

PHYSICS FOR ENGINEERING STUDENTS

Electric Fields and Magnetic Fields

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Lecture 8 — Book-style student notes

Learning focus

Electric charge, force and superposition

Electric field and electric potential

Magnetic fields and Lorentz force

Forces on currents and electric motors

Electromagnetic induction and Lenz's law

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1. Electromagnetism as a unifying framework

How electric and magnetic phenomena fit together

Electricity and magnetism were once studied as separate subjects, but modern physics treats them as two aspects of one electromagnetic interaction. Stationary charges produce electric fields. Moving charges, or electric currents, produce magnetic fields. A changing magnetic field produces an electric field, while a changing electric field contributes to a magnetic field. These connections explain how electrical energy can be generated, transmitted, converted into motion and used to carry information.

The field concept is central. Instead of imagining that two objects act on each other instantaneously across empty space, we say that a source modifies the space around it. A second object placed in that space responds locally to the field. The electric field E and magnetic field B are vector fields: at every point they have both magnitude and direction. Electric potential V is a related scalar field that often simplifies energy calculations.

Maxwell's equations in conceptual form

Law	Compact form	Physical statement
Gauss—electric	$\nabla \cdot E = \rho/\epsilon_0$	Electric charge is the source or sink of E .
Gauss—magnetic	$\nabla \cdot B = 0$	Magnetic field lines form closed loops; no isolated monopoles.
Faraday	$\nabla \times E = -\partial B/\partial t$	A changing magnetic field creates a circulating E field.
Ampère—Maxwell	$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \partial E/\partial t$	Currents and changing electric fields create B .

These equations are shown here as a map, not as a required mathematical derivation. In this lecture we use their simpler consequences: Coulomb's law, fields of point charges and currents, magnetic forces, and electromagnetic induction. The same framework underlies motors, generators, transformers, MRI, sensors, communication systems and optical electromagnetic waves.

Problem-solving principle

Begin by identifying the source, the field and the responding object. Then decide whether the relevant quantity is a vector or a scalar. Only after the geometry and direction are clear should an equation be selected. This order prevents most sign and direction errors.

2. Electric charge and Coulomb's law

The force between stationary point charges

Electric charge is a fundamental property of matter. Two signs exist: positive and negative. Like charges repel and unlike charges attract. Charge is conserved in an isolated system and quantized in integer multiples of the elementary charge $e = 1.602 \times 10^{-19} \text{ C}$. The coulomb is a large microscopic unit: one coulomb corresponds to about 6.24×10^{18} elementary charges.

If charged objects are small compared with their separation, they may be modeled as point charges. The force exerted by q_1 on q_2 is described by Coulomb's law:

$$F_{12} = [1/(4\pi\epsilon_0)] (q_1 q_2 / r^2) \hat{r}_{12} = k(q_1 q_2 / r^2) \hat{r}_{12}$$

$$k = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2, \quad \epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2/(\text{N}\cdot\text{m}^2)$$

The unit vector $\hat{r}_{12}$ points from q_1 toward q_2 . The product $q_1 q_2$ determines whether the force is along or opposite that direction. An alternative safe method is to calculate the positive magnitude $k|q_1 q_2|/r^2$ and then determine attraction or repulsion from the signs.

Inverse-square dependence

The force decreases as $1/r^2$. Doubling the separation reduces the force to one quarter; halving the separation increases it by a factor of four. This strong distance dependence is a useful reasonableness check. If a calculation gives a larger force after charges are moved farther apart without changing their magnitudes, something is wrong.

Change in distance	New force
$r \rightarrow 2r$	$F \rightarrow F/4$
$r \rightarrow 3r$	$F \rightarrow F/9$
$r \rightarrow r/2$	$F \rightarrow 4F$
$r \rightarrow r/3$	$F \rightarrow 9F$

Superposition

Every source charge acts independently. The net force on q_0 is the vector sum of all pairwise forces:

$$F_{\text{net}} = \sum_i F_{i0}$$

For collinear charges, choose a positive axis and attach a sign to each component. In two or three dimensions, resolve each force into Cartesian components, add components, and reconstruct the magnitude and direction. Never add force magnitudes unless the forces genuinely point in the same direction.

