

Thin Lenses and Lens Systems

Image Formation and Optical Design



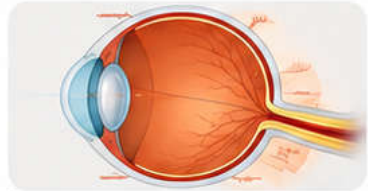
Camera Lenses



Microscope Objectives

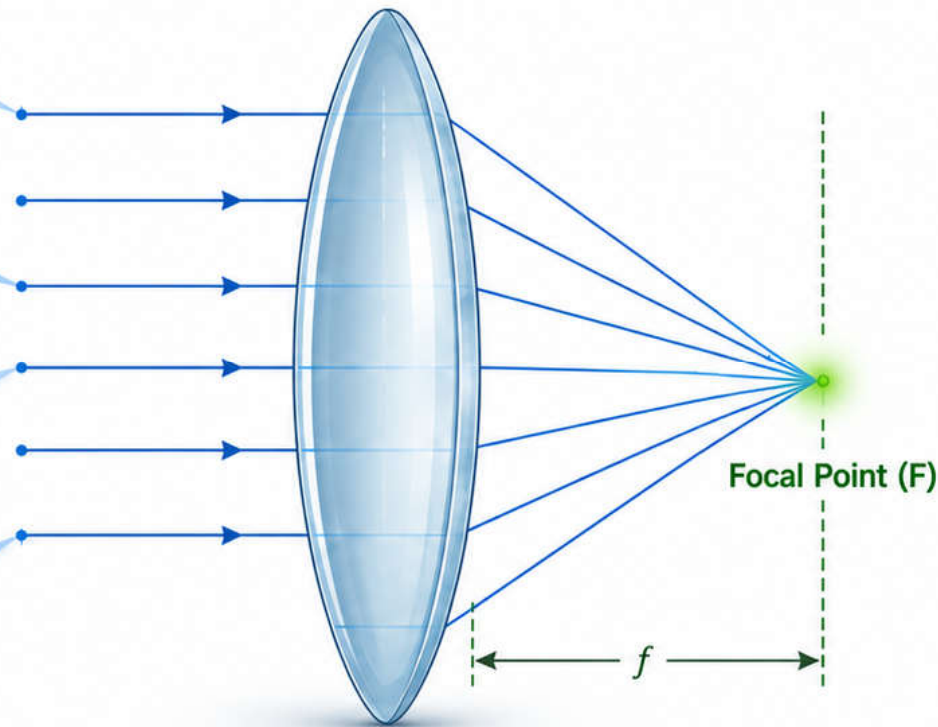


Telescopes



The Human Eye

One optical principle – countless applications



In this lecture you will learn



How lenses bend light



How images are formed



Real and virtual images



The thin lens equation



Magnification



Ray tracing techniques



Multi-lens systems



Cameras, microscopes and telescopes



Key Question

How can a simple piece of glass control light so precisely that it creates sharp images of objects millions of kilometers away—or only a few micrometers in size?



University of Szeged
Institute of Physics

Laser and Aerosol Research Laboratory

Why do we need a lens?

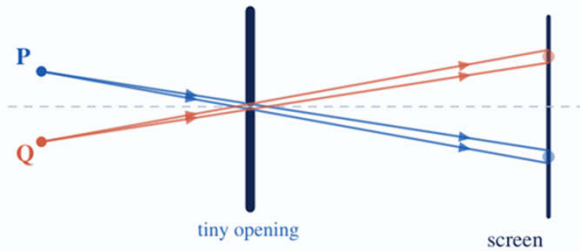
A pinhole gives a dim image; a wide opening alone makes overlapping blur. A lens collects light and separates image points.

SMALL PINHOLE

Dim but distinguishable

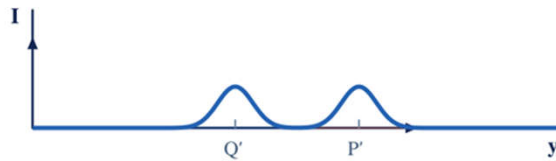


TWO SELECTED OBJECT POINTS



Few rays reach each image point.
Two faint spots form an inverted image.

SCREEN INTENSITY $I(y)$



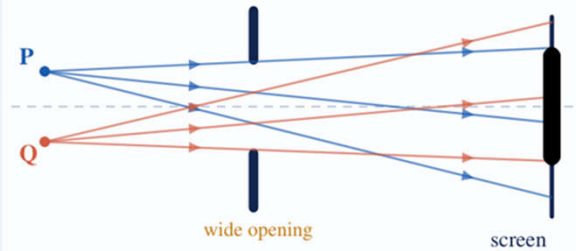
Weak, narrow peaks · less light

WIDE OPENING · NO LENS

Bright, overlapping spots

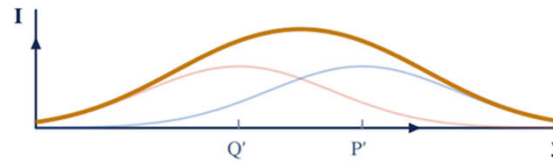


TWO SELECTED OBJECT POINTS



Many directions from P and Q overlap.
The screen shows a bright diffuse patch.

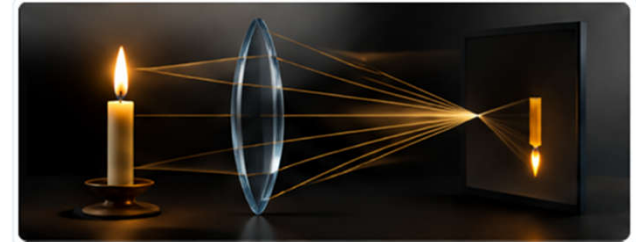
SCREEN INTENSITY $I(y)$



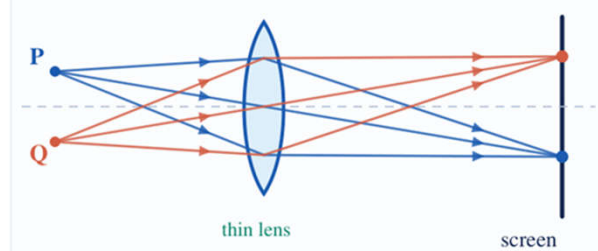
Broad profiles merge · detail is lost

WIDE OPENING · LENS

Bright, separated image points

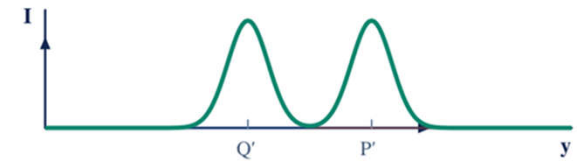


TWO SELECTED OBJECT POINTS



Many rays from P meet at P', from Q at Q'.
One inverted, bright image forms.

SCREEN INTENSITY $I(y)$



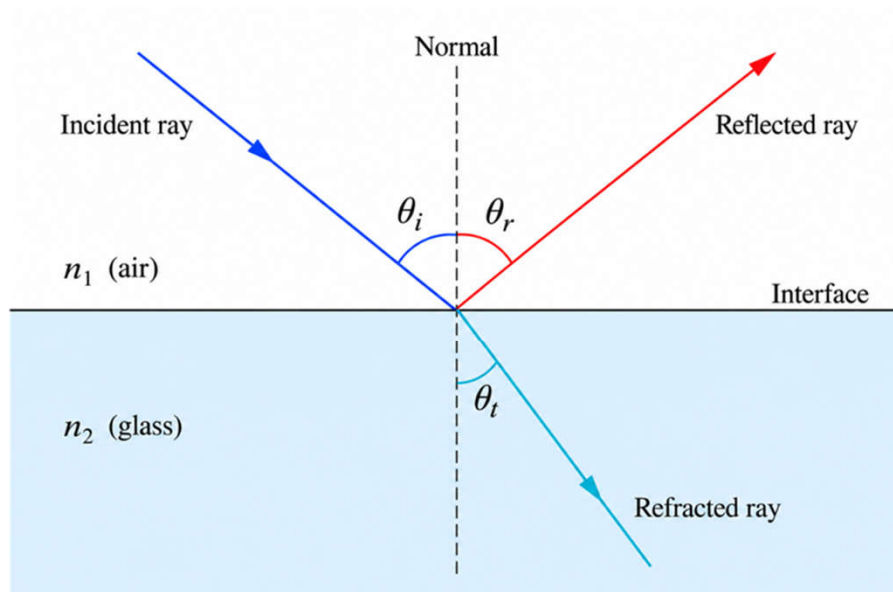
Strong, narrow peaks · details remain

KEY IDEA: a lens directs many rays from each object point to its own image point. Intensity profiles are schematic.

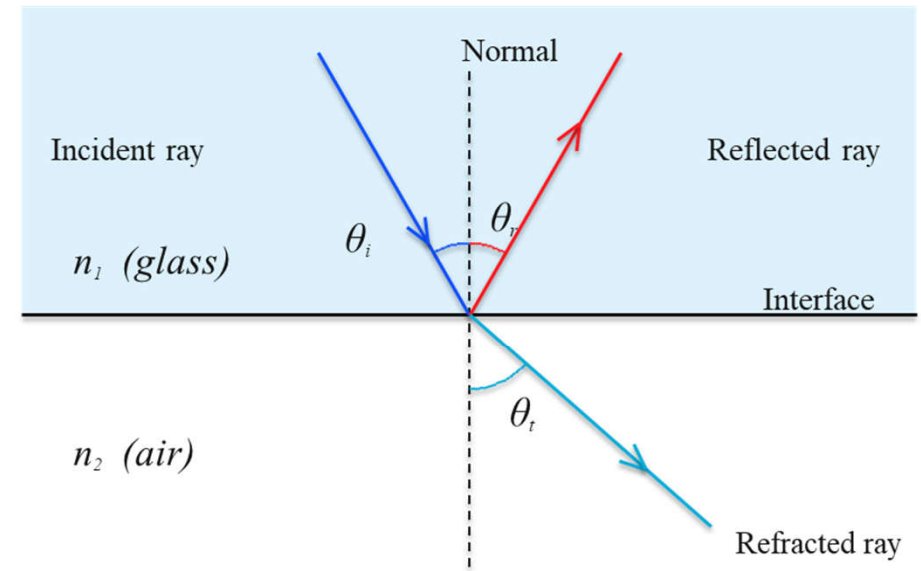
Snell's Law: Both Directions

One incident ray produces a reflected and a transmitted ray at each interface.

