

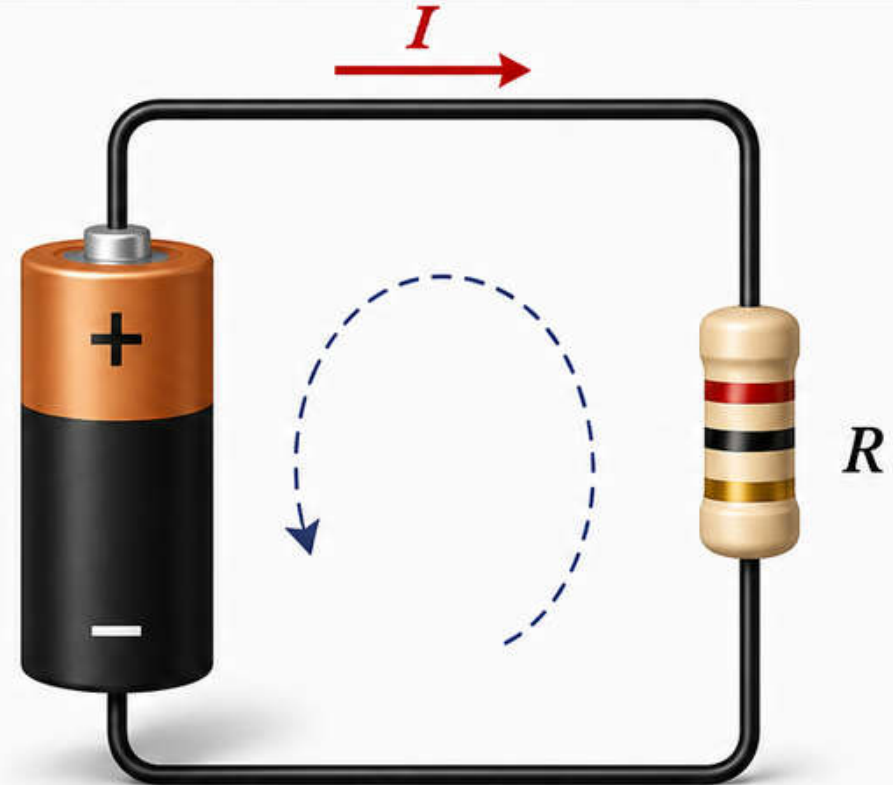
Electric Circuits

From Voltage to Current



Today's Goals

- Understand Ohm's law
- Analyze series and parallel circuits
- Apply Kirchhoff's laws
- Solve simple circuit problems



From Electric Fields to Circuits

What Makes Charges Move?

Potential difference



Charge motion



Electric current

1 What we already know

Potential difference V

A battery creates a difference in electric potential between its terminals.

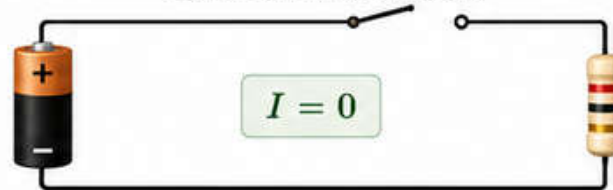


$$V = \Delta V$$

Voltage provides the “push”.

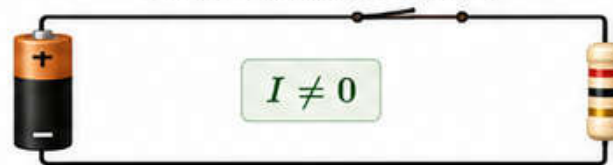
2 What happens when we close the circuit?

Open circuit (switch open)



Close the switch

Closed circuit (switch closed)



3 The new idea

Electric Current

Moving charge =
electric current



Current is the flow of electric charge through a conductor.



Think First

A battery is connected to a resistor, but the switch is open.
Is there a voltage? Is there a current?

Voltage?

YES

Current?

NO



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Understand the physics.
Master the problem-solving.

What Is Electric Current?

Electric Current = Rate of Charge Flow

$$I = \frac{\Delta Q}{\Delta t}$$

$$1 \text{ A} = 1 \text{ C/s}$$

Simple Example

If $\Delta Q = 6 \text{ C}$ passes a point
in $\Delta t = 2 \text{ s}$, then

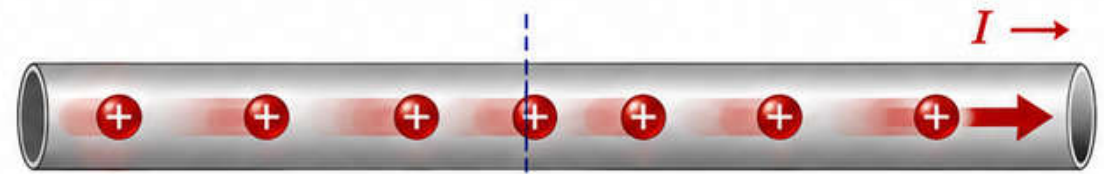
$$I = \frac{6 \text{ C}}{2 \text{ s}}$$

$$I = 3 \text{ A}$$



Physical Meaning

Current tells us how much electric charge passes a point **per unit time**.



How much charge
passes here in 1 second?

Important: Direction

Conventional current

I

defined as the direction of
positive charge flow.

Electron motion

e^-

in a metal is in the
opposite direction.



Think First

If twice as much charge passes the same point
in the same time, what happens to the current?

A It halves

B It stays
the same

C It doubles



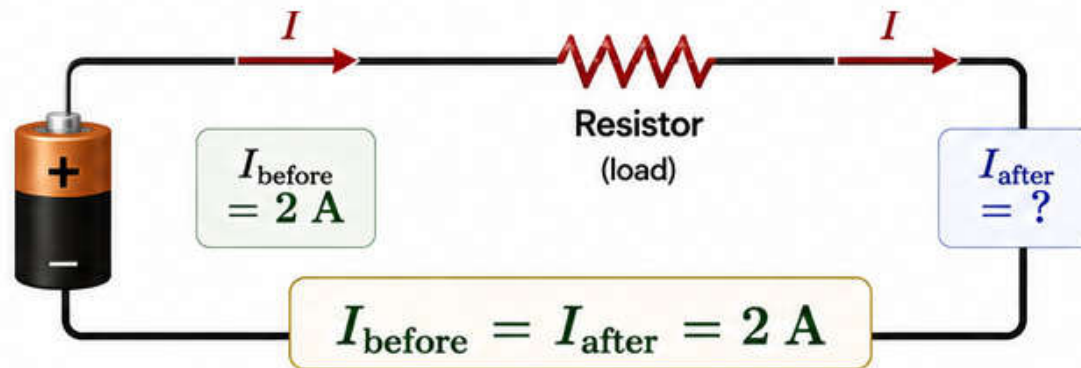
Key Message

More charge per second $\Rightarrow$ larger current



Does Current Get Used Up?

Current Is **NOT** Consumed by a Resistor



So What IS Used?

- ✗ Charge is not consumed
- ✗ Current is not consumed
- ✓ Electrical energy is transferred



In a resistor:
electrical energy $\rightarrow$ thermal energy (heat)



In a light bulb:
electrical energy $\rightarrow$ light + heat



Key Physical Idea

Current is not consumed; electrical energy is transferred.



Think First

What happens to the current after it passes through the resistor?

- (A) $I < 2 \text{ A}$ – some current was used up
 - (B) $I = 0$ – the resistor consumed the current
 - (C) $I = 2 \text{ A}$ – the current is unchanged
- $\Rightarrow I_{\text{before}} = I_{\text{after}}$

A Simple Analogy

Water in $\rightarrow$ turbine $\rightarrow$ water out



The water does not disappear.
The turbine takes energy from the flowing water.

Charge in $\rightarrow$ resistor $\rightarrow$ charge out



What Is Electrical Resistance?

Resistance Opposes Current Flow



If current is not “used up” in a resistor, what does the resistor do?
It makes it harder for current to flow.

A Simple Picture

Low resistance (easy path)



Lots of charge can pass.
Large current for a given voltage.

vs.

High resistance (narrow path)



Fewer charge can pass.
Smaller current for the same voltage.

Microscopic View

Materials scatter charge carriers as they move.
More scattering → more opposition → higher resistance.

Low resistance material



Few collisions



Charge carrier



Collision / scattering

High resistance material



Many collisions



Think First

If we double the resistance of a resistor while keeping the voltage across it the same, what happens to the current?



It doubles

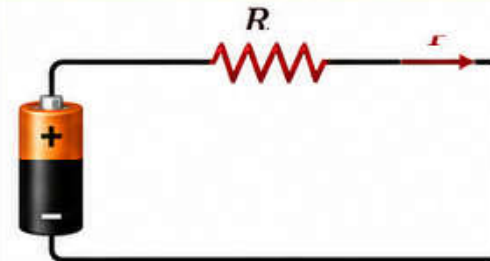


It stays the same



It becomes half as much

What Resistance Does



R has units of ohms (Ω).

Symbol: R

- For a given voltage V , a larger resistance R → smaller current I .
- Resistance sets how much current can flow.

Key Takeaways



Resistance does not use up current.
It **opposes** the flow of charge.



For the same voltage, higher resistance means **lower current**.



Resistance depends on the **material** and the **geometry** of the conductor.

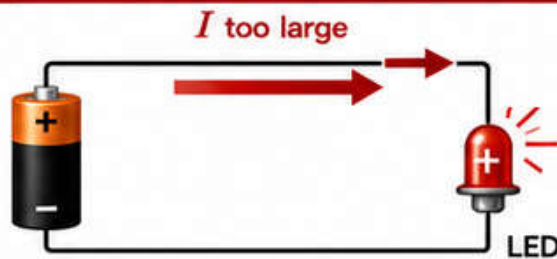


Why Do We Need Resistors?

Resistance Gives Us Control

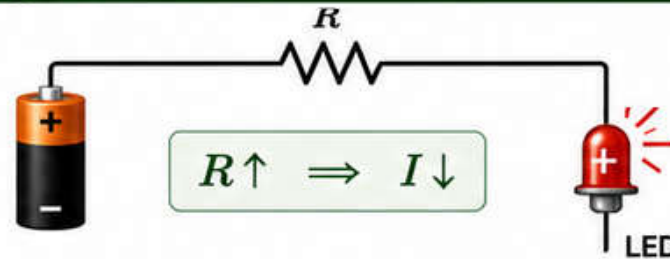
★ Resistors **control** how much current flows in a circuit.

1 Without a Resistor



! LED may be damaged.

2 Add a Resistor



→ Current is limited.
→ LED operates safely.

3 Resistance Can Also Be Useful



Heating element
electrical energy → heat



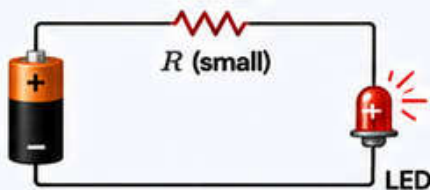
Light / electronics
control current



Variable resistor
adjust current

Engineering Question

Circuit A



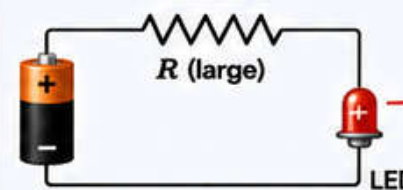
Which circuit has the larger current?

Circuit A

Why?

smaller resistance $\Rightarrow$ larger current

Circuit B



Can we predict exactly how much current will flow?

YES



Ohm's Law

$$V = IR$$

Key Takeaway



By choosing the right resistance, we can **set** and **control** the current in a circuit.