3. Electric fields

Force per unit positive test charge

The electric field at a point is defined as the force that would act per unit positive test charge:

$$\mathbf{E} = \mathbf{F}/q_0 \quad \text{and therefore} \quad \mathbf{F} = q\mathbf{E}$$

The test charge is conceptually small so that it does not disturb the source configuration. The field exists whether or not a test charge is present. Its SI unit is newton per coulomb, equivalent to volt per metre. A positive responding charge accelerates along $\mathbf{E}$; a negative responding charge experiences force opposite $\mathbf{E}$.

Field of a point charge

$$\mathbf{E}(r) = k(q/r^2) \hat{r}$$

For $q > 0$, the field points radially outward. For $q < 0$, it points radially inward. The field obeys superposition: $\mathbf{E}_{\text{total}} = \Sigma \mathbf{E}_i$. Because $\mathbf{E}$ is a vector, direction must be determined for each contribution before addition. Field lines are a visualization: the tangent gives direction and closer line density represents stronger field. Lines begin on positive charge and end on negative charge or at infinity; they never cross.

Uniform and non-uniform fields

The field of a point charge is strongly non-uniform because magnitude and direction change with position. Between large, oppositely charged parallel plates, away from the edges, the field is approximately uniform: field lines are straight, parallel and equally spaced. A charge in this region experiences nearly constant force $q\mathbf{E}$. This model is used in capacitors, particle deflection, electrostatic spraying and laboratory beam control.

Feature	Point-charge field	Parallel-plate field
Direction	Radial; changes with position	Approximately constant direction
Magnitude	$k q /r^2$	Nearly constant away from edges
Field lines	Spread or converge	Parallel and equally spaced
Classification	Non-uniform	Approximately uniform

Vector workflow

- Sketch every source charge and the observation point.
- At the observation point, draw the direction of each field contribution.
- Calculate each magnitude using its own source-to-point distance.
- Resolve contributions into components if needed.
- Add vectors and state both magnitude and direction.

4. Worked electric-field example

Two unlike charges and the superposition principle

Two charges are separated by 0.40 m: $q_1 = +2.0 \mu\text{C}$ on the left and $q_2 = -1.0 \mu\text{C}$ on the right. Point P is midway between them, so $r_1 = r_2 = 0.20 \text{ m}$. Find the net electric field at P.

Direction before calculation

The field from the positive q_1 points away from q_1 , hence to the right at P. The field from the negative q_2 points toward q_2 , also to the right. The contributions reinforce each other. This directional reasoning is more important than the arithmetic because it determines whether magnitudes add or subtract.

Individual contributions

$$E_1 = k|q_1|/r_1^2 = (8.99 \times 10^9)(2.0 \times 10^{-6})/(0.20)^2 = 4.50 \times 10^5 \text{ N/C}$$

$$E_2 = k|q_2|/r_2^2 = (8.99 \times 10^9)(1.0 \times 10^{-6})/(0.20)^2 = 2.25 \times 10^5 \text{ N/C}$$

$$E_{\text{net}} = E_1 + E_2 = 6.75 \times 10^5 \text{ N/C, to the right}$$

The answer is greater than either individual field because they point in the same direction. It is also proportional to the larger source charge. If q_2 were positive instead, its field at P would point left and the net field would be $E_1 - E_2 = 2.25 \times 10^5 \text{ N/C}$ to the right. If q_2 were $-3.0 \mu\text{C}$, its rightward field would be $6.75 \times 10^5 \text{ N/C}$ and the total would be $1.125 \times 10^6 \text{ N/C}$ to the right.

Checks and pitfalls

- Use 0.20 m, not the 0.40 m separation, in each denominator.
- Convert μC to C: $1 \mu\text{C} = 10^{-6} \text{ C}$.
- The sign of a source determines direction; the magnitude formula uses $|q|$.
- Report a direction as well as N/C.
- Check that the stronger contribution comes from the larger $|q|$ at equal distance.