RARER → DENSER



DENSER → RARER



GOVERNING LAWS

$$\theta_i = \theta_r$$

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$n = c/v \text{ (phase index)}$$

Angles from the normal.

MEANING OF SYMBOLS

n_1, n_2 : refractive indices

θ_i : angle of incidence

θ_r : angle of reflection

θ_t : angle of refraction

c, v : phase speeds

PHYSICAL CONSEQUENCES

$n_2 > n_1$: $\theta_t < \theta_i$ (toward)

$n_2 < n_1$: $\theta_t > \theta_i$ (away)

Frequency stays constant.

$$\lambda_2/\lambda_1 = v_2/v_1$$

CRITICAL ANGLE

For $n_1 > n_2$:

$$\theta_c = \sin^{-1}(n_2/n_1)$$

$\theta_i > \theta_c$: total internal reflection

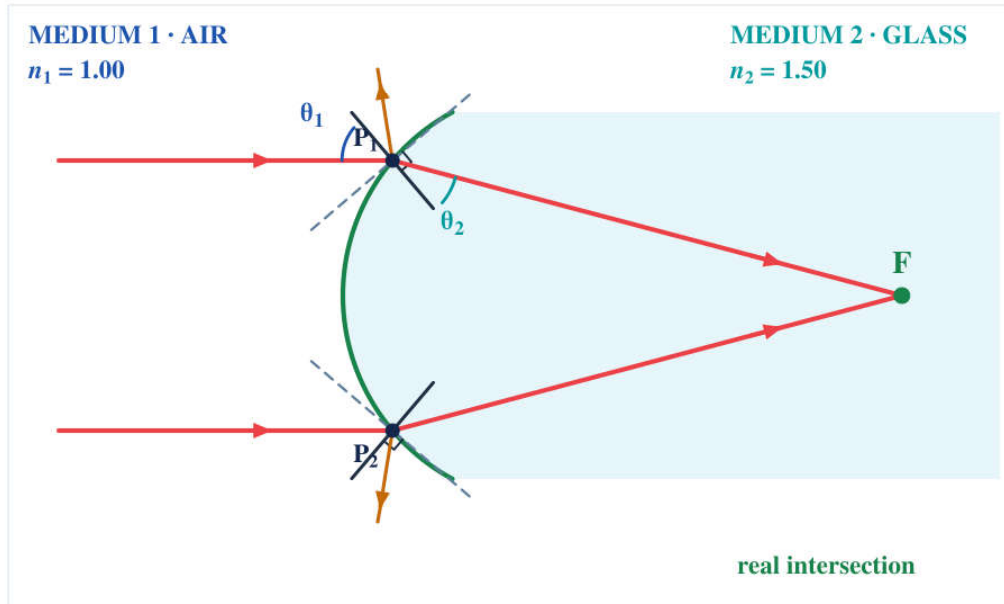
Glass → air: 41.8°

Snell's law at curved surfaces

Parallel rays meet different local normals: curvature changes the combined ray effect.

CONVEX INTERFACE

Two transmitted rays meet at a real point F.



At P_1 and P_2 , the tangent is perpendicular to the local normal.
Since $n_2 > n_1$, each transmitted ray bends toward that normal.
Symmetric rays then intersect at F in medium 2.

AT EACH POINT $n_1 \sin \theta_1 = n_2 \sin \theta_2$

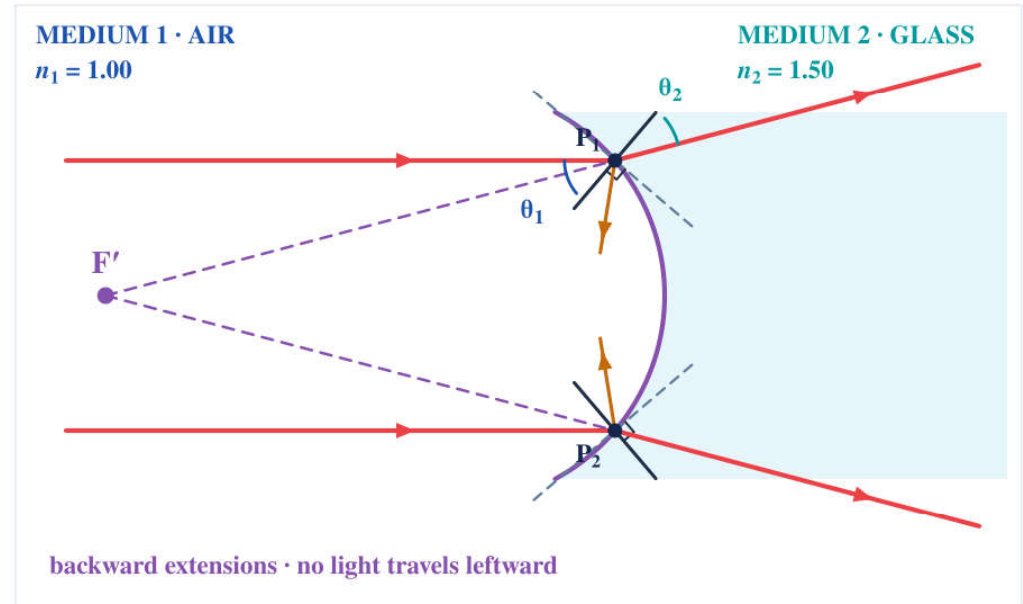
$\theta_2 < \theta_1$

Tangent $\perp$ normal

$\theta_r = \theta_i$

CONCAVE INTERFACE

The backward extensions meet at a virtual point F'.



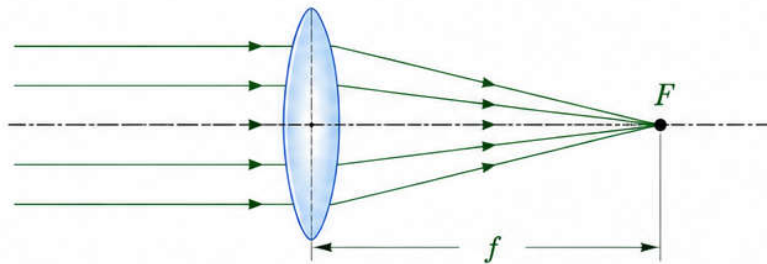
The local normals tilt oppositely to those on the convex surface.
Each ray still bends toward its own normal, yet the rays diverge.
Only the dashed backward extensions intersect at F'.

Principal Focus: What Is the Focus?

The focus is the point where parallel rays converge (or appear to diverge from).

Converging (Convex) Lens

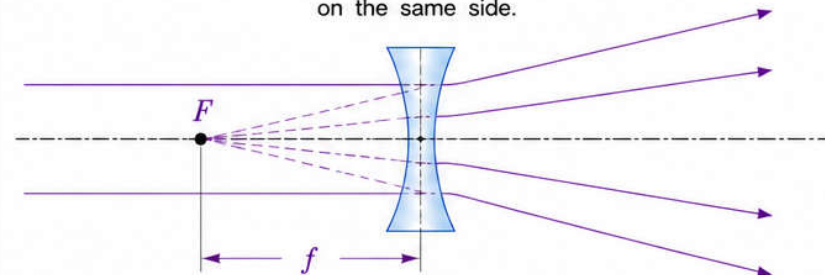
Parallel rays incident on a convex lens **converge** to a point on the other side.



The focus F is a **real point** where rays actually meet.

Diverging (Concave) Lens

Parallel rays incident on a concave lens **diverge** as if they come from a point on the same side.



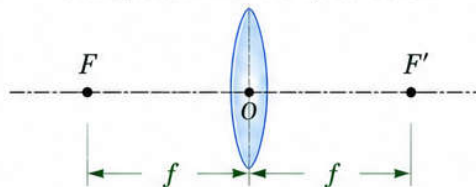
The focus F is a **virtual point** where rays appear to come from.

Key Ideas About the Focus

- The focus lies on the principal axis.
- A convex lens has a **real** focus on the far side.
- A concave lens has a **virtual** focus on the near side.
- All parallel rays (to the axis) behave this way, regardless of where they hit the lens (away from the edges).
- There are two focal points (F and F') for each lens, symmetrically located about the optical center.

Two Focal Points

Every thin lens has two focal points: F (left) and F' (right), both on the principal axis and equidistant from the optical center.



Summary Table

Property	Convex (Converging)	Concave (Diverging)
Ray behavior	Parallel rays converge	Parallel rays diverge
Focus type	Real focus	Virtual focus
Location of F	On the far side	On the near side
Focal length f	Positive ($f > 0$)	Negative ($f < 0$)



Key Takeaway

The focus is the key to image formation. It tells us where parallel light is brought together (or appears to come from). Understanding the focus is essential for using ray diagrams and the thin lens equation.



Next:
Focal Length

Focal Length and Optical Power

How strongly does a lens change the direction of light?

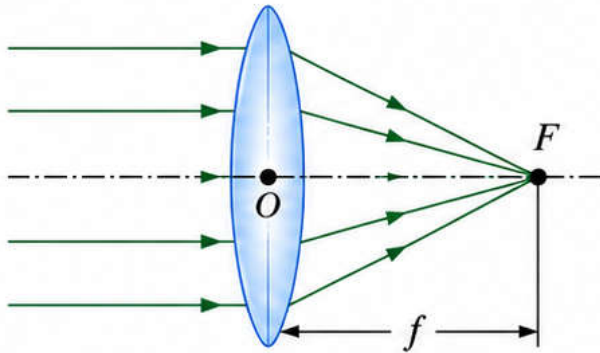
Focal Length

f : signed distance from the optical center O to the focus F .