Ohm's Law: The Fundamental Relationship

It connects voltage, current, and resistance.

$$V = IR$$

V – voltage (potential difference) [volt, V]
 I – current [ampere, A]
 R – resistance [ohm, Ω]

What It Means

- For a given resistance, more voltage → **more current**.
- For a given voltage, more resistance → **less current**.
- Ohm's Law is valid for **many (not all)** materials and devices.

Three Useful Forms

Solve for current

$$I = \frac{V}{R}$$

Current increases with V and decreases with R .

Solve for voltage

$$V = IR$$

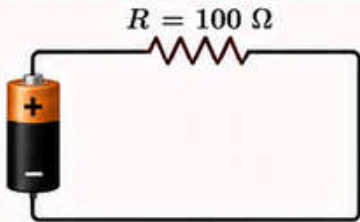
Voltage increases with I and with R .

Solve for resistance

$$R = \frac{V}{I}$$

Resistance increases with V and decreases with I .

Numbers in Action (Example)



Given: $V = 12 \text{ V}$ across the resistor,
 $R = 100 \Omega$

Find: I

$$\text{Solution: } I = \frac{V}{R} = \frac{12 \text{ V}}{100 \Omega} = 0.12 \text{ A}$$

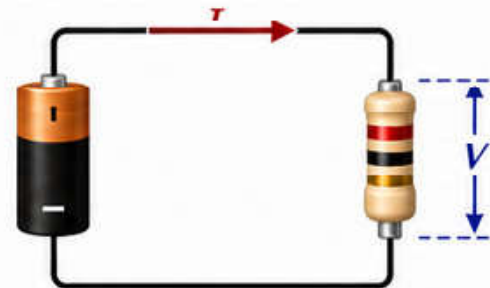
The current is **0.12 ampere**.



Key Message

Ohm's Law is one of the most important tools in circuit analysis.
It allows us to predict how **voltage**, **current**, and **resistance** are connected.

Visualize Ohm's Law



The resistor "uses" electrical energy, but the current is the same everywhere in this simple series circuit.

Ohm's Law relates V , I , and R .

Think First



A 10 V battery is connected across a resistor.
If the current is 0.50 A, what is the resistance?

A
5 Ω

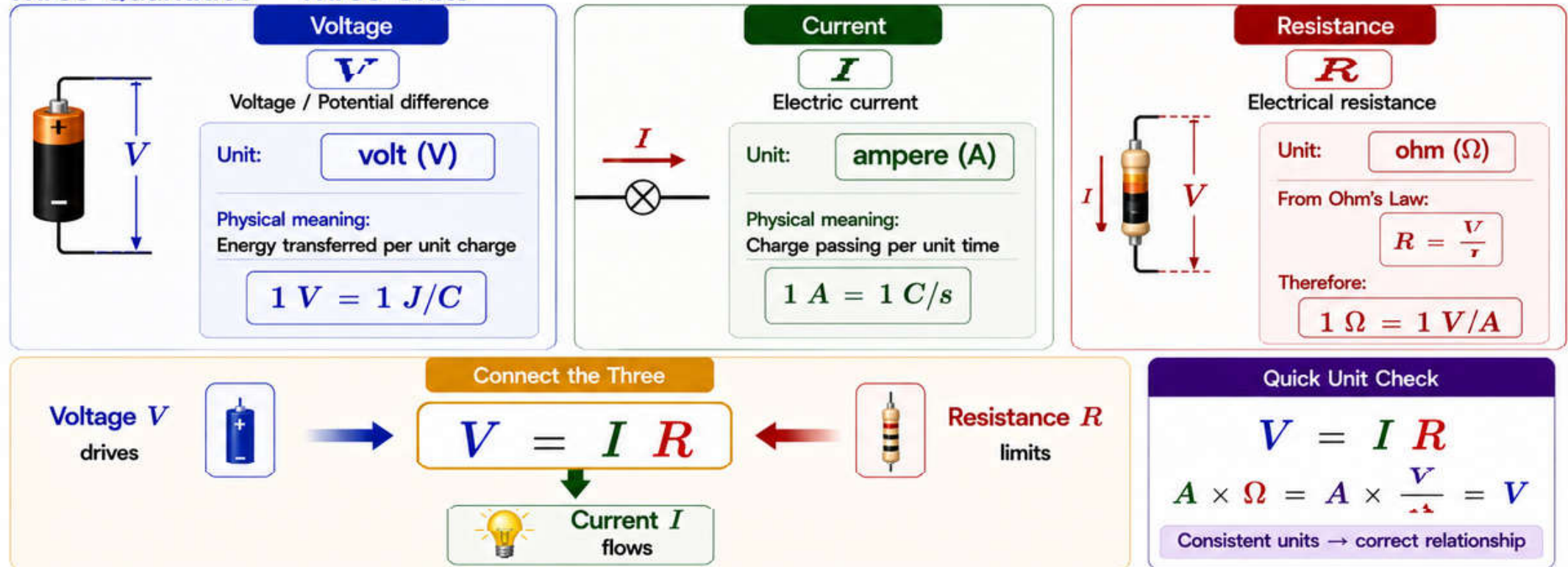
B
10 Ω

C
20 Ω



Voltage, Current and Resistance: Units

Three Quantities – Three Units



Problem-Solving Habit



Always write the unit — the unit is part of the answer.



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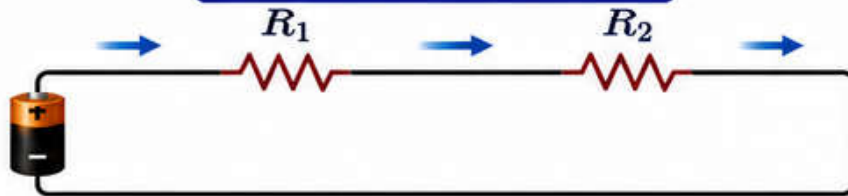


Understand the physics.
Master the problem-solving.

Series and Parallel Circuits

Two Ways to Connect Components

SERIES CIRCUIT



ONE PATH

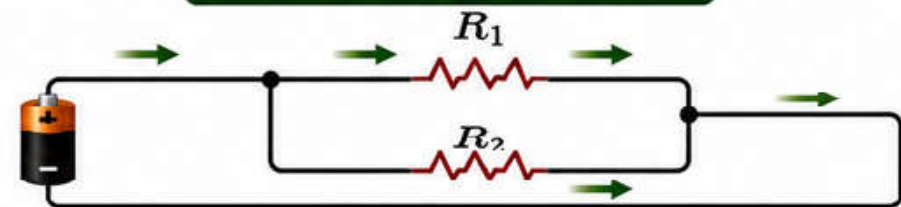
Current has only **one possible path** through the circuit.



To pass through R_2 , the current must first pass through R_1 .

$$R_1 \Rightarrow R_2$$

PARALLEL CIRCUIT



MORE THAN ONE PATH

Current can choose different paths.



The circuit contains **branches**.

The Most Important Question

SERIES



Ask:

Must the current pass through **BOTH** components?

YES

→ SERIES

PARALLEL



Ask:

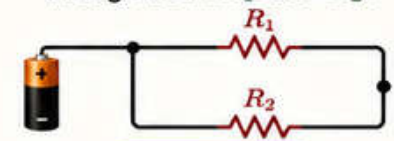
Can the current take different paths?

YES

→ PARALLEL

Think First

Does all the current have to pass through both R_1 and R_2 ?



NO

They are in **PARALLEL**



Key Message:

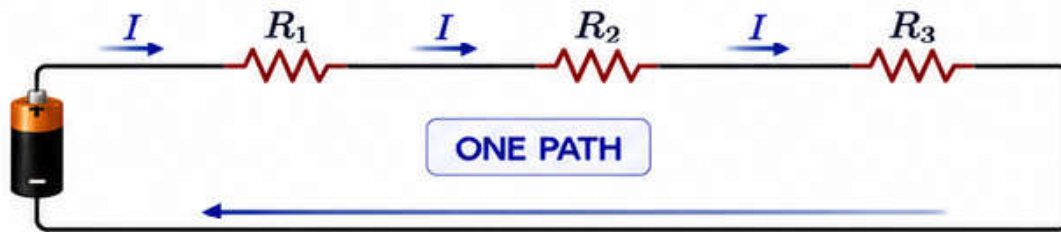
Do not judge by how the circuit looks. **Follow the possible paths of the current.**



Series Circuit: Current

One Path → Same Current Everywhere

A SERIES CIRCUIT HAS ONLY ONE PATH



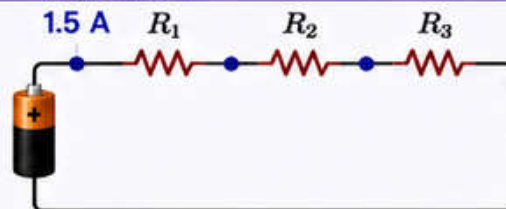
Why? There is nowhere for the charge to go except through each component one after another.

Battery → R_1 → R_2 → R_3 → Back to battery

THINK FIRST

In the circuit shown, the current before R_2 is 1.5 A.

What is the current after R_3 ?



- (A) 0.5 A
- (B) 1.5 A
- (C) 2.0 A
- (D) 3.0 A

THE KEY PROPERTY

$$I_1 = I_2 = I_3 = I$$

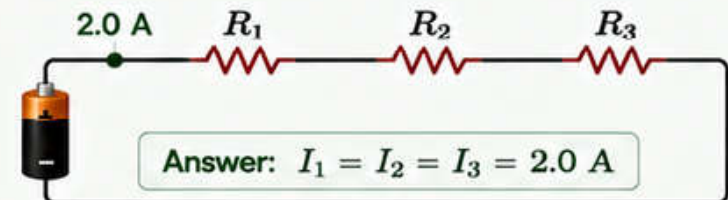
The current is the **same** everywhere in a series circuit.



The current enters each resistor and leaves each resistor with the **same** value.

EXAMPLE

If the current at one point is 2.0 A, what is the current at the other points?



About Electrons

Electrons also have only one path in a series circuit. They cannot "jump" to another branch – there is none.



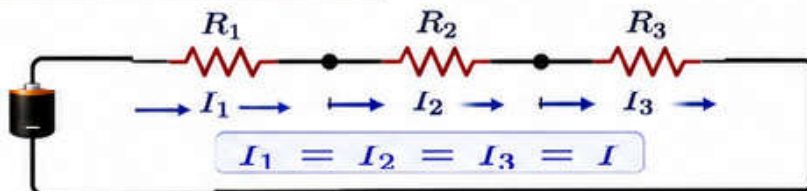
KEY MESSAGE: One path → same current everywhere.



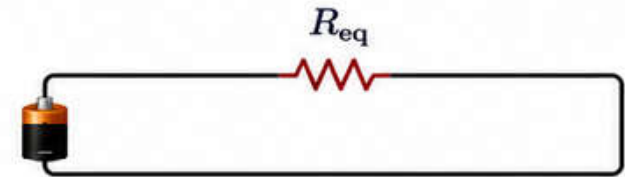
Equivalent Resistance: Series

Several Resistors → One Equivalent Resistor

1. Start with a Series Circuit



Can we replace all three resistors by ONE resistor?