This example shows a reusable strategy: separate geometry from calculation. Determine field directions visually, calculate magnitudes independently, and apply vector superposition only at the end.

5. Electric potential and potential energy

A scalar description of electric interaction

Electric potential V is electric potential energy per unit charge: $V = U/q$. Unlike E , potential is a scalar. It has magnitude and sign but no spatial direction. One volt equals one joule per coulomb. For a point charge, taking zero potential at infinity,

$$V(r) = kq/r$$

A positive source gives positive potential and a negative source gives negative potential. Potential falls as $1/r$, whereas point-charge field falls as $1/r^2$. The potential created by many point charges is the algebraic sum

$$V_{total} = \sum_i kq_i/r_i$$

Because potential is scalar, do not use vector arrows or resolve components. Retain the algebraic sign of every q_i and add ordinary signed numbers. Once V is known, a charge q placed there has potential energy $U = qV$. Thus a negative charge can have negative energy at positive potential.

Relation between field and potential

$$E = -\nabla V \quad \text{and in one-dimension} \quad E_x = -dV/dx$$

The minus sign means that E points in the direction of greatest decrease of potential. For a finite displacement from a to b , the potential difference is $\Delta V = V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{l}$. Moving a positive charge naturally along E lowers its potential energy; external work is required to move it against E .

Equipotential surfaces

Every point on an equipotential surface has the same V . Moving along it requires no electrostatic work because $\Delta V = 0$. Electric-field lines cross equipotential surfaces at right angles. Around an isolated point charge, equipotentials are concentric spheres. Between ideal parallel plates, they are planes parallel to the plates.

Quantity	Electric field E	Electric potential V
Type	Vector	Scalar
Point charge	$kq \hat{r}/r^2$	kq/r
Superposition	Vector sum	Algebraic sum
Unit	$N/C = V/m$	$V = J/C$
Physical role	Determines force $F=qE$	Determines energy $U=qV$

6. Worked potential example and electrostatic mistakes

Using signs without inventing directions

At point P, the distance to $q_1 = +3.0 \mu\text{C}$ is $r_1 = 0.30 \text{ m}$, and the distance to $q_2 = -2.0 \mu\text{C}$ is $r_2 = 0.40 \text{ m}$. The potential is

$$V_1 = kq_1/r_1 = (8.99 \times 10^9)(3.0 \times 10^{-6})/0.30 = +8.99 \times 10^4 \text{ V}$$

$$V_2 = kq_2/r_2 = (8.99 \times 10^9)(-2.0 \times 10^{-6})/0.40 = -4.50 \times 10^4 \text{ V}$$

$$V_{\text{net}} = V_1 + V_2 = +4.49 \times 10^4 \text{ V}$$

The positive contribution has greater magnitude, so the total is positive. A positive test charge at P has positive potential energy relative to infinity. Notice that no angle or vector direction appears in the calculation. Geometry matters only through the scalar distances r_1 and r_2 .

Why field and potential can behave differently

At a point of zero potential, the field need not be zero because positive and negative potentials may cancel while their field vectors reinforce. Conversely, the field can be zero at a symmetry point even though the potential is nonzero. Potential is an energy landscape; field is its local slope. Confusing them leads to incorrect claims such as “zero potential means no force.”

Common electrostatic errors

Error	Correction
Treating E as a scalar	Determine a direction for every contribution and add vectors.
Treating V as a vector	Retain charge signs and add potentials algebraically.
Using the charge separation	Use the distance from each source to the observation point.
Dropping a negative source sign in V	$V = kq/r$ uses signed q.
Using $F=qE$ without charge sign	A negative charge feels force opposite E.

Engineering relevance

Electrostatic principles appear in capacitors, laser printers, charged-particle optics, electrostatic coating, precipitators and sensors. They also explain electrostatic discharge, which can damage microelectronics. Reliable engineering requires field strength estimates for insulation and force estimates for particle control, together with potential differences that govern energy and voltage.