Converging: $f > 0$

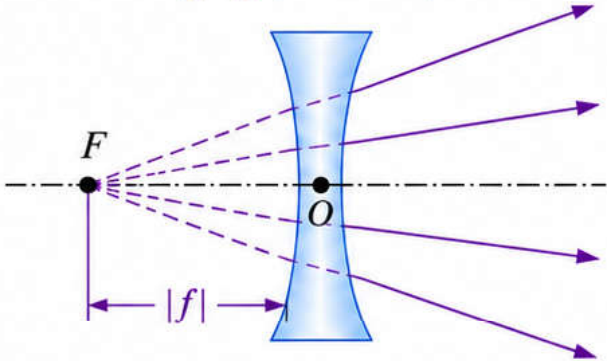
Diverging: $f < 0$

Converging (Convex) Lens



Parallel rays converge to the real focus F on the far side. $f > 0$

Diverging (Concave) Lens



Parallel rays appear to come from the virtual focus F on the same side. $f < 0$

Optical Power

The optical power P of a lens is the reciprocal of the focal length (in metres):

$$P = \frac{1}{f} \quad (f \text{ in metres})$$

Unit: diopetre (D) = m^{-1}

Converging: $P > 0$

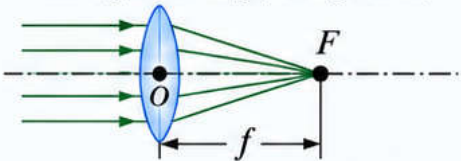
Diverging: $P < 0$

Physical Interpretation

Greater $|P|$ means stronger bending of paraxial rays and a shorter $|f|$.

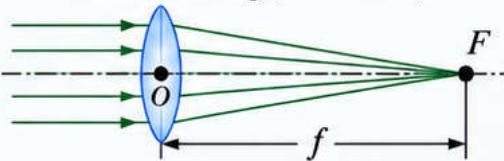
Larger $|P|$ (shorter $|f|$)

Stronger bending (stronger lens).



Smaller $|P|$ (longer $|f|$)

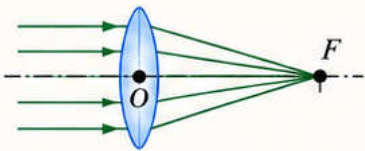
Weaker bending (weaker lens).



What the sign tells us

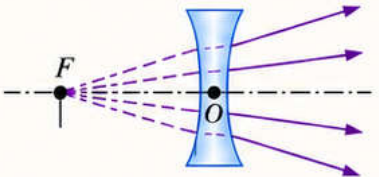
$$P > 0$$

Parallel rays meet at a real focus.



$$P < 0$$

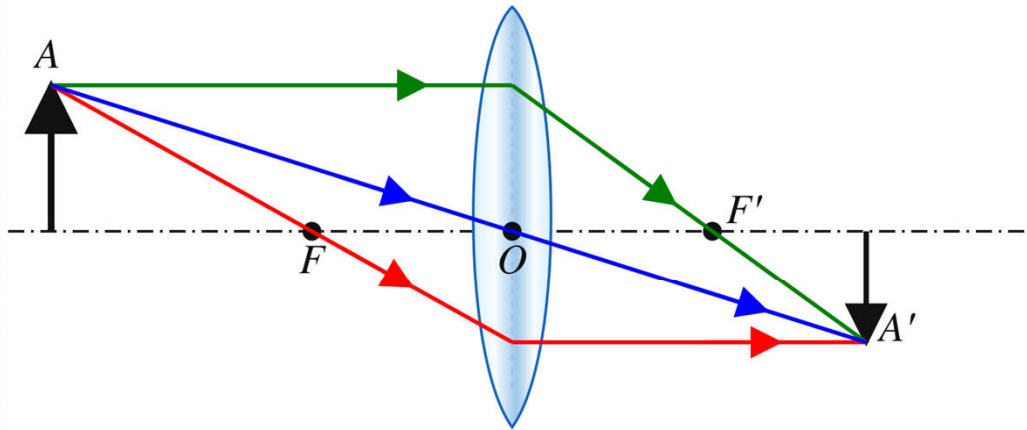
Rays spread; their backward extensions meet at a virtual focus.



Principal Rays of Thin Lenses

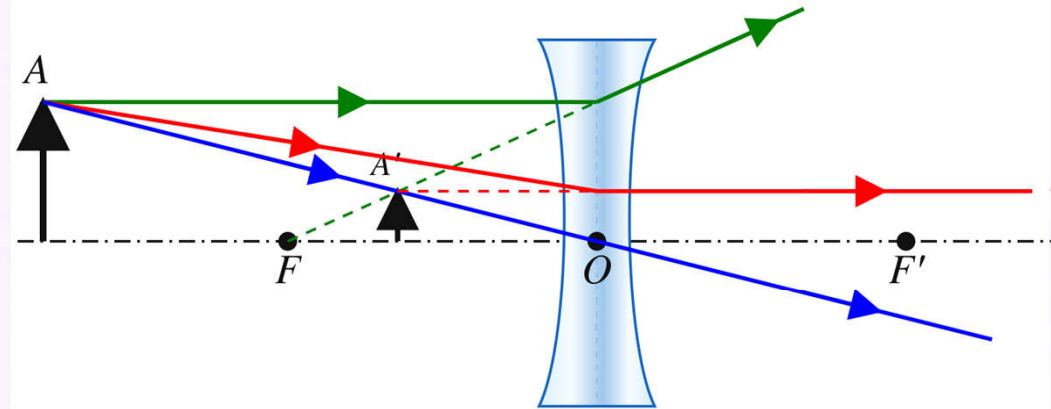
Three rays locate the image of an object point.

Converging (Convex) Lens



- 1 Parallel ray $\rightarrow$ through far focus F' .
- 2 Through near focus $F \rightarrow$ parallel.
- 3 Through $O \rightarrow$ undeviated.

Diverging (Concave) Lens



- 1 Parallel ray $\rightarrow$ appears from near focus F .
- 2 Toward far focus $F' \rightarrow$ parallel.
- 3 Through $O \rightarrow$ undeviated.

Principal optical axis:
line of symmetry through O .

O :
optical centre of the thin lens.

F, F' :
focal points.

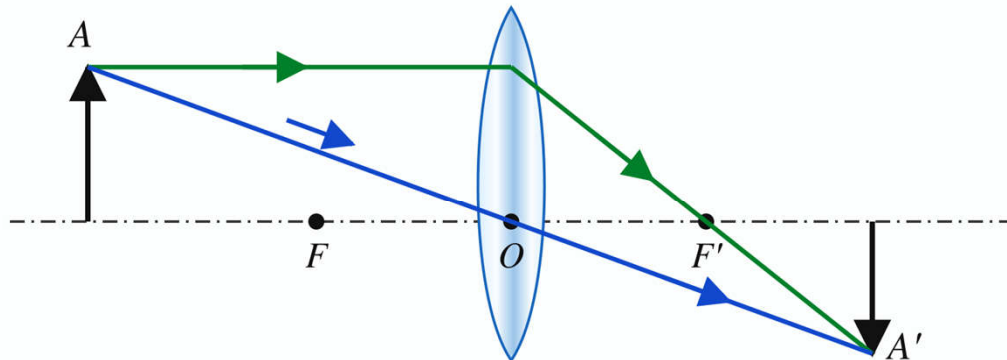
Dashed lines:
backward extensions, not physical light paths.

How to Locate an Image Point

Using two principal rays for each thin lens

Converging (Convex) Lens

Real image: actual rays intersect



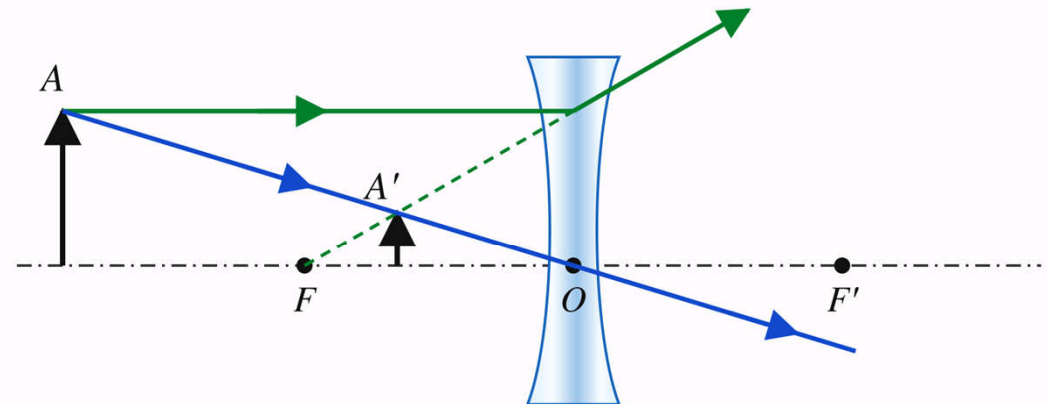
The two transmitted rays meet at A' below the axis.

- 1 Parallel ray $\rightarrow$ refracts through F' .
- 2 Ray through O $\rightarrow$ continues straight.

Image point A' : intersection of the two real rays.

Diverging (Concave) Lens

Virtual image: backward extensions meet



Extend the refracted green ray backward to meet the blue ray at A' .

- 1 Parallel ray $\rightarrow$ appears to originate at F .
- 2 Ray through O $\rightarrow$ continues straight.

Image point A' : intersection with backward extension.

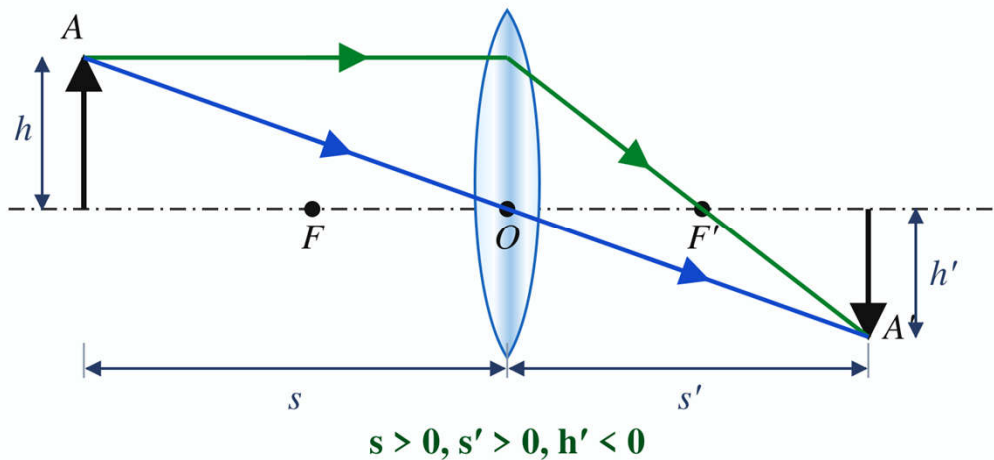
Principal optical axis: dashed horizontal line through O . F and F' : focal points.