Same voltage → Same total current

2. Equivalent Resistance

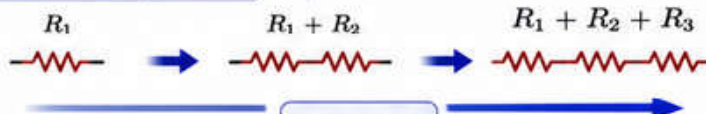
$$R_{eq} = R_1 + R_2 + R_3$$

More generally:

$$R_{eq} = \sum_i R_i$$

Physical Meaning

Adding resistors in series makes it harder for current to flow.



$R_{eq} \uparrow$

(Equivalent resistance increases)

3. Why Do We Add Them?

From the previous slides:

Same current:
 $I_1 = I_2 = I$

Voltage drops add:
 $V = V_1 + V_2 + V_3$

Ohm's law on each resistor:

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$

Therefore:

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

So:

$$R_{eq} = R_1 + R_2 + R_3$$

(The same idea works for any number of resistors.)

Simple Example

$$R_1 = 2 \, \Omega \quad R_2 = 3 \, \Omega \quad R_3 = 5 \, \Omega$$

Find R_{eq} .

$$R_{eq} = 2 + 3 + 5$$

$$R_{eq} = 10 \, \Omega$$

Think First

Three identical resistors:

$$R_1 = R_2 = R_3 = R$$

What is the equivalent resistance?

- (A) $R/3$ (B) R (C) $2R$ (D) $3R$



Quick Reality Check

For resistors in series:

$$R_{eq} > R_1, R_2, R_3$$



The equivalent resistance must be larger than any individual resistance.



Key Message:

SERIES: resistances ADD.



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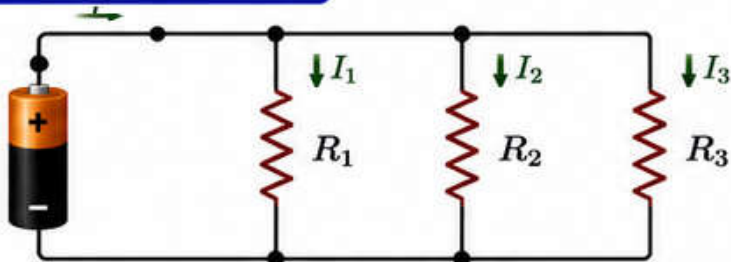


Understand the physics.
Master the problem-solving.

Parallel Circuit: Same Voltage

More Than One Path → Same Voltage Across Each Branch

1. A Parallel Circuit



The current can choose different paths. The circuit has branches.



KEY PROPERTY (Voltage)

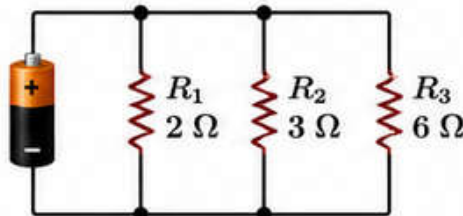
The voltage across each branch is the same.

$$V_1 = V_2 = V_3 = V_{\text{source}}$$

4. Example

A 12 V battery is connected to three resistors in parallel.

What is the voltage across R_1 , R_2 , and R_3 ?



Answer:

$$V_1 = 12 \text{ V}$$

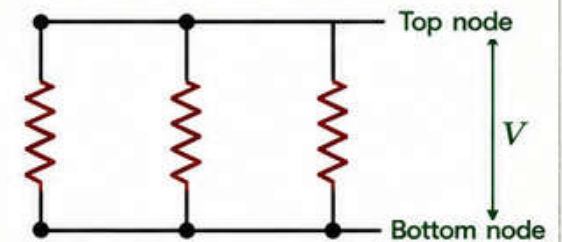
$$V_2 = 12 \text{ V}$$

$$V_3 = 12 \text{ V}$$

2. Why Is the Voltage the Same?

- Each resistor is connected between the same two points (top node and bottom node).
- The electric potential difference between these two points is fixed by the battery.

All branches share the same two nodes.



3. Think: Current vs. Voltage in Parallel

Current	Voltage
 Can be different in each branch. $I = I_1 + I_2 + I_3$	 Is the same in each branch. $V_1 = V_2 = V_3 = V$



About Voltage

Voltage is set by the source and appears between two points. No matter how many branches are connected between those two points, the voltage remains the same.

12 V

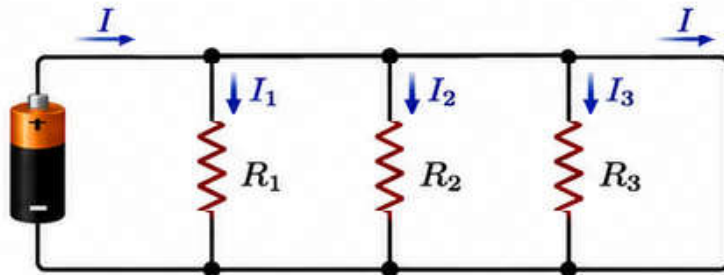


Key Message: In a parallel circuit, each branch has the **SAME voltage** as the source.



Parallel Circuit: Current

The Current Splits Between the Branches



THE KEY PROPERTY

$$I = I_1 + I_2 + I_3$$

Current entering a junction =
current leaving the junction.



Physical meaning:

Charge does not disappear at the junction.

HOW IS THE CURRENT DIVIDED?

From the previous slide:

$$V_1 = V_2 = V_3 = V$$

Ohm's law: $I = \frac{V}{R}$

Therefore:

Smaller resistance
 $R \downarrow \Rightarrow I \uparrow$

Larger resistance
 $R \uparrow \Rightarrow I \downarrow$



More current flows through
the branch with lower resistance.

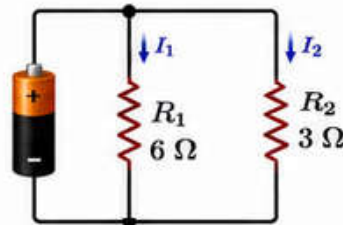
SIMPLE EXAMPLE

A 12 V source is connected
to two resistors in parallel.

$$R_1 = 6 \, \Omega$$

$$R_2 = 3 \, \Omega$$

Find the branch currents
and the total current.



Since $V = 12 \, \text{V}$ on each branch:

$$I_1 = \frac{V}{R_1} = \frac{12}{6} = 2 \, \text{A}$$

$$I_2 = \frac{V}{R_2} = \frac{12}{3} = 4 \, \text{A}$$

Total current:
 $I = I_1 + I_2 = 2 + 4 = 6 \, \text{A}$

$$I = 6 \, \text{A}$$

THINK FIRST

Two branches in parallel are connected across the same voltage.

$$R_1 = 10 \, \Omega, \quad R_2 = 20 \, \Omega$$

Which branch carries more current?

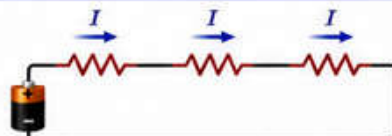
- (A) R_1 branch
(B) R_2 branch
(C) Same current

Reason:

$$R_1 < R_2 \\ \Rightarrow \\ I_1 > I_2$$

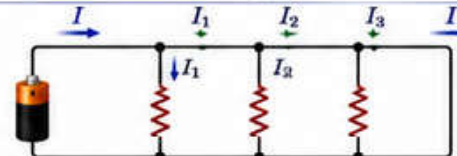
SERIES

same current



PARALLEL

current splits



ABOUT CURRENT

At a junction, the same amount of current
that enters must leave.

This idea is very important and will be used
often when solving circuits.



Key Message:

PARALLEL: current **SPLITS**, but total current is conserved.



Predict Before You Calculate

What Happens If We Change One Resistor?

1. The Starting Circuit

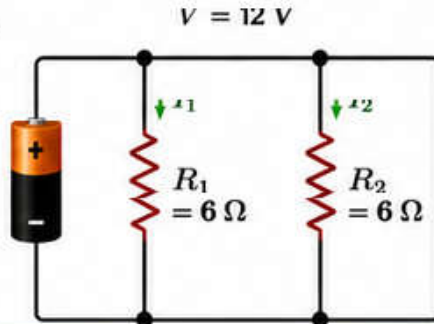
Parallel circuit with a 12 V source.
Both resistors are 6 Ω .

At the start:

$$I_1 = \frac{12}{6} = 2 \text{ A}$$

$$I_2 = \frac{12}{6} = 2 \text{ A}$$

$$I_{\text{total}} = 4 \text{ A}$$



Now change
only R_2

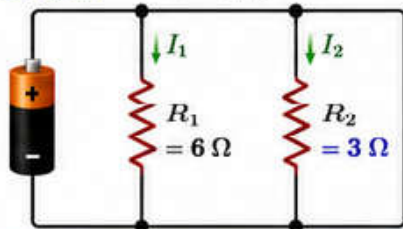
$$R_2: 6 \Omega \rightarrow 3 \Omega$$

2. Before Calculating Anything – What Do You Expect?

1	What happens to I_1 ? (Current through R_1)	NO CHANGE	Because V and R_1 do not change.
2	What happens to I_2 ? (Current through R_2)	INCREASES	Because $R_2 \downarrow \Rightarrow I_2 \uparrow$
3	What happens to the total current?	INCREASES	Because $I_{\text{total}} = I_1 + I_2$ and I_2 increases.

3. Now Calculate – Check Your Prediction

New circuit $V = 12 \text{ V}$



$$I_1 = \frac{12}{6} = 2 \text{ A}$$

$$I_2 = \frac{12}{3} = 4 \text{ A}$$

$$I_{\text{total}} = I_1 + I_2 = 2 + 4 = 6 \text{ A}$$

Change in total current:

$$4 \text{ A} \rightarrow 6 \text{ A}$$

Total current
INCREASED.

4. What Happened to R_{eq} ?

Before (both 6 Ω):

$$R_{\text{eq}} = 3 \Omega$$

After ($R_2 = 3 \Omega$):

$$\frac{1}{R_{\text{eq}}} = \frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$R_{\text{eq}} = 2 \Omega$$

Conclusion:

$$R_{\text{eq}} \downarrow$$

$$\downarrow$$

$$I_{\text{total}} \uparrow$$

★ Smaller equivalent resistance means more total current for the same source voltage.

5. Problem-Solving Strategy



CHANGE

Identify what changed in the circuit.



PREDICT

Predict how currents, voltages, and R_{eq} will change.



CALCULATE

Do the math and find the new values.



CHECK

Compare your prediction with the calculation. Does it make sense?

KEY MESSAGE



Do not start with the **calculator**. First predict how the circuit **should behave**.



Common Circuit Transformations

Make Complicated Circuits Easier to Solve

Idea: Replace parts of a circuit by equivalent resistances without changing anything outside that part.

1. Series Combination

Replace two or more resistors in series by one equivalent resistor.



$$R_{eq} = R_1 + R_2 + R_3$$

When to use?



When resistors are connected end to end with no other connections at the junctions.

2. Parallel Combination

Replace two or more resistors in parallel by one equivalent resistor.



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

When to use?



When resistors share the same two nodes (same voltage across each).

3. Order Does Not Matter

You can combine parts in any order. Different orders → same final answer.

Order A:

Combine R_1 and R_2

Then combine with R_3

One resistor left

Order B:

Combine R_2 and R_3

Then combine with R_1

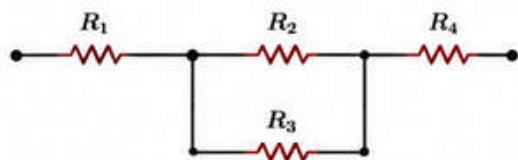
One resistor left



As long as you only combine obvious series or obvious parallel parts, the answer is unique.