7. Magnetic fields and the right-hand rule

Fields created by moving charge

Magnetic fields are produced by moving charges, electric currents and magnetized materials. A permanent magnet has north and south poles; outside the magnet, field lines are conventionally drawn from north to south. Unlike electric field lines, magnetic field lines never begin or end: they form continuous closed loops, reflecting $\nabla \cdot \mathbf{B} = 0$ and the absence of observed magnetic monopoles.

A long straight wire carrying current I creates circular field lines around the wire. Their direction is found with the right-hand grip rule: point the right thumb in the conventional current direction and the curled fingers show the direction of $\mathbf{B}$. Reversing the current reverses the field. The field magnitude outside an ideal long wire is

$$B = \mu_0 I / (2\pi r)$$

where r is the perpendicular distance from the wire, and μ_0 is the permeability of free space. The tesla is the SI unit of magnetic flux density. Field-line density is a visual indicator of magnitude, but the lines are not physical strings.

Direction symbols

A dot $\odot$ represents a vector coming out of the page, like the tip of an arrow approaching the observer. A cross $\otimes$ represents a vector going into the page, like the feathers of an arrow moving away. These symbols are essential in planar diagrams of magnetic force.

Right-hand reasoning

- Identify the conventional current direction, not electron drift direction.
- Point the right thumb along I .
- Curl the fingers around the conductor.
- At the observation point, read the local tangent direction of the circular field.
- If I reverses, reverse $\mathbf{B}$ everywhere.

Sources and applications

Earth's magnetic field guides compasses and helps shield the planet from solar charged particles. Solenoids create controlled fields for actuators and laboratory magnets. Magnetic fields enable motors, transformers, wireless power transfer, data storage, sensors and medical imaging. In every case, field direction is determined geometrically before magnitude is calculated.

8. Magnetic force on a moving charge

The Lorentz-force geometry

$$\mathbf{F} = q \mathbf{v} \times \mathbf{B} \quad \text{and} \quad F = qvB \sin\theta$$

A moving charge in a magnetic field experiences a force perpendicular to both its velocity $\mathbf{v}$ and the field $\mathbf{B}$. The angle θ is the smaller angle between $\mathbf{v}$ and $\mathbf{B}$. The force is zero for parallel or antiparallel motion and maximum for perpendicular motion. Only the velocity component perpendicular to $\mathbf{B}$ contributes.

Angle θ	$\sin\theta$	Force magnitude
0° or 180°	0	Zero
30°	0.500	$0.500 q \mathbf{v}\mathbf{B}$
45°	0.707	$0.707 q \mathbf{v}\mathbf{B}$
90°	1	Maximum: $ q \mathbf{v}\mathbf{B}$

Direction

For a positive charge, apply the right-hand cross-product rule from $\mathbf{v}$ toward $\mathbf{B}$; the thumb gives $\mathbf{F}$. For a negative charge, reverse that direction. One practical hand convention is index finger along $\mathbf{v}$, middle finger along $\mathbf{B}$ and thumb along $\mathbf{F}$, provided the fingers are kept mutually perpendicular. Always state which convention is used rather than relying on a memorized picture.

Why the magnetic force does no work

Instantaneous power is $P = \mathbf{F} \cdot \mathbf{v}$. Since $\mathbf{F}$ is perpendicular to $\mathbf{v}$, $P = 0$. Therefore a purely magnetic field changes the direction of velocity but not its speed or kinetic energy. For $\mathbf{v}$ perpendicular to a uniform $\mathbf{B}$, the force acts as a centripetal force and the trajectory is circular:

$$qvB = mv^2/R \quad \Rightarrow \quad R = mv/(qB)$$

If $\mathbf{v}$ has a component parallel to $\mathbf{B}$, that component remains unchanged while the perpendicular component circles, producing a helical path. These principles are used in mass spectrometers, particle accelerators and charged-particle detectors.