Solid lines: actual light rays. Dashed green line: virtual backward extension.

Magnification: Size and Orientation

A signed ratio derived from similar triangles

Converging lens: real image

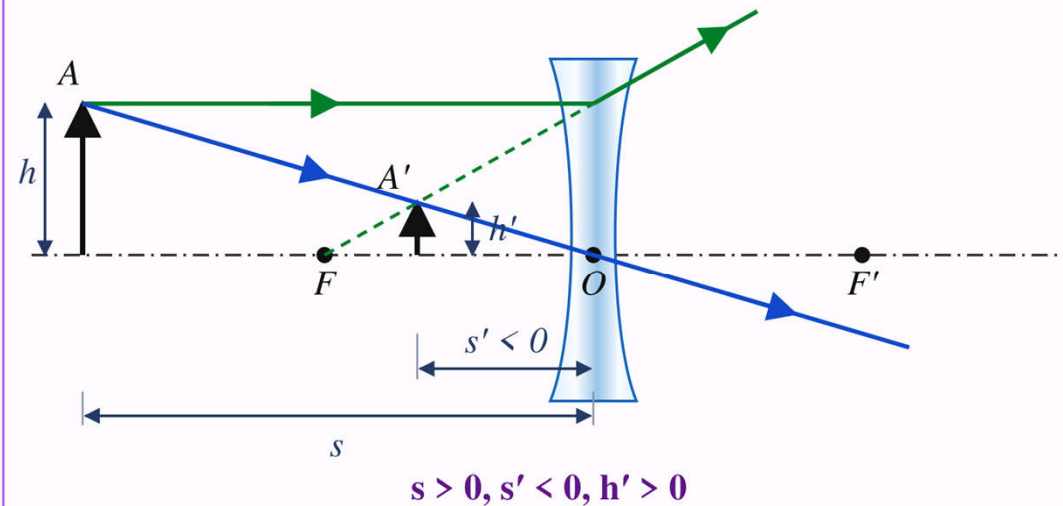


$m < 0$ The image is inverted.

$|m| < 1$ Here it is smaller than the object.

Actual rays meet: s' is positive.

Diverging lens: virtual image



$m > 0$ The image is upright.

$|m| < 1$ Here it is smaller than the object.

Back extensions meet: s' is negative.

$$m = h'/h = -s'/s$$

$|m| > 1$: enlarged; $|m| = 1$: equal; $|m| < 1$: reduced.

m is dimensionless. Upward $h > 0$; real object $s > 0$; image s' is signed.

Objects and Images: Three Ways to Classify an Image

Orientation • size • physical nature

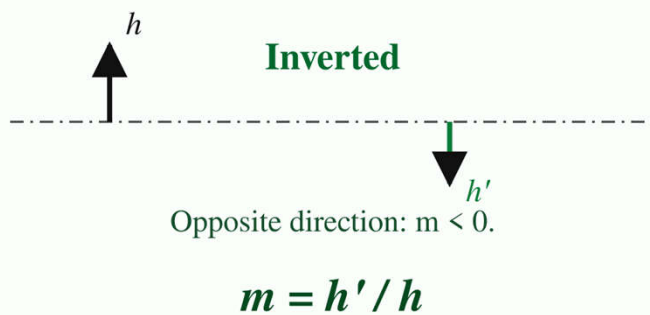
Object

A set of points from which the incident light rays originate (or appear to originate).

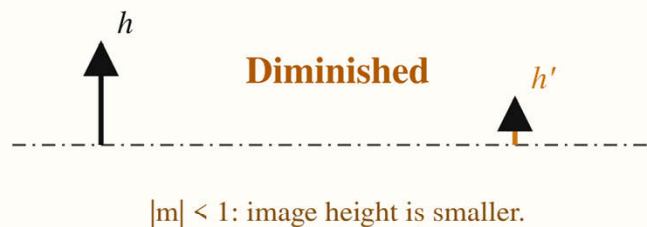
Image

The corresponding points where refracted rays meet or appear to meet when extended backward.

1. Orientation



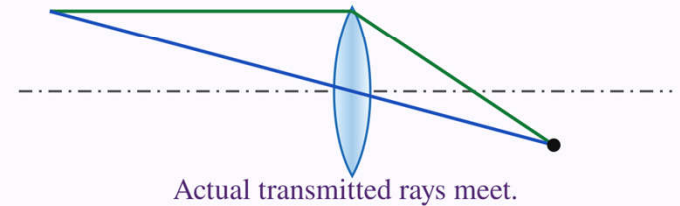
2. Relative size



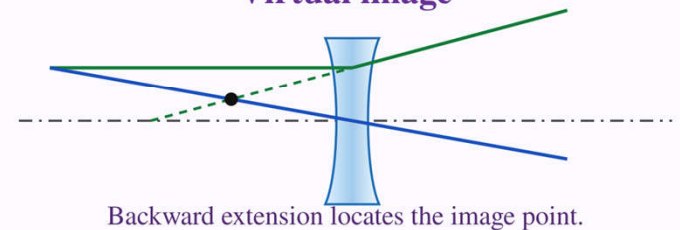
$|m| = 1$: equal size

3. Physical nature

Real image



Virtual image



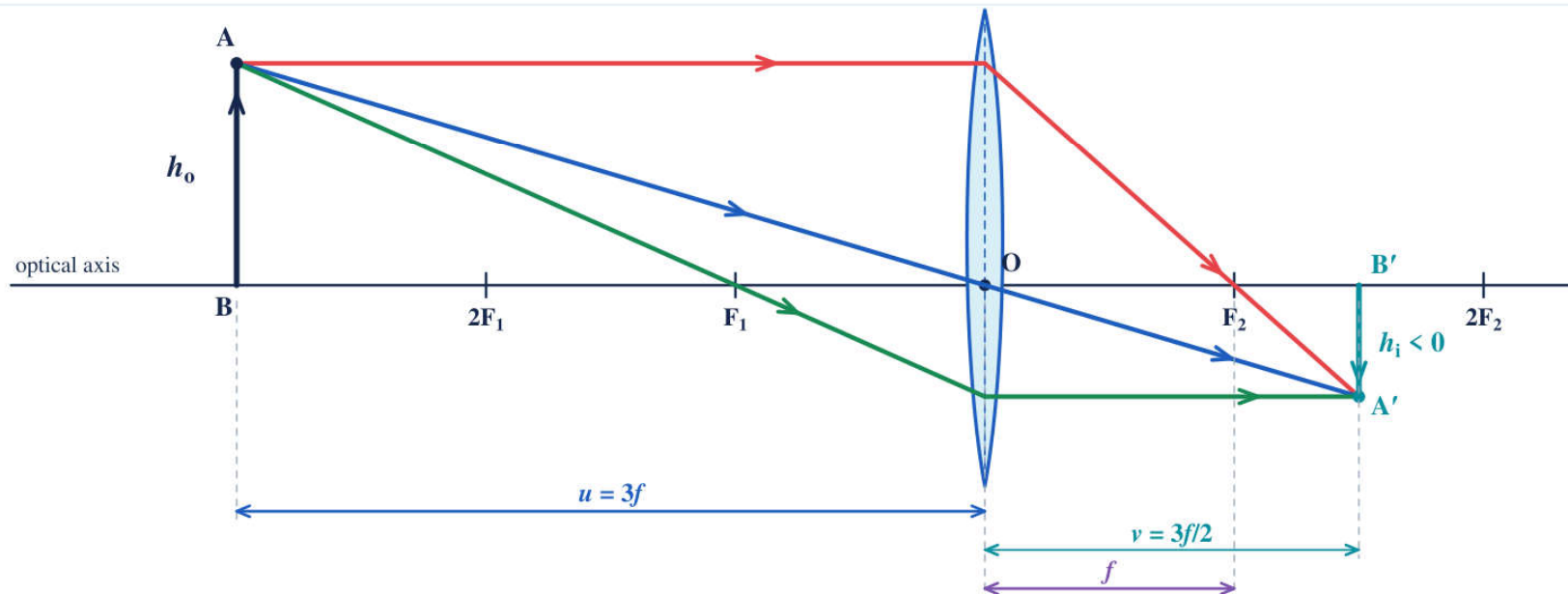
Each image has one property from each column; orientation, size and reality are separate.

For a real object and a single diverging lens: upright, diminished and virtual.

The thin-lens equation: image position and size

Three principal rays locate one image point; distances and heights give the same result algebraically.

CONVERGING THIN LENS · REAL IMAGE



THREE PRINCIPAL RAYS

- **R₁**
parallel to axis
→ passes through F₂
- **R₂**
through O
→ continues straight
- **R₃**
through F₁
→ emerges parallel

u: object distance
v: image distance
f: focal length, OF₂
O: optical center

THIN-LENS EQUATION

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$u = 3f \rightarrow v = 3f/2$$

$f > 0$ for a converging lens.

TRANSVERSE MAGNIFICATION

$$m = \frac{h_i}{h_o} = - \frac{v}{u}$$

$$m = -1/2 \rightarrow h_i = -h_o/2$$

Negative m means an inverted image.

PHYSICAL INTERPRETATION

$u = 3f > 2f \rightarrow f < v < 2f$
Image: real, inverted, half-size.
F₁ and F₂ lie one focal length from O.
Valid for a thin lens and paraxial rays.

Engineering Interpretation: Why Is Focal Length Important?

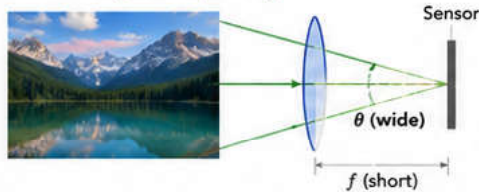


Focal length f is a key **design parameter**: it determines how an optical system maps the world into an image.