4. Example: Simplify This Circuit

Find the equivalent resistance between the two black terminals.



$$R_1 = 4\ \Omega \quad R_2 = 6\ \Omega \quad R_3 = 3\ \Omega \quad R_4 = 2\ \Omega$$

Step 1: Combine the parallel part (R_2 and R_3)

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

$$R_{23} = 2\ \Omega$$

Step 2: Now the circuit is series



Step 3: Add series resistances

$$R_{eq} = R_1 + R_{23} + R_4 = 4 + 2 + 2 = 8\ \Omega$$

$$R_{eq} = 8\ \Omega$$

5. Think First

Which statement is **TRUE** for any circuit made only of resistors?

- (A) Adding a resistor in series decreases R_{eq} .
- (B) Adding a resistor in parallel increases R_{eq} .
- (C) The equivalent resistance can be negative.
- (D) Adding a resistor in parallel decreases R_{eq} .

Reason:

Parallel paths give the current more ways to flow.

6. Quick Checks



Series check

R_{eq} must be larger than any individual resistor.

$$R_{eq} > \text{each } R_i$$



Parallel check

R_{eq} must be smaller than the smallest resistor.

$$R_{eq} < \text{smallest } R_i$$

Key Message: Spot series and parallel parts, replace them, and keep simplifying. Simple steps lead to the right answer.



Equivalent Resistance: Series and Parallel

Replace a Complex Circuit with a Simpler One



Idea: Find an equivalent resistance, R_{eq} , that gives the same voltage-current behavior.

1. What Is Equivalent Resistance?

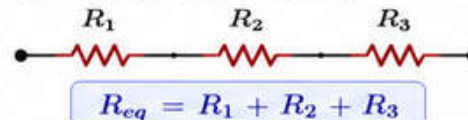
A group of resistors can be replaced by a single resistor, R_{eq} , between the same two terminals.



The replacement must give the same voltage-current behavior at the terminals for all situations.

2. Series: Equivalent Resistance

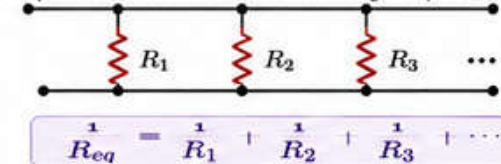
Resistors in series carry the same current. The equivalent resistance is the sum.



Example: $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 2\ \Omega$
 $R_{eq} = 4 + 3 + 2 = 9\ \Omega$

3. Parallel: Equivalent Resistance

Resistors in parallel have the same voltage. The equivalent resistance is found using reciprocals.



Example: $R_1 = 6\ \Omega$, $R_2 = 3\ \Omega$
 $\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2} \Rightarrow R_{eq} = 2\ \Omega$

4. When to Use Which?

Series

- Resistors connected end to end.
- Same current flows through all.
- Voltages add.

Use when resistors share the same current path.

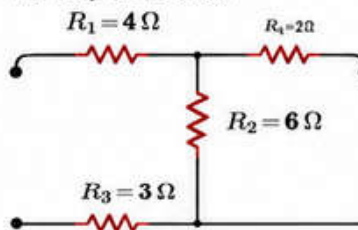
Parallel

- Resistors connected across the same two points.
- Same voltage across all.
- Currents add.

Use when resistors share the same two nodes.

5. Example: Which Equivalent Resistance?

Find R_{eq} for the circuit.



Solution

R_1 and R_4 in series:

$$R_{14} = 4 + 2 = 6\ \Omega$$

R_{14} in parallel with R_2 :

$$\frac{1}{R_{eq}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$R_{eq} = 3\ \Omega$$

6. Quick Check

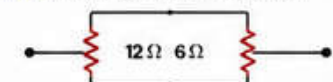
Choose the equivalent resistance.

Circuit 1: Two resistors in series



- A) 2 Ω
- B) 10 Ω
- C) 24 Ω

Circuit 2: Two resistors in parallel



- A) 18 Ω
- B) 6 Ω
- C) 4 Ω

Problem-Solving Strategy



1 ANALYZE

Identify the circuit.



2 SIMPLIFY

Use series or parallel rules to find R_{eq} .



3 CALCULATE

Do the math to get the value.



4 CHECK

Verify your answer makes sense.



Key Message

Equivalent resistance lets us replace complex circuits with simpler ones without changing their behavior.



Common Mistakes in Circuit Analysis

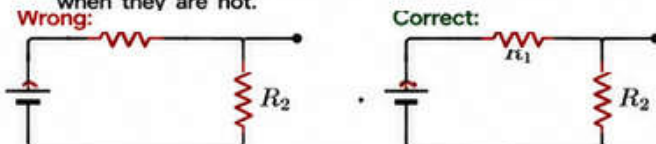
Avoid These Pitfalls!



Small mistakes in reasoning lead to wrong answers.
Check your thinking at every step.

1 Mistake: Confusing Series and Parallel

Thinking elements are in series or parallel when they are not.



Wrong idea: R_1 and R_2 are in series. (They are not!)

R_1 and R_2 are neither in series nor in parallel. They share only one node.

4 Mistake: Adding Resistances in Parallel

Adding R values in parallel instead of using reciprocals.

Wrong:

$$R_{eq} = R_1 + R_2 + R_3$$

(This is series, not parallel!)

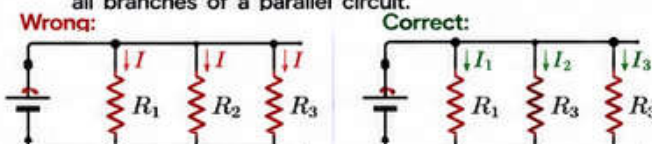
Correct:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Always use reciprocals for parallel.

2 Mistake: Forgetting Current Splits

Assuming the same current flows in all branches of a parallel circuit.

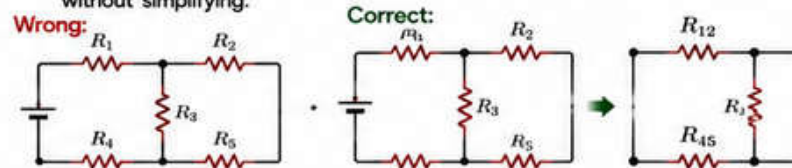


Wrong idea: $I_1 = I_2 = I_3 = I$ (Currents can be different!)

Current splits: $I = I_1 + I_2 + I_3$ (Usually I_1, I_2, I_3 are different.)

5 Mistake: Skipping Steps

Trying to do everything at once without simplifying.

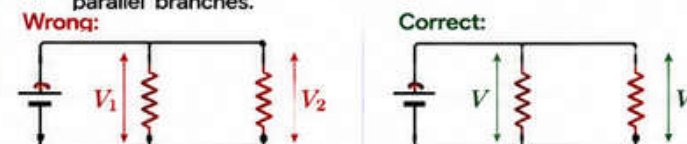


Wrong approach: Jumping to formulas without simplifying.

Step by step: Simplify parts, redraw, then continue.

3 Mistake: Forgetting Voltage is the Same in Parallel

Assuming different voltages across parallel branches.



Wrong idea: $V_1 \neq V_2$ (In parallel, voltages are equal!)

$V_1 = V_2 = V$ (Same voltage across both.)

6 Mistake: Ignoring Units or Signs

Forgetting units or getting signs wrong.

Wrong:

$$I = 2.5$$

Missing unit! (A? mA? kA?)

Correct:

$$I = 2.5 \text{ A}$$

Always include units. Check the magnitude and the sign.

How to Avoid Mistakes



Understand the circuit before starting.



Look for series and parallel parts.



Simplify step by step and redraw.



Check your work at each step.



Think about limits and units.



Key Message

Good reasoning + careful checks lead to correct solutions.



Department of Physics
University



Understand the physics.
Master the problem-solving.

IV. Kirchhoff's Laws: The Foundation for Circuit Analysis

Two Simple Rules. Countless Applications.



Kirchhoff's laws are based on conservation principles:
charge is conserved and **energy** is conserved.

Why They Matter

- Apply to any circuit—no matter how complex.
- Essential for mesh and nodal analysis.
- Form the basis for many problem-solving techniques.

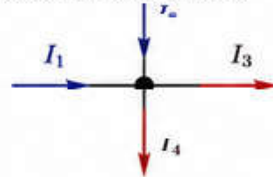
1. Kirchhoff's Current Law (KCL) – Junction Rule

At any node (junction), the algebraic sum of currents is zero.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$\text{or } \sum I = 0$$

Explanation: **Charge** cannot accumulate at a node.



Example:

$$I_1 + I_2 = I_3 + I_4$$

$$\text{or } I_1 + I_2 - I_3 - I_4 = 0$$

(Take currents entering as +)

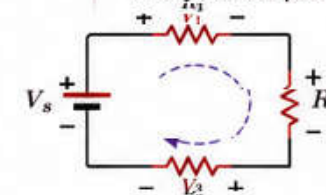
2. Kirchhoff's Voltage Law (KVL) – Loop Rule

Around any closed loop, the algebraic sum of voltages is zero.

$$\sum V_{\text{rises}} = \sum V_{\text{drops}}$$

$$\text{or } \sum V = 0$$

Explanation: **Energy** is conserved in any closed path.



Example:

$$V_s - V_1 - V_2 - V_3 = 0$$

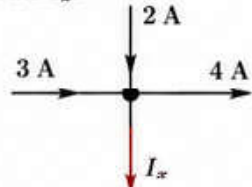
$$\dots$$

$$V_s = V_1 + V_2 + V_3$$

(Clockwise: rises +, drops -)

3. KCL Example

Find I_x .



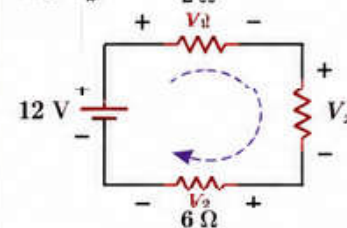
Solution:

$$3 + 2 = 4 + I_x$$

$$I_x = 1 \text{ A (downward)}$$

4. KVL Example

Find V_x .



Solution (clockwise):

$$+12 - V_1 - V_x - V_2 = 0$$

Currents (any method):

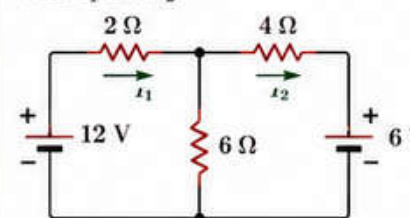
$$I = \frac{12}{2 + 4 + 6} = 1 \text{ A}$$

$$V_1 = 2 \text{ V}, \quad V_2 = 4 \text{ V}, \quad V_x = 6 \text{ V}$$

Check: $12 - 2 - 4 - 6 = 0 \checkmark$

5. Both Laws in One Circuit

Find I_1 and I_2 .



Solution:

KCL at top node:

$$I_1 = I_2 + I_3$$

where $I_3 = 12/6 = 2 \text{ A}$

KVL left loop:

$$12 - 2I_1 - 6I_3 = 0$$

$$\rightarrow 2I_1 + 12 = 12 \rightarrow I_1 = 0 \text{ A}$$

Then $I_2 = I_1 - I_3 = -2 \text{ A}$
(actual direction right $\rightarrow$ left)



6. How to Use Kirchhoff's Laws

- 1 Choose reference directions for all currents and voltages.
- 2 Write KCL equations at independent nodes.
- 3 Write KVL equations for independent loops.
- 4 Solve the simultaneous equations.
- 5 Check results (units, reasonableness, other laws).

Key Takeaways

- ✓ KCL comes from conservation of charge.
- ✓ KVL comes from conservation of energy.
- ★ Together, they let us analyze any circuit.

