constant acceleration makes $v_{\text{avg}} = \frac{1}{2}(v_0 + v) = 10 \text{ m s}^{-1}$, and $10 \text{ m s}^{-1} \times 4.0 \text{ s} = 40 \text{ m}$.

$$\Delta x = v_0 t + \frac{1}{2} a t^2 = 40 \text{ m}$$

The independent calculation confirms the result. Units reduce to metres, both velocity and displacement are positive, and the car covers a greater distance in each successive second. The acceleration model is stated explicitly; without that assumption the endpoint values alone would not determine the detailed motion.

The velocity-time graph is a straight line from 5.0 to 15.0 m s^{-1} over four seconds. Its slope is 2.5 m s^{-2} and its trapezoidal area is 40 m . The corresponding position-time graph begins with slope 5.0 m s^{-1} and curves upward because the slope increases. These three views—equations, v - t area and x - t curvature—encode the same motion.

Why two routes are useful

Using a second relation is not redundant. It tests arithmetic, sign convention and model consistency. When two valid routes disagree, the discrepancy usually reveals a substitution error, a confused interval or an equation used outside its assumptions.

12. Vertical motion and projectile motion

Near Earth's surface and without significant air resistance, free-fall acceleration is approximately constant and directed downward. If upward is positive, $a_y = -g$. A ball thrown upward has positive initial velocity but negative acceleration throughout its flight. Its velocity becomes zero at the highest point, yet acceleration remains $-g$ and immediately makes the subsequent velocity negative.

$$v_y = v_{0y} - g t$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Projectile motion separates into independent horizontal and vertical components coupled by the same time. With no air resistance, horizontal acceleration is zero and horizontal velocity is constant. Vertical motion has acceleration $-g$. The trajectory is parabolic because x changes linearly with time while y contains a quadratic time term.

$$x = x_0 + v_0 \cos\theta \cdot t$$

$$y = y_0 + v_0 \sin\theta \cdot t - \frac{1}{2} g t^2$$

The solution normally begins with the vertical event condition: landing height, maximum height or a specified vertical coordinate. Solving the vertical equation gives the relevant time, which is then used in the horizontal relation to find range. Both components must use the same time and initial event.

At the top of a projectile trajectory, only the vertical component of velocity is zero. The horizontal component remains $v_0 \cos\theta$ in the ideal model, so the projectile is still moving. Its acceleration is still downward. Confusing vertical component statements with full-vector statements is a frequent source of conceptual error.

The trajectory equation can be obtained by eliminating time between $x(t)$ and $y(t)$. This produces a quadratic dependence of y on x and explains the parabolic path. The time-parametric form is usually easier for events because both component equations share time, while the trajectory form is useful for relating height directly to horizontal position.

Launch angle and range

For equal launch and landing heights, fixed speed and negligible air resistance, complementary angles have the same range and the maximum range occurs at 45° . These conclusions change when the landing height differs, drag matters or the objective is not maximum horizontal range. Formula conditions must therefore accompany the familiar result.

13. Worked projectile and catch-up problems

A ball is launched horizontally from height h with speed v_0 . The vertical motion determines flight time because $v_{0y}=0$. Setting the landing level to $y=0$ gives $t=\sqrt{(2h/g)}$. Horizontal range is then $v_0 t$. The horizontal speed does not change the fall time in the ideal model; it changes only how far the ball travels during that time.

$$t_{flight} = \sqrt{2h/g}, \quad R = v_0 \sqrt{2h/g}$$

A catch-up problem is an event-matching problem. Write a separate position function for each object in the same frame and set the positions equal at the meeting event. Do not set the velocities equal unless the problem states that they also match. Two objects can occupy the same position while moving at different velocities.

$$x_A(t) = x_B(t)$$

After solving for time, reject roots outside the physical interval and substitute the accepted root back into either position equation. Then calculate velocities separately if required. A graph of both position functions provides an independent picture: intersections are meeting events, and the slopes at an intersection are the velocities of the two objects.

Intersections as physical events

One intersection means one meeting within the plotted interval; two intersections mean that the objects meet twice; no intersection means that the modeled motions never share a position there. Tangency represents equal position and equal velocity at one instant. Thinking geometrically before solving helps predict the number of roots and recognize extraneous numerical answers.

Relative position offers an alternative formulation. Define $\Delta x_{\text{rel}} = x_A - x_B$. A meeting occurs when $\Delta x_{\text{rel}} = 0$, and its derivative is the relative velocity $v_A - v_B$. This approach can reduce a two-object problem to one effective motion, but the sign and frame must still be defined clearly.

Event logic: Translate words such as “meets,” “returns,” “stops,” “reaches maximum height” and “lands” into explicit equations before choosing a kinematic formula.

14. Frequent errors, checks and final synthesis

Frequent errors and their corrections

Mistake	Correction
Using position as distance	State the origin and distinguish signed location from accumulated path length.
Using speed as velocity	Include direction or sign and identify the reference frame.
Reading graph height instead of slope	Identify whether the axis value, tangent slope or enclosed area is required.
Changing the positive direction	Define axes once and interpret negative results consistently.
Using constant- a equations automatically	Confirm that acceleration is constant over the modeled interval.
Mixing projectile components	Solve x and y separately but use the same time for the same event.
Accepting every algebraic root	Test each root against the time interval and physical event.
Equating velocities in a catch-up problem	Meeting requires equal positions; velocities may differ.

Compact solution checklist

- Define the frame, origin, axes, clock and event.
- Draw a motion sketch or graph before calculating.
- List knowns, unknowns and model assumptions.
- Choose the equation that matches the model and avoids unnecessary unknowns.
- Solve symbolically, then substitute values with units.
- Interpret signs and select physically valid roots.
- Check dimensions, magnitude, limits and an independent representation.

Essential equations

$$v = dx/dt, \quad a = dv/dt, \quad \Delta x = \int v dt, \quad \Delta v = \int a dt$$

$$v = v_0 + at, \quad \Delta x = v_0 t + \frac{1}{2}at^2, \quad v^2 = v_0^2 + 2a\Delta x$$

$$\boldsymbol{v}_{A/C} = \boldsymbol{v}_{A/B} + \boldsymbol{v}_{B/C}$$