

PHYSICS FOR ENGINEERING STUDENTS

Electric Circuits

From Voltage to Current and Systematic Circuit Analysis

Lecture 9 — Book-style student notes

Learning focus

Current, voltage, resistance and Ohm's law

Series and parallel resistor networks

Kirchhoff's current and voltage laws

Nodal and mesh analysis

Power balance and solution verification

University of Szeged • Institute of Physics

1. From electric fields to electric circuits

Potential difference creates the conditions for sustained charge motion

A battery separates charge and maintains an electric potential difference between its terminals. In an open circuit, voltage is present but no continuous conducting path exists, so the steady current is zero. Closing the switch completes a path. The electric field established inside the conductors drives mobile charge carriers, and an electric current appears. Voltage is therefore not current: it is the energy-per-charge difference that provides the driving condition for current.

Electric current

Electric current is the rate at which charge crosses a chosen surface:

$$I = dQ/dt \quad \text{or for an average interval} \quad I_{avg} = \Delta Q/\Delta t$$

One ampere equals one coulomb per second. If 6 C passes a point in 2 s, the average current is 3 A. Conventional current is defined in the direction in which positive charge would move. In metals, electrons drift in the opposite direction, but circuit arrows and equations normally use conventional current.

Current is not consumed

A resistor does not use up charge or current. In steady state, as much charge leaves a two-terminal component as enters it; otherwise charge would continually accumulate. What the resistor receives is electrical energy. It converts that energy mainly into thermal energy, while the current continues through the element. The water-turbine analogy is useful: the turbine extracts energy from flowing water without consuming the water itself.

A first problem-solving distinction

Quantity	Physical meaning	SI unit
Voltage V	Energy transferred per unit charge	volt: $1 \text{ V} = 1 \text{ J/C}$
Current I	Charge flow rate	ampere: $1 \text{ A} = 1 \text{ C/s}$
Resistance R	Opposition relating V and I	ohm: $1 \Omega = 1 \text{ V/A}$

Always ask whether the circuit is open or closed and whether a complete path exists. A nonzero voltage does not guarantee a nonzero current. The topology of the connection is as important as the component values.

2. Resistance and Ohm's law

The fundamental local relation for many circuit elements

Electrical resistance measures how strongly a component opposes current for a given voltage. Microscopically, mobile carriers scatter from lattice vibrations, impurities and defects. Geometry also matters: a long conductor offers more resistance, while a larger cross-sectional area offers less. For a uniform conductor, $R = \rho L/A$, where ρ is resistivity.

Ohm's law

$$V = IR \quad I = V/R \quad R = V/I$$

For an ohmic resistor at fixed temperature, current is proportional to voltage. If $V = 12 \text{ V}$ and $R = 100 \Omega$, then $I = 0.12 \text{ A}$. Doubling voltage doubles current; doubling resistance at fixed voltage halves current. Ohm's law is a constitutive relation, not a universal law for every device. Diodes, lamps during heating and many semiconductor elements have nonlinear current–voltage behavior.

Passive sign convention

A component absorbs power when current enters the terminal labeled positive. With this passive convention, $p = vi$ is positive for absorbed power and negative for delivered power. A resistor always absorbs power in the ideal model:

$$P = VI = I^2R = V^2/R$$

These forms are equivalent only when Ohm's law applies. They provide independent checks. A 10Ω resistor carrying 2 A absorbs $P = 40 \text{ W}$; the voltage drop is 20 V and VI gives the same result.

Why resistors are useful

Resistors limit current to protect components, establish operating points, divide voltage, provide heating and convert electrical signals into measurable voltage drops. A variable resistor allows adjustment. In an LED circuit, omitting a series resistor can allow excessive current because the source and LED alone may offer very small effective resistance.

- Write known values with units.
- Choose the required form of Ohm's law.
- Keep voltage polarity and current direction consistent.
- Calculate power and decide whether the element absorbs or delivers it.
- Check whether the magnitude is physically plausible.

3. Recognizing series and parallel connections

Circuit topology must be read from nodes and paths

Components are in series when the same current must pass through them: their common connection has no branch through which current can leave. Components are in parallel when both of their terminals connect to the same two nodes: they therefore share the same voltage. Visual appearance can be misleading; wires may be bent without changing the circuit. Identify nodes and possible current paths instead of judging by the drawing shape.