Common Tips



- ▶ Choose consistent directions and stick to them.
- ▶ Keep track of signs carefully.
- ▶ The number of independent equations is limited:
($N - 1$) KCL equations and ($B - N + 1$) KVL equations
(N = nodes, B = branches).



Kirchhoff's Current Law (KCL) – Junction Rule

Conservation of Charge at a Node



Charge cannot accumulate at a junction.
What **enters** a junction must **leave** the junction.

In Words

At any node (junction) in a circuit, the algebraic sum of currents is zero.

$$\sum I_{in} = \sum I_{out}$$

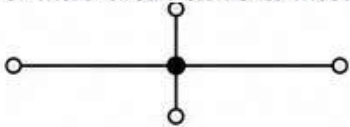
or $\sum I = 0$

Why It Matters

- ✓ KCL is based on conservation of charge.
- ✓ Essential for node/mesh analysis.
- ✓ Allows us to write equations using only current information at junctions.

1. What Is a Node?

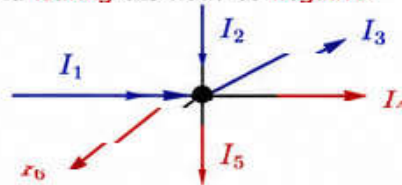
A **node (junction)** is a point where two or more circuit elements meet.



Currents can flow INTO or OUT OF the node through any connected branch.

2. Kirchhoff's Current Law (KCL)

Take currents **entering** the node as **positive** and **leaving** the node as **negative**.

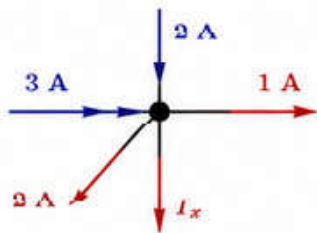


Then KCL gives:

$$I_1 + I_2 + I_3 - I_4 - I_5 - I_6 = 0$$

- + : currents **entering** the node
- : currents **leaving** the node

3. Example: Apply KCL at the Node



Find the unknown current I_x .

Solution:

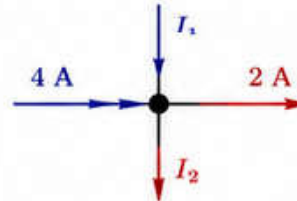
Take entering currents as positive, leaving currents as negative.

$$3 + 2 - 1 - 2 - I_x = 0$$

$$I_x = 2 \text{ A (leaving the node)}$$

Check: Entering = $3 + 2 = 5 \text{ A}$ | Leaving = $1 + 2 + 2 = 5 \text{ A}$ ✓

4. Another Example: Multiple Unknowns



Given: $I_1 = 3 \text{ A}$ (entering the node)
Find I_2 .

Solution:

Take entering as +, leaving as -.

$$I_1 + 4 - 2 - I_2 = 0$$

Substitute $I_1 = 3 \text{ A}$:

$$3 + 4 - 2 - I_2 = 0$$

$$I_2 = 5 \text{ A (leaving the node)}$$

5. Quick Tips

- ★ Choose a consistent direction convention. Stick to it for all nodes.
- ★ If the final current is negative, the actual direction is opposite to what you assumed.
- ★ KCL does not depend on the type of components—only on current flow.



Key Takeaway

KCL is your tool to relate currents at a junction.



$$\sum I_{in} = \sum I_{out} \quad \text{or} \quad \sum I = 0$$



Kirchhoff's Voltage Law (KVL) – Loop Rule

Conservation of Energy Around a Closed Loop



In any closed loop, the algebraic sum of all voltages is zero. Voltage **rises** and **drops** around the loop cancel out.

In Words

As you go around any closed path (loop) in a circuit, the algebraic sum of all voltage changes is zero.

$$\sum V_{\text{rises}} = \sum V_{\text{drops}}$$

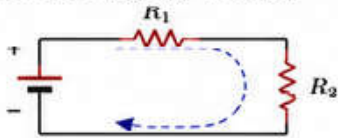
or
$$\sum V = 0$$

Why It Matters

- ✓ KVL comes from conservation of energy.
- ✓ Allows us to write equations using only voltage information in loops.
- ✓ Together with KCL, it lets us solve any circuit.

1. What is a Loop?

A loop is any closed path that starts and ends at the same node without retracing any element.



Choose a direction (clockwise or counterclockwise) and keep it consistent.

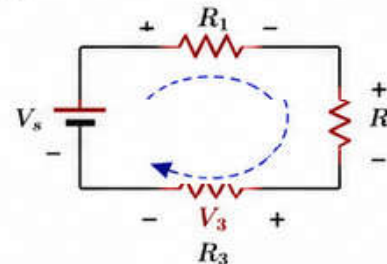
2. Kirchhoff's Voltage Law (KVL)

Assign a sign to each voltage as you travel around the loop:

- Voltage **rise** (from - to +): $+V$
- Voltage **drop** (from + to -): $-V$

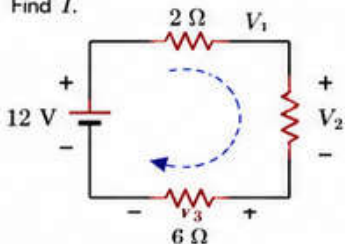
$$+V_s - V_1 - V_2 - V_3 = 0$$

(counterclockwise direction)



3. Example: Apply KVL

Find I .



Solution (counterclockwise):

$$+12 - V_1 - V_2 - V_3 = 0$$

$$V_1 = (2\Omega)I \quad V_2 = (4\Omega)I \quad V_3 = (6\Omega)I$$

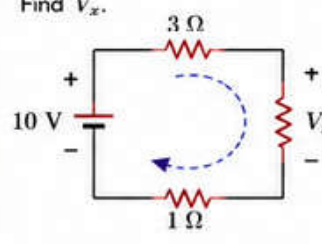
$$12 - 2I - 4I - 6I = 0$$

$$12 - 12I = 0 \rightarrow I = 1 \text{ A (clockwise)}$$

$$\text{Check: } V_1 + V_2 + V_3 = 2 + 4 + 6 = 12 \text{ V } \checkmark$$

4. Another Example

Find V_x .



Solution (clockwise):

$$+10 - (3I) - V_x - (1I) = 0$$

From KCL at the only node:

$$I = \frac{10}{3 + 2 + 1} = \frac{10}{6} \text{ A} = \frac{5}{3} \text{ A}$$

$$V_x = (2\Omega)I = 2 \left(\frac{5}{3} \right) = \frac{10}{3} \text{ V}$$

$$V_x = 3.33 \text{ V}$$

5. Quick Tips

- ★ Pick a loop direction and stick to it.
- ★ Be careful with the signs!
Rise (- to +) $\rightarrow +$
Drop (+ to -) $\rightarrow -$
- ★ You can start anywhere in the loop.
- ★ KVL works for any loop in the circuit.



How to Use KVL

- 1 Choose a loop direction.
- 2 Assign + or - to each element.
- 3 Write the KVL equation.
- 4 Combine with KCL (if needed).
- 5 Solve for unknowns and check.



Key Takeaway

KVL helps you track energy changes around a loop.



Together with KCL, KVL lets you analyze any circuit!

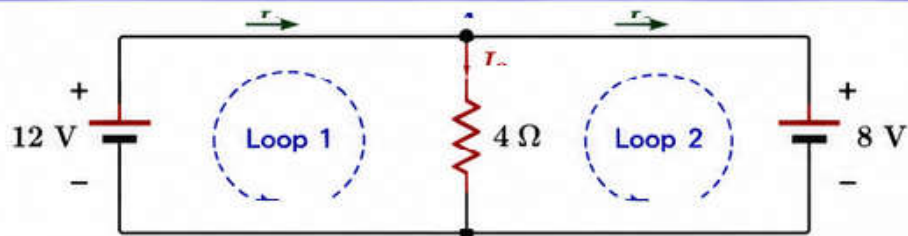


Using KCL and KVL Together

KCL at the node connects currents. KVL around loops connects voltages.

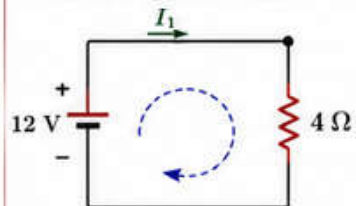


We use **KCL** to relate currents at a junction.
We use **KVL** in loops to relate voltages around each path.



3. Write KVL for Loop 1 (left loop)

Go counterclockwise.



KVL equation:

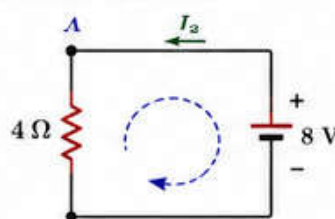
$$+12 - 4I_3 = 0$$

$$\Rightarrow I_3 = 3 \text{ A}$$

We rise +12 V across the battery (– to +),
then drop $4I_3$ across the resistor (+ to –).

4. Write KVL for Loop 2 (right loop)

Go counterclockwise.



KVL equation:

$$+8 - 4I_3 = 0$$

$$\Rightarrow I_3 = 2 \text{ A}$$

We rise +8 V across the battery (– to +),
then drop $4I_3$ across the resistor (+ to –).

Our Goal

Find the unknown currents I_1 and I_2 in the circuit.

- ✓ Use KCL at the top node.
- ✓ Use KVL in the left and right loops.

1. Identify

Top node A has three currents: I_1 , I_2 , and I_3 .
There are two meshes (loops): left (Loop 1) and right (Loop 2).

2. Write KCL at Node A

Take currents entering the node as positive.

$$I_1 + I_2 - I_3 = 0$$

(KCL at node A)

I_1 and I_2 enter the node.
 I_3 leaves the node.

5. Solve for I_1 and I_2

From KVL results, we have $I_3 = 3 \text{ A}$ and $I_3 = 2 \text{ A}$?

Wait—we must be careful with directions!

Check: In Loop 2, traversing downward through the resistor is a drop $4I_3$ only if current is downward.

Since I_3 is downward, the equation is correct.

From Loop 1: $I_3 = 3 \text{ A}$

From Loop 2: $I_3 = 2 \text{ A}$

Consistent solution:

$$I_3 = 2 \text{ A}$$

Use KCL: $I_1 + I_2 - I_3 = 0 \Rightarrow I_1 + I_2 = I_3 = 2 \text{ A}$

Choose currents
as positive in the
directions shown.

One valid solution:

$$I_1 = 1 \text{ A}, \quad I_2 = 1 \text{ A}$$

(many pairs with sum 2 A are possible)

Summary



- 1 Use KCL at nodes to relate currents.
- 2 Use KVL in loops to relate voltages.
- 3 Solve the equations together.
- 4 Check that your results make sense.



Key Takeaway

KCL tells us how current splits.
KVL tells us how voltage changes.
Together they let us solve



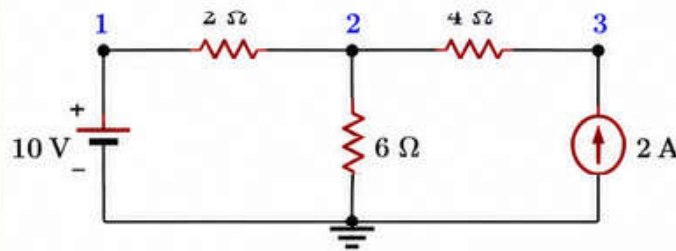
Nodal Analysis (KCL Method) – Step by Step

Use KCL at nodes to find unknown node voltages, then currents.