Electric versus magnetic force

The electric force $q\mathbf{E}$ acts even on a stationary charge and may do work. The magnetic force requires motion, depends on angle, and is perpendicular to motion. In the full Lorentz law the contributions combine: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

9. Worked magnetic-force example

A proton moving perpendicular to a field

A proton with charge $q = +1.60 \times 10^{-19} \text{ C}$ moves at $v = 2.0 \times 10^6 \text{ m/s}$ through a uniform magnetic field $B = 0.40 \text{ T}$. The velocity is perpendicular to the field, so $\theta = 90^\circ$ and $\sin\theta = 1$.

$$F = qvB \sin\theta$$

$$F = (1.60 \times 10^{-19})(2.0 \times 10^6)(0.40)(1) = 1.28 \times 10^{-13} \text{ N}$$

$$\text{To two significant figures: } F = 1.3 \times 10^{-13} \text{ N}$$

The force is small because the charge is microscopic, but it can strongly curve the trajectory over time. Its direction is obtained from the right-hand rule for a positive proton. If the velocity is upward and B points to the right on the page, $v \times B$ points into the page. The essential rule is that F is perpendicular to both vectors. For an electron with the same velocity, the magnitude would be the same but the direction opposite, out of the page.

Reasonableness checks

- $\theta = 90^\circ$ must give the largest force for the stated q , v and B .
- The result must have units of newtons: $\text{C} \cdot (\text{m/s}) \cdot \text{T} = \text{N}$.
- Doubling speed or B doubles F ; changing θ introduces $\sin\theta$.
- Changing the sign of q changes only direction, not magnitude.
- Because $F \perp v$, kinetic energy should remain constant in a magnetic-only region.

Circular motion estimate

Using the proton mass $m_p = 1.67 \times 10^{-27} \text{ kg}$, the radius for perpendicular motion is

$$R = m_p v / (qB) = (1.67 \times 10^{-27})(2.0 \times 10^6) / [(1.60 \times 10^{-19})(0.40)] \approx 5.2 \times 10^{-2} \text{ m}$$

Thus a force of only 10^{-13} N bends the proton into a circle of radius about 5.2 cm. Microscopic forces can produce easily measurable beam deflections because particle masses are extremely small.

10. Force on a current-carrying wire and electric motors

From microscopic charge force to macroscopic motion

A current consists of moving charges, so a wire in a magnetic field experiences the combined magnetic force on its charge carriers. For a straight active length L in a uniform field,

$$\mathbf{F} = I \mathbf{L} \times \mathbf{B} \quad \text{and} \quad F = BIL \sin\theta$$

The vector L points along conventional current and has magnitude equal to the length inside the field. The force is zero when the wire is parallel to B and maximum when perpendicular. Reversing either I or B reverses F .

Worked wire example

A 0.50 m wire carries $I = 3.0$ A to the right in a uniform $B = 0.80$ T upward. Since $\theta = 90^\circ$,

$$F = (0.80 \text{ T})(3.0 \text{ A})(0.50 \text{ m}) \sin 90^\circ = 1.20 \text{ N}$$

The right-hand rule applied from current toward B gives the force perpendicular to the page. Reversing the current would reverse the page direction while leaving magnitude unchanged.

Motor principle

In a rectangular coil, opposite sides carry currents in opposite directions. Magnetic forces on those sides are equal and opposite but act along different lines, forming a couple and producing torque. For N turns of area A in a uniform field, the torque magnitude is

$$\tau = NIAB \sin\phi$$

where ϕ is the angle between the coil's area vector and B . In a simple direct-current motor, a split-ring commutator reverses current every half-turn so that torque continues in the same rotational sense. Electrical energy is converted into mechanical rotation through magnetic forces. Practical motors use multiple coils and poles to reduce torque ripple and improve efficiency.

Applications and limitations

The same force principle operates in loudspeakers, moving-coil meters, galvanometers and electromagnetic actuators. The simple BIL equation assumes a straight conductor, uniform field and a clearly defined active length. For curved conductors or spatially varying fields, differential forces must be integrated.