Connection	Topological test	Shared quantity	Conservation statement
Series	One path; unbranched common node	Same current	Voltage drops add
Parallel	Same two end nodes	Same voltage	Branch currents add

Series current and voltage

In a steady one-path circuit, charge cannot disappear between elements, so $I_1 = I_2 = \dots = I$. The source voltage equals the algebraic sum of resistor drops. The resistor with larger R has the larger voltage drop because $V_i = IR_i$.

Parallel voltage and current

Every branch spans the same pair of nodes, so $V_1 = V_2 = \dots = V_{\text{source}}$. Current divides at the junction according to branch resistance: $I_i = V/R_i$. Lower resistance carries larger current. At the junction, conservation of charge requires $I_{\text{total}} = \sum I_i$.

Conceptual example

A 12 V source feeds 6 Ω and 3 Ω branches in parallel. The branch currents are 2 A and 4 A, giving 6 A total. Current is not divided equally because the resistances differ. Both branches nevertheless have exactly 12 V across them in the ideal circuit.

Prediction before calculation

If one parallel resistance decreases while the ideal source voltage stays fixed, current in the unchanged branch stays fixed, current in the changed branch increases, and total current increases. The network equivalent resistance therefore decreases. Making this prediction before using formulas is an effective error detector.

4. Equivalent resistance

Replacing a resistor network without changing terminal behavior

Equivalent resistance R_{eq} is defined so that the replacement draws the same total current from the same terminal voltage as the original network. It simplifies the external behavior without claiming that internal branch currents are identical.

Series combination

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

Series resistances add because the same current flows through every element and voltage drops add. R_{eq} must be larger than every individual resistance. For $2\ \Omega$, $3\ \Omega$ and $5\ \Omega$ in series, $R_{eq} = 10\ \Omega$.

Parallel combination

$$1/R_{eq} = 1/R_1 + 1/R_2 + \cdots + 1/R_n$$

$$\text{For two resistors: } R_{eq} = R_1 R_2 / (R_1 + R_2)$$

Parallel conductances add because each branch has the same voltage and currents add. R_{eq} must be smaller than the smallest branch resistance. Two equal resistors R in parallel give $R/2$; n equal resistors give R/n .

Mixed-network reduction

Simplify only combinations that are unambiguously series or parallel. Replace one group by its equivalent, redraw the remaining network and repeat. For example, if $6\ \Omega$ and $3\ \Omega$ are parallel, their equivalent is $2\ \Omega$. If this block is in series with $4\ \Omega$ and $2\ \Omega$, the overall resistance is $4 + 2 + 2 = 8\ \Omega$.

Sanity check	Expected result
Adding a resistor in series	R_{eq} increases
Adding a branch in parallel	R_{eq} decreases
Series network	$R_{eq} > \text{largest individual } R$
Parallel network	$R_{eq} < \text{smallest individual } R$

Do not combine elements merely because they touch. Series elements must share the same current with no branching at their common node. Parallel elements must share both nodes. When neither condition holds, use Kirchhoff's laws, nodal analysis or mesh analysis.

5. Kirchhoff's laws

Conservation of charge and energy in arbitrary circuits

Kirchhoff's current law — KCL

$$\Sigma I = 0 \text{ at any node} \quad \text{or} \quad \Sigma I_{\text{entering}} = \Sigma I_{\text{leaving}}$$

KCL expresses conservation of charge. In steady state, a circuit node cannot accumulate charge indefinitely. Choose a sign convention—such as currents leaving positive—and use it consistently. If a solved current is negative, the actual direction is opposite the assumed arrow; the mathematics remains valid.

Kirchhoff's voltage law — KVL

$$\Sigma \Delta V = 0 \text{ around every closed loop}$$

KVL expresses energy conservation. Choose a traversal direction. Crossing a source from negative to positive is a voltage rise; crossing from positive to negative is a drop. Across a resistor, moving with the assumed current is a drop $-IR$; moving against it is a rise $+IR$. Any starting point and either loop direction work if signs remain consistent.

Law	Applied to	Conservation principle	Typical use
KCL	Node or junction	Charge	Relate branch currents; nodal analysis
KVL	Closed loop	Energy	Relate voltages; mesh analysis

Small examples

If 3 A and 2 A enter a node while 1 A and an unknown I_x leave, KCL gives $3 + 2 - 1 - I_x = 0$, hence $I_x = 4$ A leaving. For a 12 V source and series drops of 2 V, 4 V and V_x , KVL gives $12 - 2 - 4 - V_x = 0$, so $V_x = 6$ V.