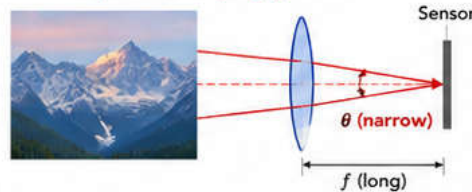
1. Field of view in cameras

- **Short focal length (wide-angle)** → **wide field of view**
More of the scene fits on the sensor.
- **Long focal length (telephoto)** → **narrow field of view**
Distant objects appear larger.

Wide-angle lens (short f)



Telephoto lens (long f)



Focal length links object angle θ , image size y , and sensor size. It is not just "magnification" — it sets the **geometry of the entire system**.

2. Focal length in other optical systems

Microscopes



Objective lens has a **short** f and large numerical aperture (NA).



High magnification and resolution.

Telescopes



Angular magnification depends on the ratio of focal lengths.



$$M_{ang} \approx \frac{f_{objective}}{f_{eyepiece}}$$

Human eye



The eye's lens adjusts its effective focal length to **focus on the retina**.



Sharp image on the **retina**.

Projectors



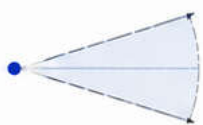
Lens focal length determines image size and **throw distance**.



Controls screen size and **brightness**.

3. What focal length controls

Field of view (angle)



Short f → wider view
Long f → narrower view

Image size on sensor



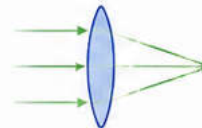
For the same object angle, **longer** f → larger image

Depth of field



Depends on f , aperture (N), and focusing distance.

Light gathering



Related to aperture diameter relative to focal length ($f/\#$).

System design



f determines size, weight, performance and cost.



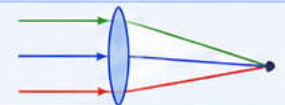
Key takeaway

- Focal length sets how an optical system **sees the world**.
- It affects angle, size, brightness, and many other **performance factors**.
- The same thin-lens equations apply, but **engineering choices** determine how the system is used.



We now understand that focal length is a central design parameter in real optical systems.

Next: instead of only equations, we will use **geometry**. We begin with **three special rays** that let us locate images easily.



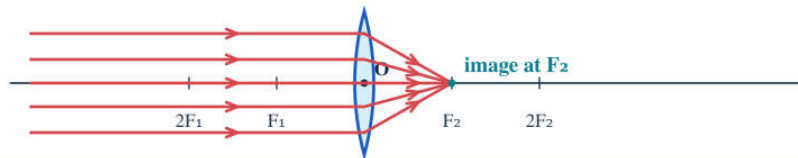
Five image-forming cases of a converging lens

For a finite object, trace the principal rays from its tip; their intersection locates the image tip.

1. AT INFINITY

$$u \rightarrow \infty$$

$$v = f; m \rightarrow 0$$



Parallel rays converge at the rear focal point.

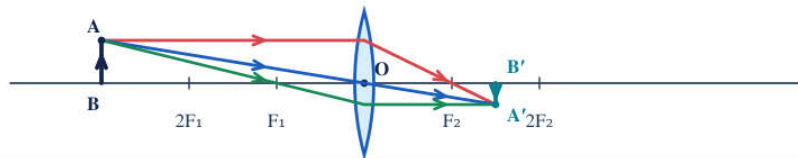
The image is real and inverted; a distant object appears tiny.

One parallel bundle represents one distant object point.

2. BEYOND $2f$

$$u > 2f$$

$$f < v < 2f; -1 < m < 0$$



The rays meet between F_2 and $2F_2$.

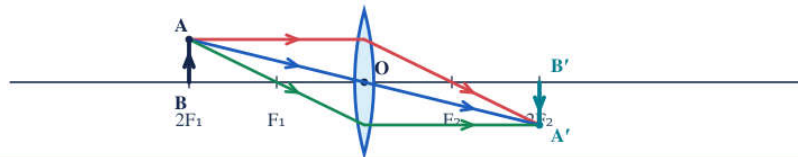
The image is real, inverted and smaller than the object.

Example: $u = 3f$ gives $v = 3f/2$ and $m = -1/2$.

3. AT $2f$

$$u = 2f$$

$$v = 2f; m = -1$$



The image forms at $2F_2$, equally far from the lens.

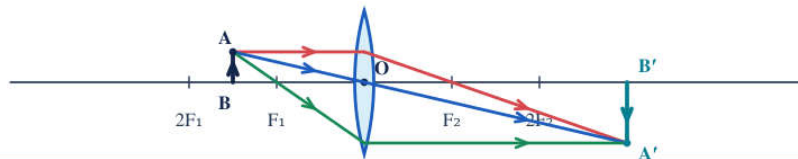
It is real, inverted and the same size as the object.

The two triangles have equal bases and heights.

4. BETWEEN f AND $2f$

$$f < u < 2f$$

$$v > 2f; m < -1$$



The rays meet beyond $2F_2$.

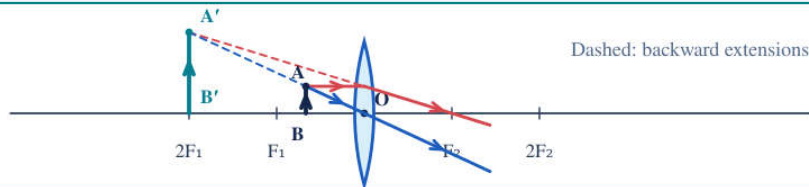
The image is real, inverted and enlarged.

Example: $u = 3f/2$ gives $v = 3f$ and $m = -2$.

5. INSIDE f

$$0 < u < f$$

$$v < 0; m > 1$$



The emerging rays diverge. Trace them backward to meet.

Their apparent image is virtual, upright and enlarged.

Example: $u = 2f/3$ gives $v = -2f$ and $m = +3$.

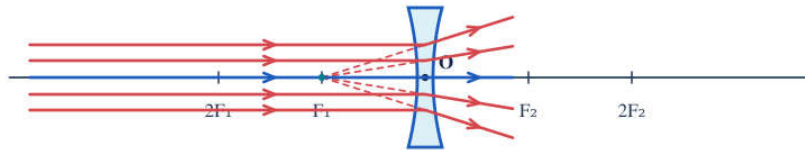
Five object positions for a diverging lens

Follow two rays from the object tip; extend the emerging rays backward to find the virtual image.

1. AT INFINITY

$$u \rightarrow \infty$$

$$v \rightarrow f = -|f|; m \rightarrow 0$$

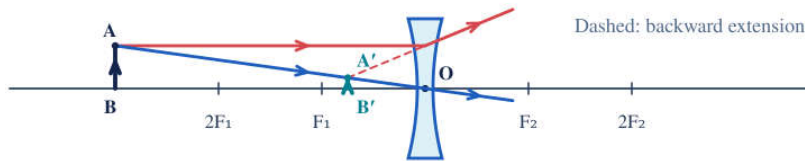


An incident parallel bundle spreads after the lens.
The backward extensions meet at the near focus F_1 .
A distant object gives a very small virtual image.

2. BEYOND $2|f|$

$$u > 2|f|$$

$$-|f| < v < -2|f|/3$$

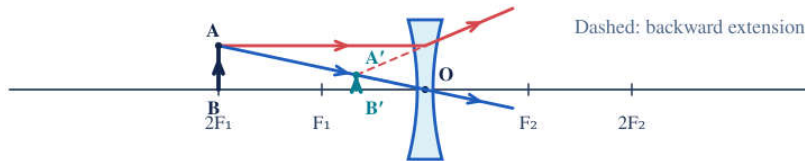


The apparent image lies between F_1 and the lens.
It is virtual, upright and smaller than the object.
Example: $u = 3|f|$ gives $v = -3|f|/4$, $m = +1/4$.

3. AT $2|f|$

$$u = 2|f|$$

$$v = -2|f|/3; m = +1/3$$

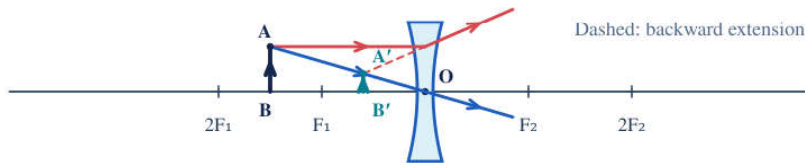


The virtual image remains on the object side.
It lies between F_1 and O, closer to the lens.
Its height is one third of the object height.

4. BETWEEN $|f|$ AND $2|f|$

$$|f| < u < 2|f|$$

$$-2|f|/3 < v < -|f|/2$$

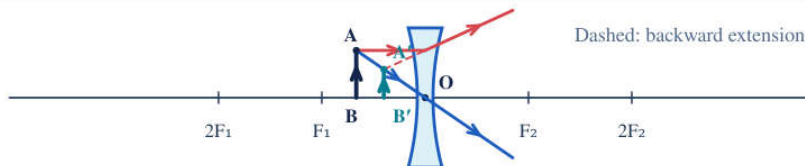


The image shifts toward the lens as u decreases.
It stays virtual, upright and diminished.
Example: $u = 3|f|/2$ gives $v = -3|f|/5$, $m = +2/5$.