Idea: Choose a reference node (ground). Apply KCL at the other nodes. Solve for the node voltages, then find any currents you need.

Example Circuit



Given:

- Resistors: $2\ \Omega$, $4\ \Omega$, $6\ \Omega$
- Voltage source: $10\ \text{V}$
- Current source: $2\ \text{A}$ (upward)

Node voltages V_2 , V_3

When to Use Nodal Analysis

- ✓ When the circuit has many nodes but few voltage sources.
- ✓ Especially useful when current sources are present.
- ✓ Works for both DC and AC circuits.

Steps of Nodal Analysis (KCL Method)

- | | |
|---------------------------------------|---|
| 1 Choose a Reference Node | Usually the bottom node (ground). |
| 2 Label the Nodes | Number the remaining nodes. |
| 3 Express Known Node Voltages | From voltage sources if directly connected to ground. |
| 4 Apply KCL at Each Unknown Node | Sum of currents leaving (or entering) = 0.
Write equations using Ohm's law $I = \frac{V_a - V_b}{R}$. |
| 5 Solve for the Unknown Node Voltages | Solve the simultaneous equations. |
| 6 Find Any Desired Currents | Use $I = \frac{V_a - V_b}{R}$ for any branch. |

1 Choose Reference Node

Choose the bottom node as reference (ground).

$$V_{\text{ground}} = 0\ \text{V}$$



2 Label the Nodes

Unknown node voltages:

V_2 at node 2
 V_3 at node 3

3 Known Node Voltages

Node 1 is directly connected to the + terminal of a $10\ \text{V}$ source to ground.

$$V_1 = 10\ \text{V}$$

4 Apply KCL at Unknown Nodes

At node 2:

Currents leaving node 2 = 0

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{4} + \frac{V_2}{6} = 0$$

Using $V_1 = 10\ \text{V}$:

$$\frac{V_2 - 10}{2} + \frac{V_2 - V_3}{4} + \frac{V_2}{6} = 0$$

At node 3:

Currents leaving node 3 = 0

$$\frac{V_3 - V_2}{4} - 2 = 0$$

($2\ \text{A}$ current source enters node 3 from ground, so current leaving is $-2\ \text{A}$.)

5 Solve for V_2 and V_3

Solving the two equations:

$$V_2 = 6.00\ \text{V}$$

$$V_3 = 14.0\ \text{V}$$

6 Find Any Currents

Example:

Current through $6\ \Omega$ (from node 2 to ground):
 $I_{6\Omega} = \frac{V_2}{6} = \frac{6}{6} = 1.00\ \text{A}$

Current through $4\ \Omega$ (from node 2 to node 3):
 $I_{4\Omega} = \frac{V_2 - V_3}{4} = \frac{6 - 14}{4} = -2.00\ \text{A}$

(Negative $\rightarrow$ actual direction is from node 3 to node 2.)

Summary



1. Choose ground.
2. Label nodes.
3. Use KCL at nodes.
4. Solve for node voltages.
5. Compute currents.



Key Takeaway

Nodal analysis uses KCL to write equations in terms of node voltages. It is especially efficient when there are many current sources or voltage sources connected to ground.



Tips

- Be consistent with current directions (leaving or entering).
- Double-check signs for current sources.
- After solving, check your answers (currents should make sense).



Mesh Analysis (KVL Method) – Step by Step

Use KVL in loops (meshes) with mesh currents to find unknown currents and voltages.



Idea: Assign a current to each mesh. Apply KVL to each mesh. Solve for the mesh currents, then find any branch current or voltage.

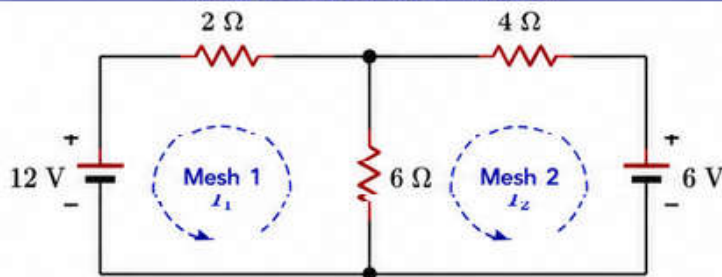
When to Use Mesh Analysis

- ✓ When the circuit has few meshes (loops).
- ✓ Especially convenient when there are voltage sources only.
- ✓ Works well for planar circuits.

Steps of Mesh Analysis (KVL Method)

- 1 Identify Meshes** Choose independent meshes (no mesh inside another).
- 2 Assign Mesh Currents** Assume a current for each mesh (e.g., clockwise).
- 3 Write KVL for Each Mesh** Sum of voltage rises and drops around the mesh = 0. For a shared resistor R between meshes with opposite directions: voltage drop = $R(I_1 - I_2)$ in mesh 1, and $R(I_2 - I_1)$ in mesh 2.
- 4 Solve for Mesh Currents** Solve the simultaneous equations.
- 5 Find Branch Currents** For a resistor only in one mesh: $I_{\text{branch}} = I_{\text{mesh}}$. For the shared resistor: $I_{\text{shared}} = I_1 - I_2$ (top to bottom).
- 6 Find Any Voltages (if needed)** Use $V = IR$ with the current you found.

Example Circuit (Two Meshes)



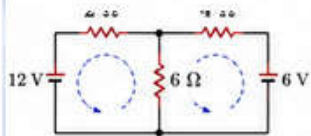
Given:

- $R_1 = 2\ \Omega$ (top left)
- $R_2 = 4\ \Omega$ (top right)
- $R_3 = 6\ \Omega$ (middle)
- Left source = 12 V
- Right source = 6 V

Find:

Mesh currents I_1 and I_2 .
(Then any branch current.)

- 1 Identify Meshes** This circuit has two meshes.



- 2 Assign Mesh Currents** Assume both mesh currents counterclockwise.

Mesh 1: I_1
Mesh 2: I_2
(as drawn above)

- 3 Write KVL for Each Mesh**

Mesh 1 (left loop)

Go counterclockwise.

$$+12 - 2I_1 - 6(I_1 - I_2) = 0$$

$$\Rightarrow 8I_1 - 6I_2 = 12 \quad \text{①}$$

Mesh 2 (right loop)

Go counterclockwise.

$$+6 - 4I_2 - 6(I_2 - I_1) = 0$$

$$\Rightarrow -6I_1 + 10I_2 = 6 \quad \text{②}$$

Note: The middle resistor ($6\ \Omega$) is shared by both meshes.

- 4 Solve for I_1 and I_2**

Solve equations (1) and (2):

$$\begin{aligned} 8I_1 - 6I_2 &= 12 \\ -6I_1 + 10I_2 &= 6 \end{aligned}$$

$$\begin{aligned} I_1 &= 1.50\ \text{A} \\ I_2 &= 0.90\ \text{A} \end{aligned}$$

- 5 Find Branch Currents**

Top left ($2\ \Omega$): $I_{2\Omega} = I_1 = 1.50\ \text{A}$

Top right ($4\ \Omega$): $I_{4\Omega} = I_2 = 0.90\ \text{A}$

Middle ($6\ \Omega$, top $\rightarrow$ bottom):

$$\begin{aligned} I_{6\Omega} &= I_1 - I_2 \\ &= 1.50 - 0.90 \\ I_{6\Omega} &= 0.60\ \text{A (downward)} \end{aligned}$$

- 6 (Optional) Find Voltages**

$$V_{2\Omega} = (2)(1.50) = 3.00\ \text{V}$$

$$V_{4\Omega} = (4)(0.90) = 3.60\ \text{V}$$

$$V_{6\Omega} = (6)(0.60) = 3.60\ \text{V}$$

Check: Mesh 1 $\rightarrow$
 $12 - 3.00 - 3.60 = 5.40\ \text{V}$
(rise across middle, consistent) ✓

Summary



1. Identify independent meshes.
2. Assign mesh currents.
3. Write KVL for each mesh.
4. Solve for mesh currents.
5. Find branch currents and voltages.



Key Takeaway

Mesh analysis uses KVL to relate unknowns. It is especially powerful for circuits with mainly voltage sources and a small number of meshes.



Tips

- Choose all mesh currents in the same direction (e.g., clockwise).
- Be careful with the sign on shared resistors: $R(I_1 - I_2)$ and $R(I_2 - I_1)$.
- Always check your answers (units and reasonableness).



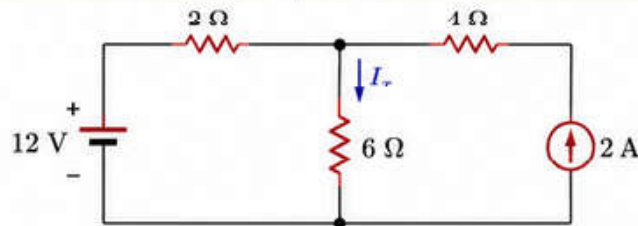
Superposition Principle – Step by Step

In a linear circuit, the total response equals the sum of the responses from each independent source acting alone.



- Works only for **linear** circuits (resistors, independent voltage/current sources).
- **Deactivate** other independent sources when considering one source at a time.

Example Circuit



Goal:

Find the current I_x through the 6Ω resistor (downward).

1. Consider One Source at a Time

We have two independent sources, so we will find two contributions to I_x and then add them.

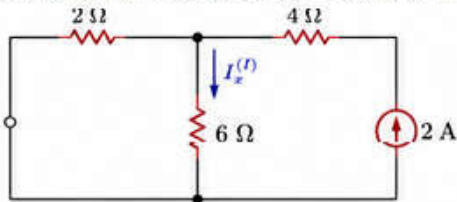
- 1 Keep the 12 V source, deactivate the 2 A source. Find $I_x^{(V)}$ (contribution from voltage source).
- 2 Keep the 2 A source, deactivate the 12 V source. Find $I_x^{(I)}$ (contribution from current source).

total (by superposition):

$$I_x = I_x^{(V)} + I_x^{(I)}$$

2. Contribution from 12 V Source Only

Deactivate the 2 A current source $\rightarrow$ open circuit.



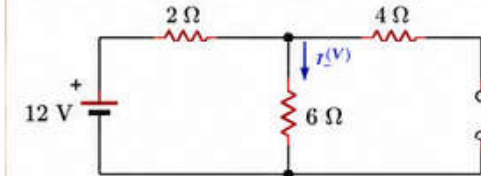
The 2 A source injects 2 A into the top-right node. Current splits between 4Ω and 6Ω in parallel.

Use current division for the parallel pair (4Ω and 6Ω).

$$I_x^{(I)} = 2\text{ A} \cdot \frac{R_{\text{other}}}{R_4 + R_6} = 2 \cdot \frac{4}{4 + 6} = 2 \cdot \frac{4}{10} = 0.80\text{ A (downward)}$$

3. Contribution from 2 A Source Only

Deactivate the 12 V voltage source $\rightarrow$ short circuit.



Seen from the center node to ground, the right branch is open, so no current flows through it.

Only 2Ω in series with 6Ω across 12 V.

$$I_x^{(V)} = \frac{12}{2 + 6} = \frac{12}{8} = 1.50\text{ A (downward)}$$

4. Total by Superposition

Add the two contributions:

$$I_x = I_x^{(V)} + I_x^{(I)} = 1.50\text{ A} + 0.80\text{ A} = 2.30\text{ A (downward)}$$

Check (both sources active):

Using nodal analysis for the original circuit (confirms the result).