Equation discipline

- Draw current arrows and voltage polarities.
- Label nodes and choose ground.
- Write KCL only after declaring the current sign convention.
- Write KVL while physically tracing the chosen loop direction.
- Solve simultaneous equations and interpret negative results.
- Verify at unused nodes or loops.

6. Nodal analysis

A KCL-based method using node voltages as unknowns

Nodal analysis is efficient when a circuit has many nodes or current sources. Select one reference node, called ground, and set its potential to zero. Label every remaining node voltage relative to ground. Express each resistor current by the voltage difference across it divided by resistance, then apply KCL at every independent unknown node.

General branch current

$$\text{Current from node a to node b: } I_{ab} = (V_a - V_b)/R$$

The numerator is a voltage difference, not merely one node voltage, unless the other terminal is ground. This is one of the most common errors in nodal equations.

Standard procedure

- Choose ground, preferably a common node.
- Label unknown node voltages $V_1, V_2, \dots$
- Mark source arrows and known source-connected node voltages.
- Choose all currents leaving or all currents entering as positive.
- Write KCL at each unknown node using $(V_a - V_b)/R$.
- Solve the linear system, then calculate any requested branch currents.
- Check KCL and power balance.

Short nodal example

Suppose $V_1 = 10 \text{ V}$ is known. Node V_2 connects to V_1 through 2Ω , to node V_3 through 4Ω and to ground through 6Ω . With currents leaving V_2 positive,

$$(V_2 - 10)/2 + (V_2 - V_3)/4 + V_2/6 = 0$$

A second KCL equation at V_3 completes the system. Current-source arrows enter with negative sign under a leaving-positive convention and leave with a positive sign. The choice is arbitrary, but consistency is not.

When nodal analysis is attractive

The method directly yields physically meaningful potentials and handles current sources naturally. It often requires fewer equations than branch-current methods. Voltage sources between two unknown nodes may require a supernode, while sources connected to ground immediately set a node voltage.

7. Mesh analysis

A KVL-based method using loop currents

Mesh analysis applies to planar circuits and is especially efficient when there are few meshes and mainly voltage sources. Assign a mesh current to each smallest independent loop, usually all clockwise or all counterclockwise. Write KVL for every mesh and solve the simultaneous equations.

Shared resistors

If a resistor belongs to one mesh only, its drop is $R_i I_i$. If it is shared by meshes i and j whose currents oppose each other in the branch, the branch current is $I_i - I_j$ in the direction of I_i . The voltage drop in mesh i is $R(I_i - I_j)$, while mesh j uses $R(I_j - I_i)$.

Two-mesh equation pattern

$$(R_1 + R_s)I_1 - R_s I_2 = V_1$$

$$-R_s I_1 + (R_2 + R_s)I_2 = V_2$$

The diagonal coefficient is the total resistance around that mesh. The off-diagonal coefficient is minus the shared resistance. Source signs follow the selected traversal direction. After solving mesh currents, branch currents and voltages follow from differences and Ohm's law.

Worked coefficient example

For a left mesh with $2\ \Omega$ and shared $6\ \Omega$ driven by 12 V , and a right mesh with $4\ \Omega$ and the same shared $6\ \Omega$ driven by 6 V , one consistent formulation is

$$8I_1 - 6I_2 = 12, \quad -6I_1 + 10I_2 = 6$$

Solving gives $I_1 = 3.55\text{ A}$ and $I_2 = 2.73\text{ A}$ for this sign assignment, so the shared-branch current is $I_1 - I_2 = 0.82\text{ A}$ in the I_1 direction. Substitution verifies both equations within rounding. If source polarities or traversal signs differ, numerical signs change; the diagram and equations must be treated as one consistent system.

Comparison with nodal analysis

Method	Unknowns	Natural sources	Best suited to
Nodal	Node voltages	Current sources	Many nodes, few essential unknown voltages
Mesh	Mesh currents	Voltage sources	Planar circuits with few loops

8. Worked example: mixed DC resistor circuit

Node voltages, branch currents and equivalent resistance

A 24 V source feeds a 6 Ω resistor leading to node A. Node A connects to ground through 12 Ω and to node B through 4 Ω . Node B connects to ground through 8 Ω . Find V_A , V_B , all resistor currents and the source current.

Recognize the structure

From node A to ground there are two paths: a direct 12 Ω branch and a series path 4 Ω + 8 Ω = 12 Ω . These two 12 Ω branches are parallel, giving 6 Ω from A to ground. Including the source-side 6 Ω resistor, the total equivalent resistance is 12 Ω . Thus $I_{\text{source}} = 24/12 = 2.00$ A. This reduction already predicts the main result.