5. INSIDE $|f|$

$$0 < u < |f|$$

$$-|f|/2 < v < 0; 1/2 < m < 1$$



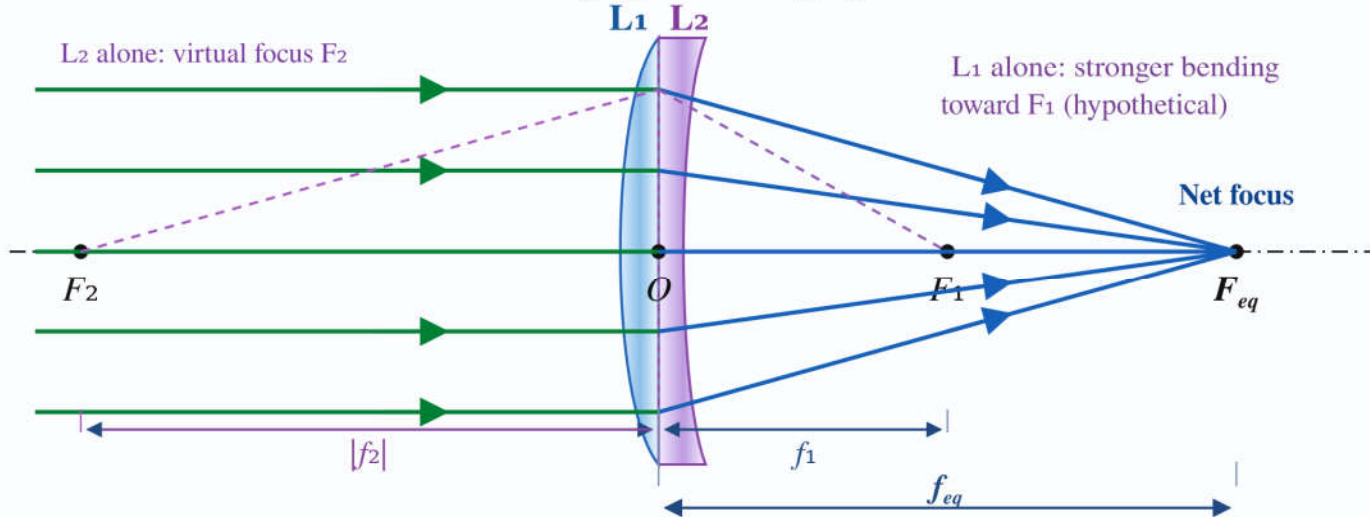
The image lies between the object and the lens.
It remains virtual, upright and smaller than the object.
Example: $u = 2|f|/3$ gives $v = -2|f|/5$, $m = +3/5$.

Two Thin Lenses in Contact

Their optical powers add when the separation is negligible.

Parallel rays through the touching pair

Converging L₁ + diverging L₂



Equivalent single lens

Same emergent rays and focal point

$$P_{eq} = P_1 + P_2$$

$$P = 1/f \text{ (f in metres)}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$f_1 = +10 \text{ cm}$$

$$f_2 = -20 \text{ cm}$$

$$\text{Net power: } +10 \text{ D} - 5 \text{ D} = +5 \text{ D}$$

$$\text{Effective focal length: } +20 \text{ cm}$$

1 Sign and strength

P₁ > 0 bends rays toward the axis.

P₂ < 0 opposes that convergence.

Net power > 0: the pair converges.

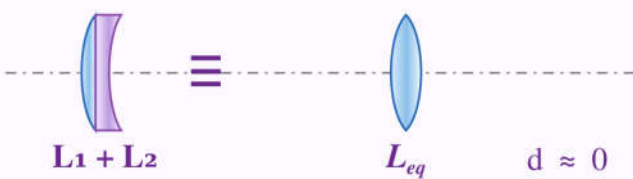
2 Same emergent direction

At each height, angular changes add:

$$\Delta\theta_{eq} \approx -y/f_1 - y/f_2 = -y/f_{eq}$$

The pair acts as one thin lens.

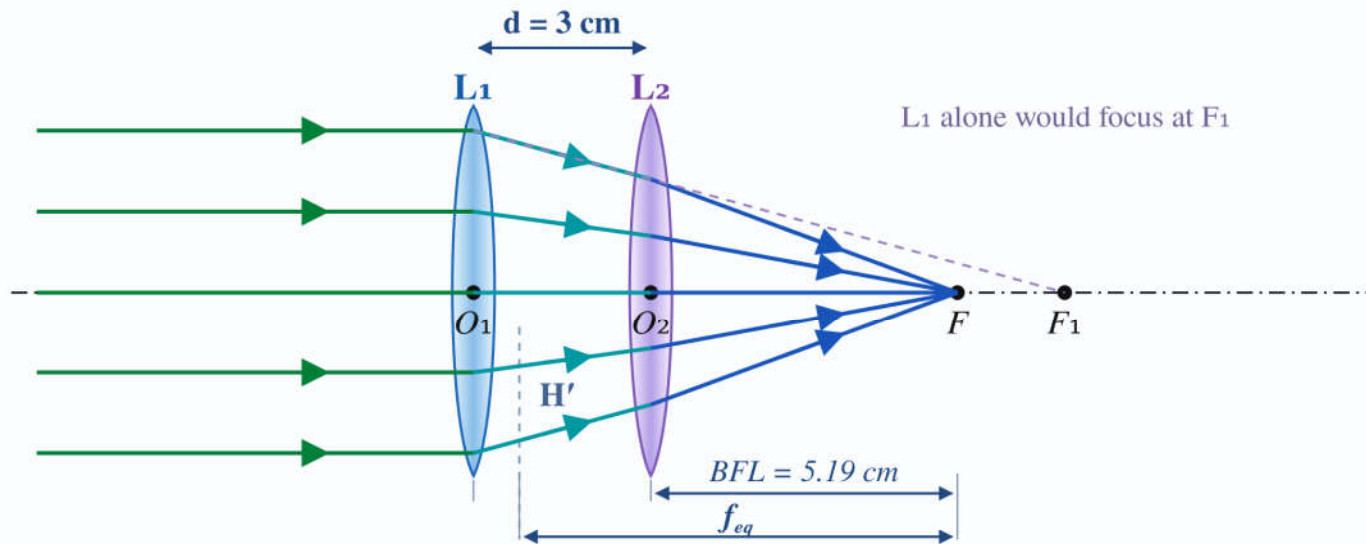
3 Graphical equivalence



Two Thin Lenses Separated by a Distance

The first lens changes the ray height at the second lens.

Two refractions and one final focus



Equivalent power

For a gap d in the same medium:

$$P_{eq} = P_1 + P_2 - d \cdot P_1 \cdot P_2$$

$$f_1 = +10 \text{ cm}; \quad f_2 = +20 \text{ cm}$$

$$d = 3 \text{ cm} = 0.03 \text{ m}$$

$$P_1 = +10 \text{ D}; \quad P_2 = +5 \text{ D}$$

$$P_{eq} = 10 + 5 - 0.03 \times 10 \times 5$$

$$= +13.5 \text{ D}$$

$$f_{eq} = 1/P_{eq} = 7.41 \text{ cm}$$

$$\text{BFL} = 5.19 \text{ cm from } L_2$$

1 Trace the rays in sequence

L1 bends parallel rays toward its F_1 .

At L2 the rays are already converging.

L2 bends them again toward final F.

2 Why separation matters

At L2, the ray height is

$$y_2 = y_1 (1 - d/f_1).$$

The correction is $-d \cdot P_1 \cdot P_2$.

3 Two focal distances

Effective f is measured from H' .

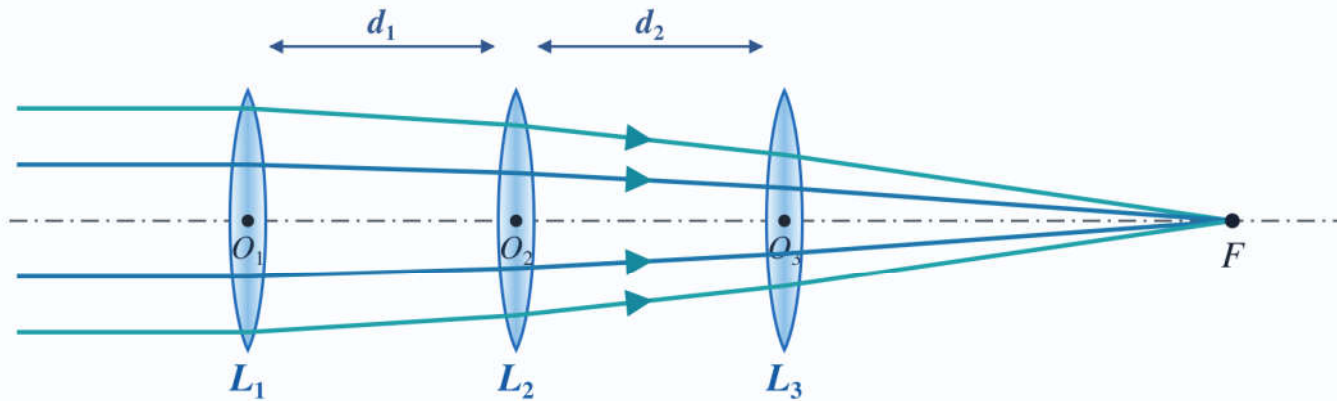
Back focal length starts at L_2 .

They coincide only when $d \rightarrow 0$.

Multiple Thin Lenses: General Method

Paraxial matrix method: travel through a gap, then bend at a lens

Follow the same rays through every lens



L_j = lens; O_j = optical centre; d_j = next gap; F = final focus.

Two elementary operations

A. Travel a distance d

$$y' = y + d\theta; \quad \theta' = \theta$$

$$T(d) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \quad \text{height changes; angle stays}$$

B. Cross a thin lens

$$y' = y; \quad \theta' = \theta - y/f$$

$$L(f) = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \quad \text{height stays; angle changes}$$

$r = (y, \theta)^T$: y height; θ angle, up positive (radians).

d gap; $f > 0$ converging, $f < 0$ diverging; ' means after.

1 Apply the operations in order

$$r_1 = L(f_1) r_{\text{in}}$$

$$r_{j+1} = L(f_{j+1}) T(d_j) r_j$$

r_{in} before L_1 ; r_j just after L_j .

j numbers lenses; $r_{\text{out}} = r_N$ after the last lens.

2 Read the system result

$$r_{\text{out}} = M r_{\text{in}}; \quad M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \begin{array}{l} A, B, C, D: \text{entries of } M. \\ A, D: \text{dimensionless}; B: \text{length}. \\ C: \text{inverse length}. \end{array}$$

$$f_{\text{eq}} = -1/C$$

effective focal length, from rear principal plane

$$\text{BFL} = -A/C$$

back focal length, from the last lens.

Worked example 1: one screen, two lens positions

A fixed object–screen distance makes lens placement a geometric problem.

PROBLEM

An object stands at $x = 0$.
A screen stands at $x = 90$ cm.
A lens with $f = 20$ cm slides between them.
Find every lens position giving a sharp image.
Compare image size and orientation.