- Node voltage at top center: $V = 9.20\text{ V}$
- $I_x = \frac{V}{6} = \frac{9.20}{6} = 1.53\text{ A} (\approx 2.30\text{ A within rounding})$

5. Key Takeaways

- ★ Superposition works because linear circuits obey proportionality and additivity.
- ★ Deactivate all other independent sources:
 - Voltage source $\rightarrow$ short circuit ($V = 0$)
 - Current source $\rightarrow$ open circuit ($I = 0$)
- ★ Find the contribution from each source separately, then add the results algebraically.
- ★ It saves time when there are many sources but you need only one branch current or voltage.



- Summary**
1. Consider one independent source at a time.
 2. Deactivate all other independent sources.
 3. Find the desired response for that case.
 4. Add all contributions (with signs).



Important

Superposition is valid **ONLY** for linear circuits. It does NOT apply to circuits with nonlinear elements (e.g., diodes, transistors in active region).



Tips

- Keep the assumed directions the same in every case.
- A negative result simply means the actual direction is opposite.
- Use it mainly when the number of sources is small or moderate.

Final Answer

$$I_x = 2.30\text{ A (downward)}$$



WORK EXAMPLE 1 – MIXED DC CIRCUIT

1) PROBLEM

Objective:

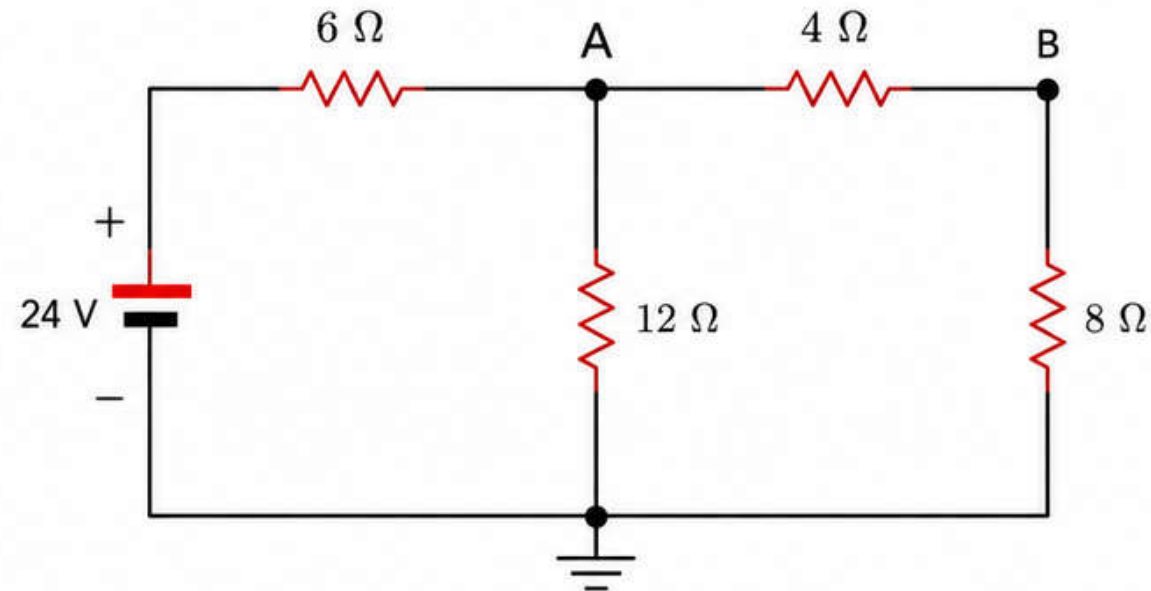
Use Ohm's law, series-parallel combinations, and KCL/KVL to find node voltages, branch currents, and the total current supplied by the source.

Tasks:

1. Find the node voltages V_A and V_B with respect to ground.
2. Find the current in each resistor.
3. Find the total current supplied by the 24 V source.

Given:

- DC source: $V_S = 24\text{ V}$
- $R_1 = 6\ \Omega$ (between source + and node A)
- $R_2 = 4\ \Omega$ (between node A and node B)
- $R_3 = 12\ \Omega$ (between node A and ground)
- $R_4 = 8\ \Omega$ (between node B and ground)
- **Ground** is the bottom node (0 V reference)



WORK EXAMPLE 1 – MIXED DC CIRCUIT

2) ANALYTICAL SOLUTION

1 Step 1: Choose ground and identify node voltages.

Ground is the bottom node (0 V). Unknown node voltages: V_A and V_B .

2 Step 2: Write KCL at node B.

Currents leaving node B:

$$\frac{V_B - V_A}{4} + \frac{V_B - 0}{8} = 0 \quad (\text{divide by } 1/4)$$

$$(V_B - V_A) + \frac{1}{2} V_B = 0 \Rightarrow -V_A + \frac{3}{2} V_B = 0 \Rightarrow V_A = \frac{3}{2} V_B \quad (1)$$

3 Step 3: Write KCL at node A.

Currents leaving node A (toward 24 V, toward B, and toward ground):

$$\frac{V_A - 24}{6} + \frac{V_A - V_B}{4} + \frac{V_A - 0}{12} = 0$$

Multiply by 12:

$$2(V_A - 24) + 3(V_A - V_B) + V_A = 0$$

$$2V_A - 48 + 3V_A - 3V_B + V_A = 0$$

$$6V_A - 3V_B = 48$$

$$2V_A - V_B = 16 \quad (2)$$

4 Step 4: Solve for V_A and V_B .

From (1): $V_A = \frac{3}{2} V_B$. Substitute into (2):

$$2\left(\frac{3}{2} V_B\right) - V_B = 16 \Rightarrow 3V_B - V_B = 16 \Rightarrow 2V_B = 16$$

$$V_B = 8 \text{ V} \Rightarrow V_A = \frac{3}{2} (8) = 12 \text{ V}$$

5 Step 5: Find current in each resistor (Ohm's law).

Through 6 Ω (from source + to node A):

$$I_{6\Omega} = \frac{24 - V_A}{6} = \frac{24 - 12}{6} = 2.00 \text{ A}$$

Through 4 Ω (from A to B):

$$I_{4\Omega} = \frac{V_A - V_B}{4} = \frac{12 - 8}{4} = 1.00 \text{ A}$$

Through 12 Ω (from A to ground):

$$I_{12\Omega} = \frac{V_A - 0}{12} = \frac{12}{12} = 1.00 \text{ A}$$

Through 8 Ω (from B to ground):

$$I_{8\Omega} = \frac{V_B - 0}{8} = \frac{8}{8} = 1.00 \text{ A}$$

6 Step 6: Total current supplied by the source.

The source current equals the current through the 6 Ω resistor

(only path from source to the circuit):

$$I_{\text{source}} = I_{6\Omega} = 2.00 \text{ A}$$

Check with KVL (outer loop)

Total rise of 24 V equals total drops around the outer loop

(through 6 Ω , 4 Ω , and 8 Ω):

$$24 = (2.00)(6) + (1.00)(4) + (1.00)(8) = 12 + 4 + 8 = 24 \text{ V} \quad \checkmark$$

Equivalent resistance seen by the source

$$R_{eq} = \frac{V_s}{I_{\text{source}}} = \frac{24}{2.00} = 12 \Omega$$

(Also: $12 \Omega \parallel (4 \Omega + 8 \Omega) = 12 \parallel 12 = 6 \Omega$; total = $6 \Omega + 6 \Omega = 12 \Omega$) $\checkmark$

Summary of Results

$$V_A = 12 \text{ V}$$

$$V_B = 8 \text{ V}$$

$$I_{6\Omega} = 2.00 \text{ A}$$

$$I_{4\Omega} = 1.00 \text{ A}$$

$$I_{12\Omega} = 1.00 \text{ A}$$

$$I_{8\Omega} = 1.00 \text{ A}$$

$$I_{\text{source}} = 2.00 \text{ A}$$

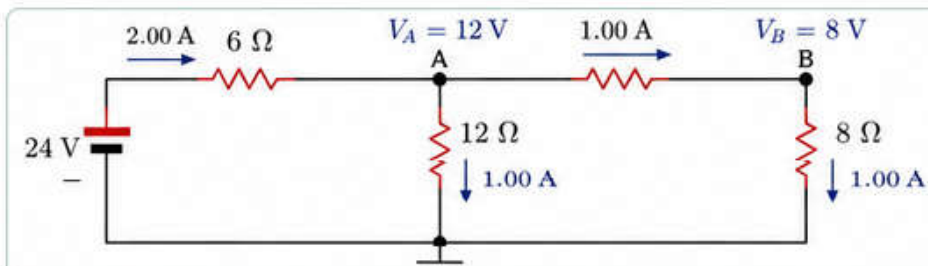
WORK EXAMPLE 1 – MIXED DC CIRCUIT

3) DISCUSSION



1. Physical interpretation of the results

- $V_A = 12\text{ V}$ and $V_B = 8\text{ V}$: Node A is at a higher potential than node B, so current flows from A to B through the $4\ \Omega$ resistor.
- Currents are all positive in the assumed directions:
 - From the source through $6\ \Omega$ into node A (2.00 A).
 - From node A to ground through $12\ \Omega$ (1.00 A).
 - From node A to node B through $4\ \Omega$ (1.00 A).
 - From node B to ground through $8\ \Omega$ (1.00 A).
- The $6\ \Omega$ resistor carries the total source current, then the current splits at node A into two paths: one to ground ($12\ \Omega$) and one toward node B ($4\ \Omega$), and finally it goes through $8\ \Omega$ to ground.



2. Consistency checks

- KCL verification
 - At node A: $2.00\text{ A in} = 1.00\text{ A (to } 12\ \Omega) + 1.00\text{ A (to } 4\ \Omega) = 2.00\text{ A}$ ✓
 - At node B: $1.00\text{ A in} = 1.00\text{ A (to } 8\ \Omega)$ ✓
- KVL verification (outer loop)
 - Rise = 24 V , drops = $(2.00)(6) + (1.00)(4) + (1.00)(8) = 12 + 4 + 8 = 24\text{ V}$ ✓
- Equivalent resistance check
 - $R_{eq} = 12\ \Omega \Rightarrow I_{source} = \frac{24}{12} = 2.00\text{ A}$ ✓



3. Power balance check

In a DC circuit, total power delivered by the source must equal the total power absorbed by the resistors.

Element	Current (A)	Resistance (Ω)	Power Absorbed $P = I^2 R$ (W)
$6\ \Omega$	2.00	6	$(2.00)^2(6) = 24$
$4\ \Omega$	1.00	4	$(1.00)^2(4) = 4$
$12\ \Omega$	1.00	12	$(1.00)^2(12) = 12$
$8\ \Omega$	1.00	8	$(1.00)^2(8) = 8$
Total absorbed			48 W

Power delivered by source: $P_{source} = V_s I_{source} = 24 \times 2.00 = 48\text{ W}$
Delivered = Absorbed = 48 W ✓



4. What this example teaches

- Combine KCL and Ohm's law to write node equations.
- Solving for node **voltages** first makes current calculations straightforward.
- **Signs** and assumed directions matter—always verify with KCL.
- **Power balance** is an excellent final check for correctness.



5. Key takeaway

By applying KCL at the unknown nodes, solving for node voltages, and then using Ohm's law, we found all branch currents and verified the solution using KVL, equivalent resistance, and power balance.

Always check your results using multiple methods!

WORK EXAMPLE 2 – NODAL ANALYSIS WITH CURRENT SOURCES

1) PROBLEM

Objective:

Use nodal analysis (KCL) to find node voltages and branch currents in a circuit that includes independent current sources.