Nodal equations

$$\text{Node B: } (V_B - V_A)/4 + V_B/8 = 0 \quad \Rightarrow \quad V_A = 3V_B/2$$

$$\text{Node A: } (V_A - 24)/6 + (V_A - V_B)/4 + V_A/12 = 0$$

Solving gives $V_A = 12$ V and $V_B = 8$ V. These values are consistent with the two equal 12 Ω branches: the source-side 6 Ω and the A-to-ground equivalent 6 Ω divide 24 V equally, placing node A at 12 V. Along the 4 Ω –8 Ω series branch, the 12 V divides in the ratio 4:8, leaving $V_B = 8$ V.

Branch currents

$$I_6 = (24 - 12)/6 = 2.00 \text{ A}$$

$$I_{12} = 12/12 = 1.00 \text{ A}, \quad I_4 = (12 - 8)/4 = 1.00 \text{ A}, \quad I_8 = 8/8 = 1.00 \text{ A}$$

At node A, 2 A enters and splits into two 1 A currents. At node B, 1 A enters and 1 A leaves. KCL is satisfied at both nodes. The outer-loop KVL check is $24 - (2)(6) - (1)(4) - (1)(8) = 0$.

9. Power balance as an independent verification

Delivered power must equal absorbed power

A correct DC solution should satisfy conservation of power. With passive sign convention, resistors absorb positive power I^2R . An ideal source delivering current has negative absorbed power, or equivalently positive delivered power.

Power in the mixed-circuit example

Element	Current	Power absorbed
6 Ω	2.00 A	$(2.00)^2(6) = 24 \text{ W}$
4 Ω	1.00 A	$(1.00)^2(4) = 4 \text{ W}$
12 Ω	1.00 A	$(1.00)^2(12) = 12 \text{ W}$
8 Ω	1.00 A	$(1.00)^2(8) = 8 \text{ W}$

$$P_{\text{resistors}} = 24 + 4 + 12 + 8 = 48 \text{ W}$$

$$P_{\text{source, delivered}} = VI_{\text{source}} = (24 \text{ V})(2.00 \text{ A}) = 48 \text{ W}$$

Delivered and absorbed powers agree. This check is stronger than simply repeating the same algebra because it combines every voltage and current result. A power mismatch indicates at least one wrong current, polarity, resistance or arithmetic operation.

Three layers of verification

- Local check: apply KCL at every node.
- Path check: apply KVL around one or more independent loops.
- Global check: verify total delivered power equals total absorbed power.
- Topology check: compare the source current with $V_{\text{source}}/R_{\text{eq}}$ when reduction is possible.
- Limit check: ask how the answer changes if a resistance becomes zero or infinite.

Physical interpretation

The source supplies 48 joules each second. The 6 Ω resistor absorbs half because it carries the full 2 A before current splits. Each downstream branch carries only 1 A, so its power depends directly on its resistance. Energy is converted to heat, while charge and steady current are conserved through the network.

10. Worked example: nodal analysis with current sources

Careful source directions and voltage differences

Node A connects to ground through 4 Ω , to node B through 2 Ω , and receives 3 A from an upward current source. Node B connects to ground through 6 Ω and sends 1 A downward through a current source. Ground is the bottom node. Use currents leaving each node as positive.

Node equations

$$\text{Node A: } V_{A/4} + (V_A - V_B)/2 - 3 = 0 \quad \Rightarrow \quad 3V_A - 2V_B = 12$$

$$\text{Node B: } V_{B/6} + (V_B - V_A)/2 + 1 = 0 \quad \Rightarrow \quad -3V_A + 4V_B = -6$$

Adding the two equations gives $2V_B = 6$, hence $V_B = 3 \text{ V}$. Substitution gives $V_A = 6 \text{ V}$.

Resistor currents

$$I_{4\Omega} = V_{A/4} = 1.50 \text{ A downward}$$

$$I_{2\Omega} = (V_A - V_B)/2 = 1.50 \text{ A from A to B}$$

$$I_{6\Omega} = V_{B/6} = 0.50 \text{ A downward}$$

At node A, the 3 A source current entering splits into 1.50 A through 4 Ω and 1.50 A toward B. At node B, 1.50 A enters from A; 0.50 A leaves through 6 Ω and 1.00 A leaves through the current source. Both KCL equations are satisfied.