WHAT MUST BE FOUND

- 1 All permitted object distances u
- 2 Corresponding screen distances v
- 3 Image orientation and height ratio m
- 4 Why two placements may exist

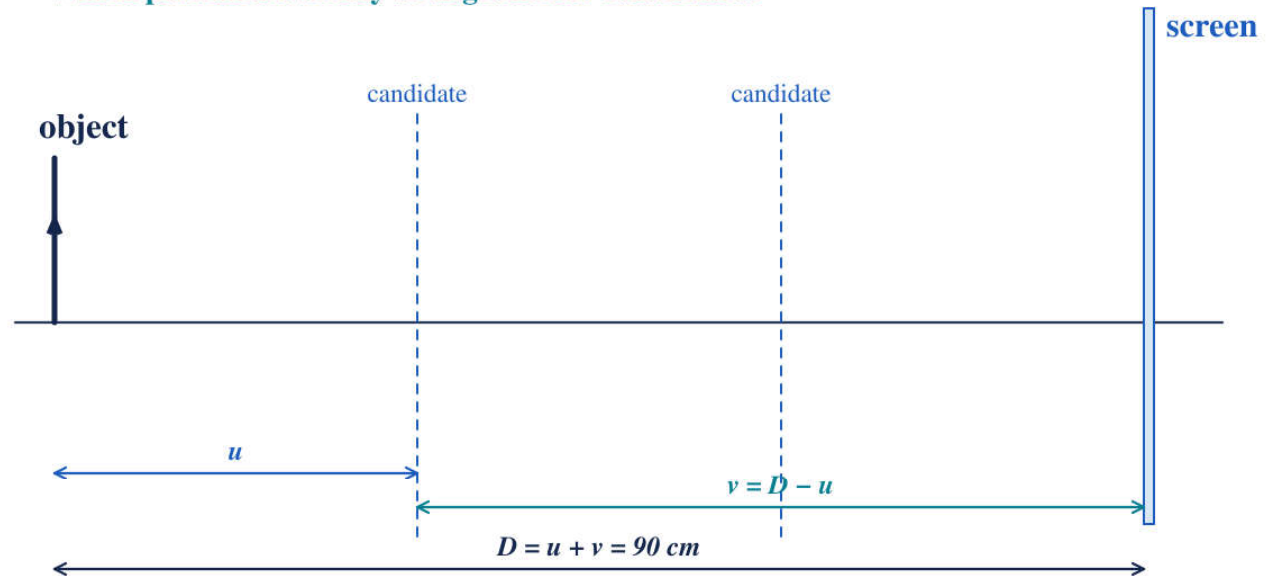
Constraint: $u + v = D = 90$ cm.

For a sharp screen image, v must be positive.
Use the thin-lens and paraxial model.

FIXED OBJECT AND SCREEN

The lens moves; the object and screen do not.

Which placements satisfy both geometric constraints?



The screen is fixed: changing u automatically changes v .
A screen can record only a real image, so $v > 0$.

Worked example 1: solve for both lens positions

Use the screen constraint before substituting numerical values.

ANALYTICAL SOLUTION

Start with the thin-lens equation.

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Set $v = D - u$; combine the fractions.

$$u(D - u) = fD$$

Expand and move terms to the left.

$$u^2 - Du + fD = 0$$

Insert $D = 90 \text{ cm}$ and $f = 20 \text{ cm}$.

$$u^2 - 90u + 1800 = 0$$

$$u = (90 \pm \sqrt{900})/2$$

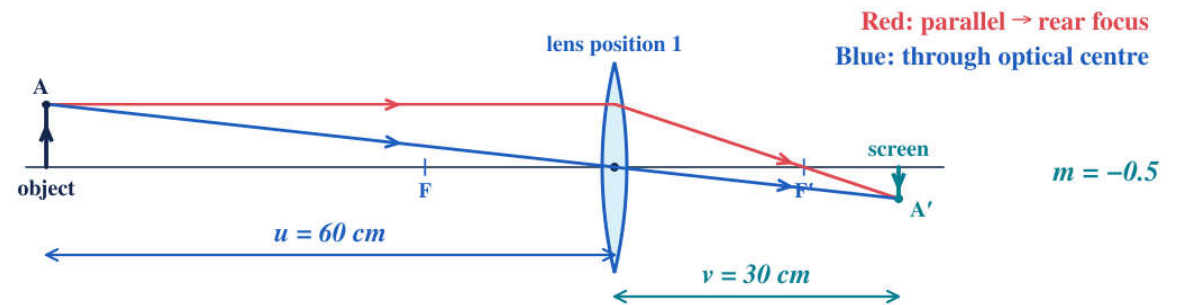
$$u_1 = 60 \text{ cm} \quad u_2 = 30 \text{ cm}$$

$$v_1 = 30 \text{ cm} \quad v_2 = 60 \text{ cm}$$

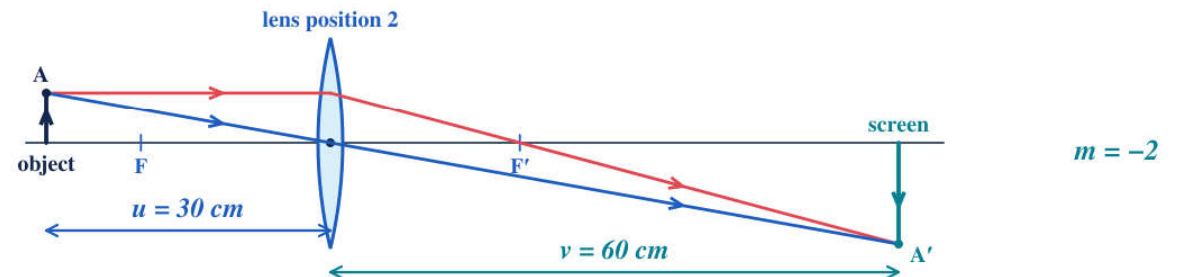
$$m_1 = -1/2 \quad m_2 = -2$$

Both have $u > f$ and $v > 0$: real images.

TWO EXACT RAY CONSTRUCTIONS



The larger object distance gives the smaller image.



Moving the lens toward the object enlarges the image.

Worked example 1: what do the two roots mean?

Interpret the exchange of object and image distances, then test when solutions exist.

SWAPPING OBJECT AND IMAGE DISTANCES

Lens position 1

$$u = 60 \text{ cm } v = 30 \text{ cm}$$

$$m = -1/2 \cdot \text{real, inverted}$$

The image is half as tall.



Lens position 2

$$u = 30 \text{ cm } v = 60 \text{ cm}$$

$$m = -2 \cdot \text{real, inverted}$$

The image is twice as tall.



Lens displacement between the positions: $|60 - 30| = 30 \text{ cm}$.

EXISTENCE AND PHYSICAL INTERPRETATION

Real lens positions require a nonnegative discriminant:

$$\Delta = D^2 - 4fD = D(D - 4f) \geq 0$$

$$\text{Therefore } D \geq 4f = 80 \text{ cm.}$$

$$D > 4f$$

Two distinct lens positions; here $D = 90 \text{ cm}$.

$$D = 4f$$

One position at $u = v = 2f$; equal-size image.

$$D < 4f$$

No lens position brings a real image onto the screen.

The two magnification magnitudes differ by a factor of four.

Worked example 3: two separated thin lenses

The first image becomes the object used in the second lens equation.

PROBLEM

Two converging lenses stand 60 cm apart.

L_1 has $f_1 = 12$ cm; L_2 has $f_2 = 16$ cm.

An object is 18 cm before L_1 .

Locate the intermediate and final images.

Find their orientation and magnification.

Stage 1: original object $\rightarrow$ intermediate image

Stage 2: intermediate image $\rightarrow$ final image

Keep all positions on one optical axis.

Does L_2 receive a real object or a virtual one?

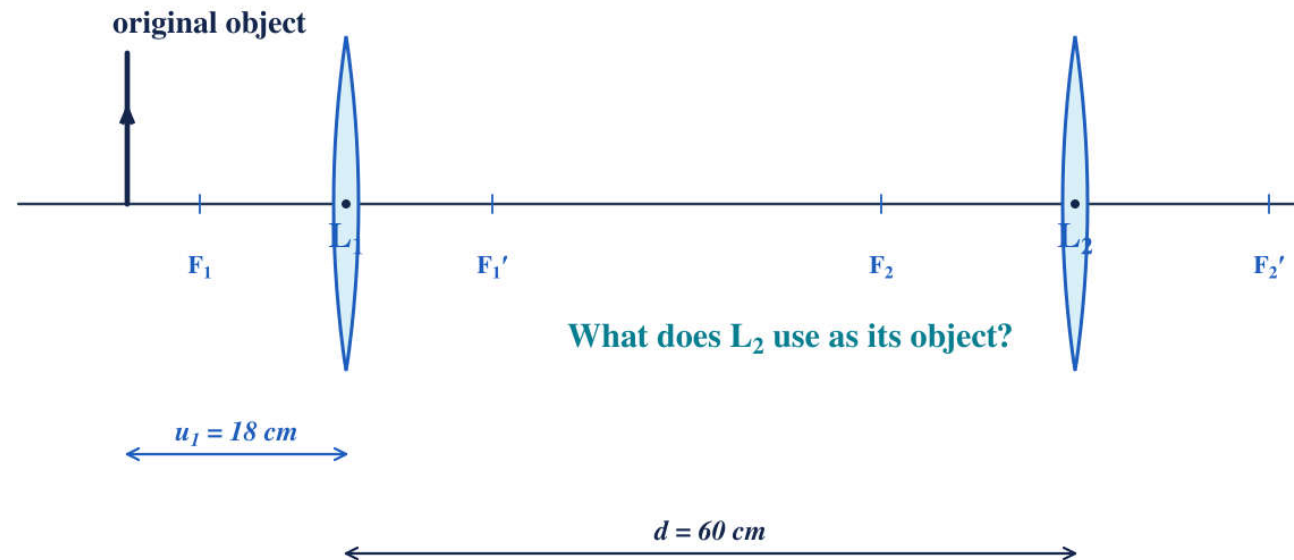
Can the final image be projected on a screen?

Unknown: v_1 , u_2 , v_2 , m_1 and m_2 .

GIVEN GEOMETRY

Follow the first image through the second lens.

Where is the first image?