11. Electromagnetic induction

Faraday's law, magnetic flux and Lenz's law

Electromagnetic induction occurs whenever the magnetic flux through a conducting loop changes. Magnetic flux measures how much magnetic field passes through an oriented area:

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A} \quad \text{and for uniform B:} \quad \Phi_B = BA \cos\varphi$$

The angle φ is between $\mathbf{B}$ and the area normal, not the plane itself. Flux is maximum when $\mathbf{B}$ is perpendicular to the loop and zero when $\mathbf{B}$ lies in the loop plane. Its unit is the weber, $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

Faraday's law

$$\varepsilon = -N d\Phi_B/dt$$

The induced electromotive force ε is proportional to the rate of flux change and to the number N of turns. Flux can change because $\mathbf{B}$ changes, the loop area changes, the loop rotates, or the magnet and coil move relative to each other. If flux is constant, $\varepsilon = 0$ even when the flux itself is large.

Lenz's law

The minus sign expresses Lenz's law: the induced current creates a magnetic field that opposes the change in flux, not necessarily the original field. If flux into the page increases, the induced field points out of the page. If flux into the page decreases, the induced field points into the page to resist that decrease. This opposition follows from energy conservation; an induced current that reinforced the change would create energy without external work.

Flux behavior	Induced response
Increasing	Creates a field opposing the increase
Decreasing	Creates a field supporting the old flux and opposing the decrease
Constant	No induced emf
Faster change	Larger $ \varepsilon $

Engineering systems

Generators use mechanical motion to change flux and produce voltage. Transformers use alternating current to create changing flux that induces voltage in another winding. Inductive sensors detect motion or material properties, while eddy-current brakes use induced currents whose magnetic effects oppose motion. Wireless charging transfers energy through time-varying magnetic coupling.

12. Integrated problem-solving guide and formula summary

How to choose the right model and avoid common mistakes

Decision workflow

- Identify what creates the field: stationary charge, current, magnet or changing flux.
- Identify what responds: charge, current-carrying wire or conducting loop.
- Decide whether the target is force, field, potential, energy, flux or induced voltage.
- Draw all directions and define angles before substitution.
- Choose vector addition for $\mathbf{E}$ and force, algebraic addition for V .
- Apply the correct right-hand rule and reverse for negative charge when necessary.
- Check units, limiting cases and physical plausibility.

Core equations

Coulomb: $F = k|q_1q_2|/r^2$ *Point field:* $\mathbf{E} = kq \hat{r}/r^2$ *Potential:* $V = kq/r$

Electric force: $\mathbf{F} = q\mathbf{E}$ *Field-potential:* $E = -\nabla V$

Moving charge: $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ *Wire:* $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$ *Torque:* $\tau = NIAB \sin\phi$

Flux: $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ *Faraday-Lenz:* $\varepsilon = -N d\Phi_B/dt$

Frequent mistakes

Mistake	Better practice
Forgetting the angle	Identify the two vectors and use the angle between them.
Assuming magnetic force is always maximum	Check whether $\theta = 90^\circ$; parallel motion gives zero.
Using the right hand for a negative charge without reversal	Find the positive-charge direction, then reverse it.
Confusing $\mathbf{E}$ and V	$\mathbf{E}$ is vector force per charge; V is scalar energy per charge.
Adding fields algebraically	Add $\mathbf{E}$ vectors by components.
Adding potentials as vectors	Add signed scalar V contributions.
Opposing the field instead of its change in Lenz's law	Determine whether flux is increasing or decreasing first.

Final synthesis

Stationary charge establishes an electric field and potential. Moving charge creates magnetic effects and responds to magnetic fields through a transverse force. Currents convert these microscopic forces into macroscopic force and torque. Changing magnetic flux generates voltage, completing a chain from charge to field, motion and energy conversion.

Successful electromagnetic problem solving is mostly geometry and interpretation. Equations are compact, but their signs, vector directions and angles encode the physics. Draw first, predict second, calculate third and interpret last. When the sketch, algebra, units and physical expectation agree, the solution is trustworthy.

Self-check questions

- Why can the electric potential be zero at a point where the electric field is not zero?
- How does doubling the distance from a point charge affect its field and its potential?
- Why does a magnetic field change the direction of a charged particle without changing its speed?
- What changes in the force on a wire if the current direction is reversed?
- Which four physical changes can alter the magnetic flux through a loop?
- In Lenz's law, what exactly is opposed: the magnetic field or its change?