Tasks:

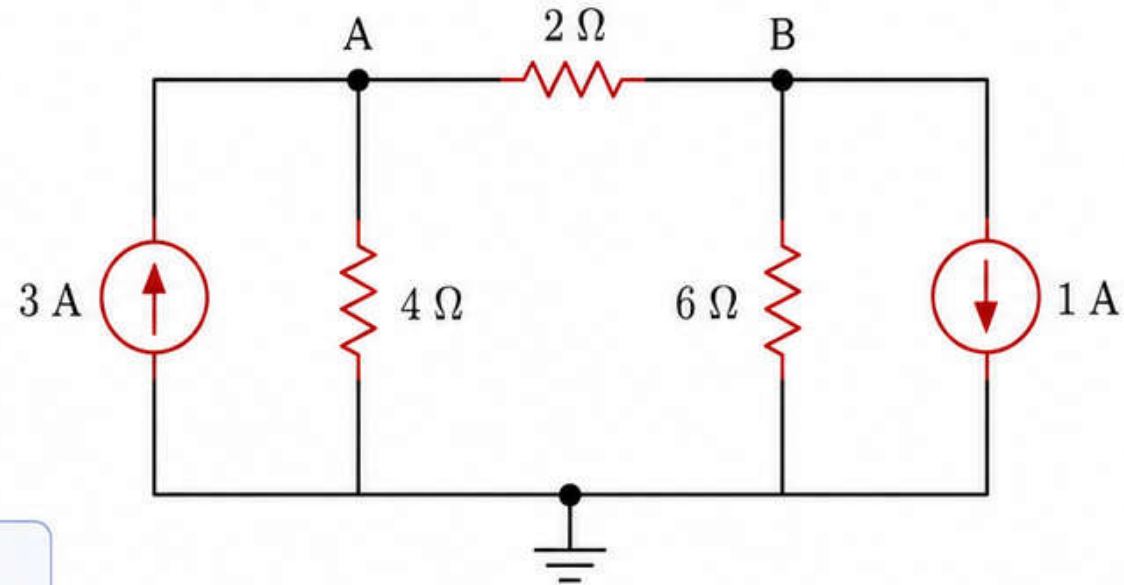
1. Find the node voltages V_A and V_B with respect to ground.
2. Find the current through each resistor.
3. Find the current delivered by each current source.

Given:

- Current source on the left: 3 A upward (from ground to top node)
- Current source on the right: 1 A downward (from top node to ground)
- Resistors: $R_1 = 4\ \Omega$ (from node A to ground),
 $R_2 = 2\ \Omega$ (between node A and node B),
 $R_3 = 6\ \Omega$ (from node B to ground)
- Ground is the bottom node (0 V reference)

Find:

V_A , V_B , I_{R1} , I_{R2} , I_{R3} , current delivered by each source.



WORK EXAMPLE 2 – NODAL ANALYSIS WITH CURRENT SOURCES

2) ANALYTICAL SOLUTION

1. Choose ground and identify node voltages.

Ground is the bottom node (0 V). Unknown node voltages: V_A and V_B .

2. Write KCL at node A.

Define currents leaving node A as positive.

Currents leaving node A:

- Through $4\ \Omega$ resistor to ground: $\frac{V_A}{4}$
- Through $2\ \Omega$ resistor to node B: $\frac{V_A - V_B}{2}$
- Through left current source: arrow is upward (from ground to A), so it enters node A. With "leaving" positive, enter as -3 A .

Apply KCL (sum leaving = 0):

$$\frac{V_A}{4} + \frac{V_A - V_B}{2} - 3 = 0$$

Multiply by 4:

$$3V_A - 2V_B = 12 \quad (1)$$

3. Write KCL at node B.

Define currents leaving node B as positive.

Currents leaving node B:

- Through $6\ \Omega$ resistor to ground: $\frac{V_B}{6}$
- Through $2\ \Omega$ resistor to node A: $\frac{V_B - V_A}{2}$
- Through right current source: arrow is downward (from B to ground), so it leaves node B. With "leaving" positive, enter as $+1\text{ A}$.

Apply KCL (sum leaving = 0):

$$\frac{V_B}{6} + \frac{V_B - V_A}{2} + 1 = 0$$

Multiply by 6:

$$-3V_A + 4V_B = -6 \quad (2)$$

4. Solve the two equations for V_A and V_B .

$$\text{From (1): } 3V_A - 2V_B = 12$$

$$\text{From (2): } -3V_A + 4V_B = -6$$

$$\text{Add (1) and (2): } 2V_B = 6 \Rightarrow V_B = 3\text{ V}$$

Substitute $V_B = 3\text{ V}$ into (1):

$$3V_A - 2(3) = 12 \Rightarrow 3V_A = 18 \Rightarrow V_A = 6\text{ V}$$

5. Find current through each resistor (assume directions as shown).

Through $4\ \Omega$ (A to ground):

$$I_{R1} = \frac{V_A - 0}{4} = \frac{6}{4} = 1.50\text{ A}$$

Through $2\ \Omega$ (A to B):

$$I_{R2} = \frac{V_A - V_B}{2} = \frac{6 - 3}{2} = 1.50\text{ A}$$

Through $6\ \Omega$ (B to ground):

$$I_{R3} = \frac{V_B - 0}{6} = \frac{3}{6} = 0.50\text{ A}$$

6. Current delivered by each current source.

- Left source (3 A upward from ground to node A):
Arrow enters node A $\rightarrow$ it delivers 3.00 A to the circuit.
- Right source (1 A downward from node B to ground):
Arrow leaves node B $\rightarrow$ it delivers 1.00 A to the circuit.

Summary of Results

Node voltages:

$$V_A = 6\text{ V}, \quad V_B = 3\text{ V}$$

Resistor currents (actual directions as shown):

$$\begin{aligned} I_{R1} &= 1.50\text{ A} \quad (\text{downward}) \\ I_{R2} &= 1.50\text{ A} \quad (\text{A} \rightarrow \text{B}) \\ I_{R3} &= 0.50\text{ A} \quad (\text{downward}) \end{aligned}$$

Current delivered by sources:

$$\begin{aligned} \text{Left source: } & 3.00\text{ A} \\ \text{Right source: } & 1.00\text{ A} \end{aligned}$$

MISUNDERSTANDINGS AND COMMON MISTAKES

Avoiding Typical Errors in DC Circuit Analysis

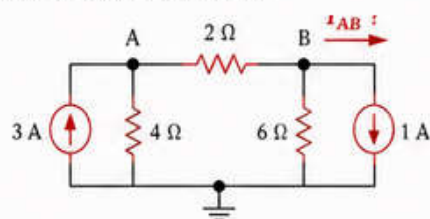


Good analysis starts with correct thinking!

i Even with the right formulas, it's easy to make conceptual or algebraic mistakes. Here are the most common ones—and how to avoid them.

1 Incorrect Current Directions

Assuming directions without consistency leads to sign errors and confusion.

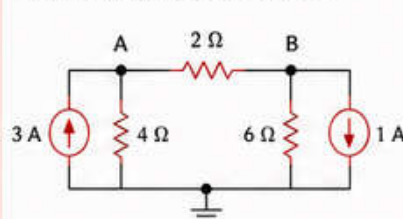


X If you guess current directions randomly, you may write KCL equations incorrectly.

✓ How to avoid it: Choose a consistent convention (e.g., all currents leaving the node are positive).

2 Forgetting Source Directions

Misreading current source arrows is a very common and serious mistake.

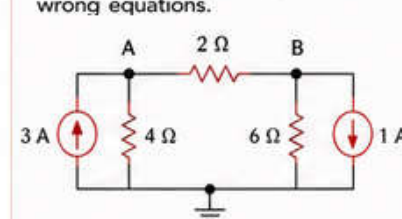


X Treating the 3 A source as leaving node A instead of entering it flips the sign in KCL.

✓ How to avoid it: Always read the arrow: arrow INTO the node = negative (with "leaving" positive convention).

3 Mixing Up Node Voltages and Voltage Differences

Using the wrong voltage in Ohm's law leads to wrong equations.

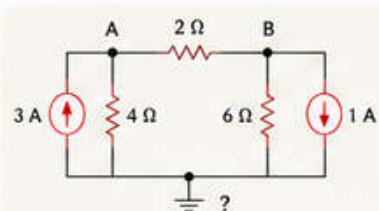


X Using V_A instead of $V_A - V_B$ across the 2Ω resistor is incorrect.

✓ How to avoid it: Across a resistor between two nodes: use the difference ($V_{from} - V_{to}$).

4 Forgetting the Ground Reference

Writing equations without a clear reference leads to ambiguity.



X If the ground (0 V) is not defined, node voltages have no reference.

✓ How to avoid it: Always choose and mark a ground node before writing equations.

5 Algebraic Sign Errors

Small sign mistakes in KCL or algebra lead to wrong node voltages.

Example at node A:

$$\text{Correct KCL (leaving positive):} \\ \frac{V_A}{4} + \frac{V_A - V_B}{2} - 3 = 0$$

Common mistake:

$$\frac{V_A}{4} + \frac{V_A - V_B}{2} + 3 = 0$$

X Wrong sign for the current source changes the entire solution.

✓ How to avoid it: Be systematic with signs and double-check each term.

6 Not Verifying the Results

Accepting results without checks can hide mistakes.

KCL

Check KCL at each node:
Sum of currents leaving = 0.

KVL

Check KVL around one or more loops:
Sum of voltage rises = sum of drops.

P

Check power balance:
Total power delivered = total power absorbed.

R_{eq}

Check equivalent resistance seen by the source, if possible (sanity check).



BEST PRACTICES



Be Consistent

- Choose current directions and stick with them.
- Use the same sign convention everywhere.



Draw and Label Clearly

- Mark node voltages (V_A , V_B , ...).
- Show source directions.
- Label resistor values.



Go Step by Step

- Write KCL equations first.
- Solve for node voltages.
- Then compute currents.



Always Check

- KCL at each node.
- KVL around a loop.
- Power balance.



Trust, but Verify

- Multiple checks = confidence in your results.

ELECTRIC CIRCUITS – SUMMARY

Key Concepts, Methods and Tools

SUMMARY

1 FUNDAMENTALS

Current: $I = \Delta Q / \Delta t$

Ohm's law: $V = I R$

Power: $P = V I$

3 KIRCHHOFF'S LAWS

KCL: $\sum I = \sum I$

or, with signed currents: $\sum I = 0$

KVL: algebraic sum of ΔV around a loop = 0

5 SUPERPOSITION

Keep one independent source active at a time.

Voltage source → short; current source → open.

Add signed contributions to the required response.

2 SERIES AND PARALLEL

Series: same current; $R_{eq} = \sum R$

Parallel: same voltage

$1 / R_{eq} = \sum (1 / R)$

4 NODAL AND MESH ANALYSIS

Nodal: choose ground → apply KCL

Mesh: assign mesh currents → apply KVL

Solve the equations, then recover branch values

6 RELIABLE PROBLEM-SOLVING

Predict → draw and label → choose the law

Solve systematically → check units and signs

Verify with KCL, KVL, R_{eq} , or power balance

MODEL → EQUATIONS → SOLUTION → PHYSICAL CHECK

• What is asked?

• Label values, sources,
and directions.

• Theorems (Thévenin, etc.)
• Superposition if useful

• Solve step by step.
• Find unknowns.

• Understand what results
mean physically.

• KVL in loops
• Power balance

CONFIDENCE
IN RESULTS

★ Master the fundamentals. Practice consistently. Think systematically. → That is how you become an excellent circuit analyst!