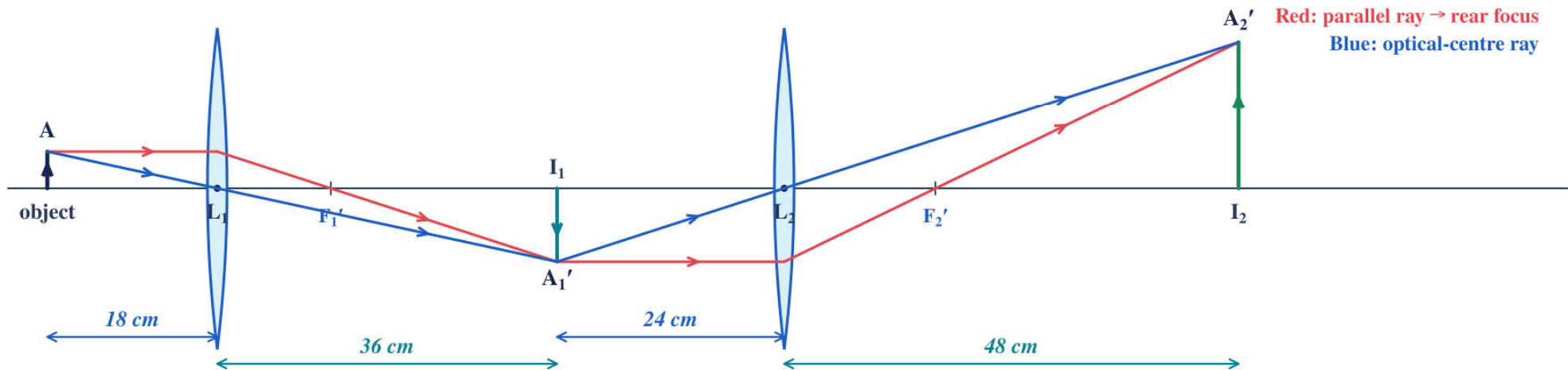


What does L_2 use as its object?

Worked example 3: propagate the image through both lenses

The second calculation begins with the position of the intermediate image.

RAY CONSTRUCTION: ONE OBJECT POINT THROUGH TWO LENSES



STAGE 1 · LENS L_1

Use $u_1 = 18$ cm and $f_1 = 12$ cm:

$$1/12 = 1/18 + 1/v_1 \rightarrow v_1 = 36 \text{ cm}$$

$$m_1 = -v_1/u_1 = -2$$

I_1 is real, inverted and twice as tall.

It forms 36 cm beyond L_1 .

STAGE 2 · LENS L_2

I_1 now serves as a real object for L_2 :

$$u_2 = d - v_1 = 60 - 36 = 24 \text{ cm}$$

$$1/16 = 1/24 + 1/v_2 \rightarrow v_2 = 48 \text{ cm}$$

$$m_2 = -v_2/u_2 = -2$$

I_2 is real, 48 cm beyond L_2 .

Worked example 3: interpret the final image

Follow the image orientation across stages and track what a screen can capture.

MAGNIFICATION AND ORIENTATION

$$m = m_1 m_2 = (-2)(-2) = +4$$

Two inversions restore the upright orientation.

Original object

height $h_o > 0$



After L_1

$h_1 = -2h_o$ (inverted)



After L_2

$h_2 = +4h_o$ (upright)



A screen 48 cm beyond L_2 records the final image.

WHY THE SEPARATION MATTERS

The intermediate image is 36 cm beyond L_1 .

Hence L_2 sees a real object at $u_2 = d - 36$ cm.

At $d = 60$ cm:

$u_2 = 24$ cm $>$ $f_2 = 16$ cm, giving a real I_2 .

At $d = 52$ cm:

$u_2 = f_2$; rays leave L_2 parallel.

For 36 cm $< d < 52$ cm:

$0 < u_2 < f_2$; the final image is virtual.

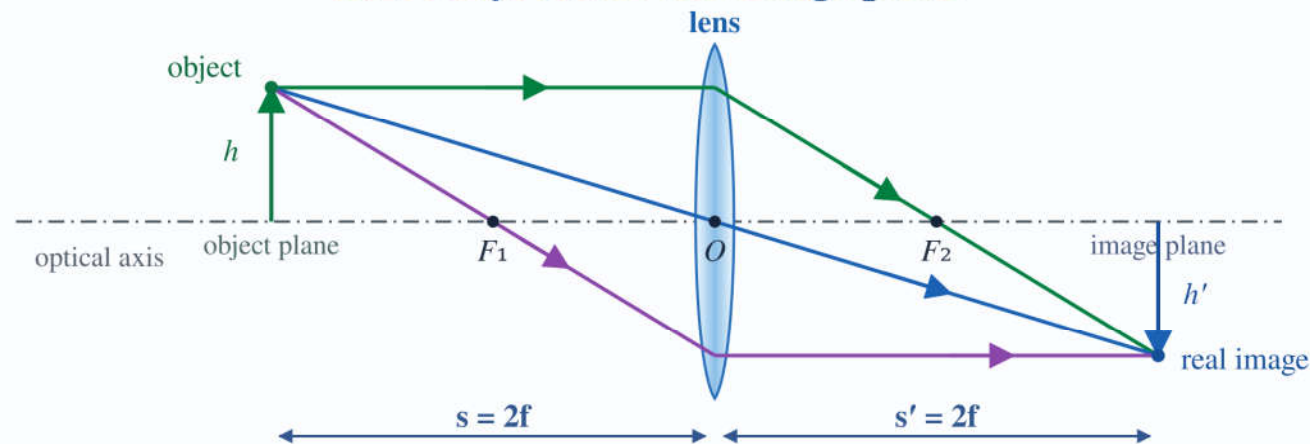
The image type changes when u_2 crosses f_2 .



Summary I: Imaging with One Thin Lens

Geometry, equations, signs, and the physical meaning of the image

Three rays locate one image point



This construction has $s = s' = 2f$, so the real image is inverted and equal in size.

Relations for a thin lens

s : object distance s' : image distance

f : signed focal length O : lens centre

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = \frac{h'}{h} = -\frac{s'}{s}$$

h and h' : signed object and image heights.

$$P = \frac{1}{f} \quad f \text{ in metres, } P \text{ in dioptres (D).}$$

Paraxial rays; distances measured from O .

1 Named rays

Parallel to axis $\rightarrow$ through rear focus F_2 .

Through $O \rightarrow$ straight line (thin lens).

Through front focus $F_1 \rightarrow$ parallel.

Two correct rays suffice to locate the tip.

2 Signed distances and lens type

$s > 0$: real object to the left.

$s < 0$: virtual object to the right.

$s' > 0$: real image to the right.

$s' < 0$: virtual image to the left.

$f > 0$ converging; $f < 0$ diverging.

3 Orientation and size

$m < 0$: inverted; $m > 0$: upright.

$|m| > 1$ enlarged; $|m| < 1$ reduced.

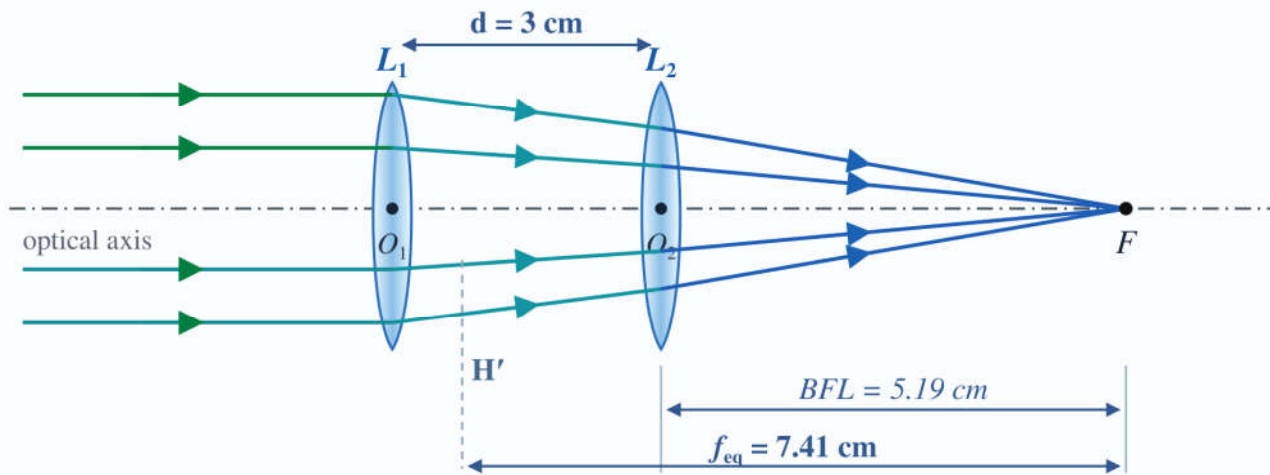
Real image: outgoing rays meet.

Virtual image: backward extensions meet.

Summary II: Systems of Thin Lenses

Sequence, equivalent power, and the reference plane for focal length

The second lens receives converging rays



Contact and separation

Lenses in contact ($d = 0$)

$$P_{eq} = P_1 + P_2$$

Lenses separated by d in one medium

$$P_{eq} = P_1 + P_2 - d \cdot P_1 \cdot P_2$$

$$P_j = 1/f_j; \quad d \text{ and } f \text{ in m, } P \text{ in D.}$$

$$P_1 = 10 \text{ D}; \quad P_2 = 5 \text{ D}; \quad d = 0.03 \text{ m}$$

$$P_{eq} = 13.5 \text{ D}; \quad f_{eq} = 7.41 \text{ cm}$$

BFL from L_2 ; f_{eq} from H' .

1 Construct successive images

$$1/s_j + 1/s'_j = 1/f_j$$

$$s_{j+1} = d_j - s'_j$$

The preceding image becomes the next lens's object.

$$m_{total} = \prod (-s'_j/s_j)$$

Use signed distances even for virtual intermediate images.

2 Trace any paraxial ray

$$r = (y, \theta)^T; \quad r_{j+1} = L(f_{j+1}) T(d_j) r_j$$

y : ray height; θ : angle. $T(d)$ changes y ; $L(f)$ changes θ .

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \begin{aligned} f_{eq} &= -1/C \\ \text{BFL} &= -A/C \end{aligned}$$

M : system matrix; A, C : entries ($C \neq 0$); H' : rear principal plane.