Important interpretation of a current source

A source arrow states current direction, not necessarily whether the source delivers or absorbs power. Power requires the voltage across the source and passive polarity. In this example the left source injects current into node A; the right source draws current from node B to ground. Whether each delivers energy depends on its terminal voltage and current-entry polarity.

Common failure modes

- Reading a source arrow backward.
- Using V_A instead of $V_A - V_B$ across the 2 Ω resistor.
- Forgetting to define ground before writing node voltages.
- Changing the leaving-positive convention midway.
- Calling a negative solution an error instead of reversing the assumed direction.

11. Common mistakes and a robust workflow

Most circuit errors are conceptual or algebraic, not computational

Mistake	Why it fails	Correction
Confusing series and parallel	A visual layout does not define topology.	Identify shared nodes and current paths.
Assuming current is used up	Charge is conserved in steady state.	Track energy conversion separately from charge flow.
Adding parallel resistances directly	Parallel currents, not resistances, add.	Add reciprocals or conductances.
Ignoring current-source arrows	The KCL source term receives the wrong sign.	Read each arrow relative to the chosen node convention.
Using node voltage across a two-node resistor	Ohm's law needs terminal voltage difference.	Use $(V_a - V_b)/R$.
Skipping checks	A plausible number may still be wrong.	Use KCL, KVL, Req and power balance.

Integrated circuit-analysis workflow

- Redraw the circuit clearly and label all nodes.

- Choose ground, current arrows and voltage polarities.
- Predict qualitative behavior: larger or smaller currents, bounds for Req.
- Simplify obvious series and parallel groups.
- Choose Ohm's law, nodal analysis or mesh analysis according to topology.
- Write equations symbolically before inserting numbers.
- Solve step by step and preserve units.
- Interpret negative results as direction information.
- Verify with conservation laws and power.

Sign discipline

Reference directions are assumptions, not claims. It is acceptable to guess an unknown current direction. A negative answer simply means that the actual direction is opposite. What is unacceptable is silently changing arrows during the derivation. Keep the original reference system until interpretation at the end.

Unit discipline

Use volts, amperes and ohms consistently. Prefixes must be converted: $1 \text{ mA} = 10^{-3} \text{ A}$ and $1 \text{ k}\Omega = 10^3 \Omega$. Conveniently, volts divided by kilo-ohms give milliamperes, but this must be used consciously. Always attach a unit to the final answer.

12. Formula sheet and self-check

A compact reference for solving DC resistor circuits

$$\text{Current: } I = dQ/dt \quad \text{Ohm: } V = IR \quad \text{Power: } P = VI = I^2R = V^2/R$$

$$\text{Series: } R_{eq} = \Sigma R_i \quad \text{Parallel: } 1/R_{eq} = \Sigma (1/R_i)$$

$$\text{KCL: } \Sigma I = 0 \text{ at a node} \quad \text{KVL: } \Sigma \Delta V = 0 \text{ around a loop}$$

$$\text{Branch current: } I_{ab} = (V_a - V_b)/R$$

Fast diagnostic questions

- Is the circuit closed?
- Which points are the same node?
- Must two elements carry the same current?
- Do two elements share the same two terminal nodes?
- Is the quantity being added a vector, signed scalar, voltage drop or current?
- Does the equivalent resistance satisfy series or parallel bounds?
- Do KCL, KVL and power balance all agree?

Self-check problems

- Three equal resistors R are in series. What is R_{eq} and how does the source voltage divide?
- Three equal resistors R are in parallel. What is R_{eq} and how does total current divide?
- A 12 V source is connected to $4\ \Omega$ and $8\ \Omega$ in series. Find current and each voltage drop.
- A 12 V source is connected to $4\ \Omega$ and $8\ \Omega$ in parallel. Find branch and total currents.
- Why can a current source absorb power in one circuit and deliver power in another?
- What does a negative node voltage or negative branch current physically mean?

Final synthesis

Circuit analysis translates physical conservation laws into algebra. Ohm's law describes component behavior; topology identifies shared current or voltage; KCL preserves charge at nodes; KVL preserves energy around loops. Nodal and mesh methods organize these laws for larger networks. Equivalent resistance and power balance provide independent checks.

A reliable analyst does not begin with a calculator. First understand the circuit, define references and predict behavior. Then choose the simplest valid method, solve systematically and verify from several viewpoints. This combination of physics, topology, algebra and checking is the core problem-solving lesson of electric circuits.