

PHYSICS FOR ENGINEERING STUDENTS

Diffraction and Interference

Slit and fringes

Lecture 6 — Book-style student notes

Learning focus

Wave nature of light

Huygens' principle

Coherence

Path difference

Intensity distribution

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1. The wave nature of light

Geometrical optics describes light with rays and is extremely successful when apertures and obstacles are much larger than the wavelength. It predicts reflection, refraction and image formation, but it cannot explain every observation. When light passes through a narrow opening, it spreads into the region behind the aperture. When waves from two openings overlap, alternating bright and dark bands appear. These phenomena are diffraction and interference, direct consequences of the wave nature of light.

Diffraction is the redistribution of a wave when an aperture or obstacle restricts its wavefront. Interference is the intensity pattern created when coherent waves superpose. They are not unrelated effects: a single-slit diffraction pattern is itself produced by interference among contributions from different points across one aperture. A double-slit pattern is interference between two principal sources, usually modulated by the diffraction envelope of the finite slit widths.

Central strategy: begin with the physical sources, determine the path difference to the observation point, convert it to phase difference, and then decide whether the contributions reinforce or cancel.

The scale that controls diffraction

The ratio of aperture size a to wavelength λ determines how strongly a wave spreads. If a is many wavelengths wide, diffraction angles are small and the ray model is often adequate. If a is comparable with λ , spreading becomes pronounced. This is why audible sound bends noticeably around ordinary doorways while visible light, whose wavelength is only a few hundred nanometres, usually produces obvious diffraction only at very small apertures.

This chapter first develops the single-slit pattern using Huygens' principle and pairwise cancellation. It then treats Young's double-slit experiment, derives fringe positions and spacing, and shows how measurements of an interference pattern can reveal wavelengths or microscopic separations.

2. Huygens' principle and the origin of diffraction

Huygens' principle states that every point on a wavefront can be regarded as a source of secondary wavelets. After a short time, the new wavefront is the envelope tangent to these wavelets. For an unobstructed plane wave in a homogeneous medium, the wavelets reconstruct another plane wave, so the front remains plane. Curvature does not appear spontaneously.

What changes at a slit

When an opaque barrier contains a slit, only points inside the opening transmit the incident wave. These points emit secondary wavelets into the region beyond the barrier. The blocked parts of the original wavefront do not contribute. Because the active source region is finite, the envelope spreads laterally instead of remaining perfectly plane. This spreading is diffraction.

The aperture therefore acts as a new distributed source. Each position across the slit contributes a field at an observation point P. Contributions from different positions travel slightly different distances and arrive with different phases. The observed field is their vector sum, and the observed intensity is proportional to the squared magnitude of that total field.

Path difference $\Delta r \rightarrow$ phase difference $\Delta\phi = (2\pi/\lambda)\Delta r \rightarrow$ resultant amplitude $\rightarrow$ intensity

Fraunhofer and Fresnel regimes

The lecture focuses on the Fraunhofer, or far-field, pattern. In this regime the observation distance is sufficiently large, or lenses are used, so rays from different parts of the slit to a given screen point are approximately parallel. A single angle θ then describes the observation direction. In the Fresnel, or near-field, regime, wavefront curvature and detailed distances must be retained, and the pattern changes with propagation distance.

Model boundary: the familiar formula $a \sin\theta = m\lambda$ for single-slit minima is a Fraunhofer result for a uniformly illuminated slit. Applying it blindly in the near field can give the wrong pattern.

Coherence across the aperture

A monochromatic plane wave illuminates all points of the slit with a stable phase relationship. That coherence allows systematic cancellation. If phases fluctuated randomly, the stable dark and bright structure would wash out. Coherence is therefore not an optional mathematical detail; it is the physical condition that makes an interference pattern observable.

3. Geometry of a single slit: path and phase difference

Consider a slit of width a illuminated normally by monochromatic light of wavelength λ . Let y measure position across the aperture and let P be observed at angle θ from the forward direction. In the far field, two points separated by a small distance dy across the slit have a path difference

$$d(\Delta r) = dy \sin\theta$$

The corresponding phase difference is

$$d\phi = (2\pi/\lambda)dy \sin\theta$$

Across the full width a , the path difference between the extreme rays is $a \sin\theta$ and the total phase change is

$$\Delta\phi_{\text{total}} = (2\pi a/\lambda) \sin\theta$$

At $\theta = 0$, every contribution travels essentially the same distance, so all fields arrive in phase and the central intensity is maximum. Away from the axis, phase changes continuously across the aperture. The total field must be obtained by adding contributions with every intermediate phase, not merely the two edge rays.

Why edge rays alone are insufficient

A common misconception is to look only at the top and bottom of the slit. For the first minimum, their path difference is one wavelength, so those two edge contributions are in phase. This does not mean the total field is maximum. The cancellation occurs among pairs distributed throughout the aperture. The complete aperture—not just its endpoints—must be considered.

From path difference to an angular pattern

The path difference depends on $\sin\theta$, so the pattern is naturally described as a function of angle. A distant screen converts this angular distribution into positions. If the screen is a distance L away and the angles are small, $\tan\theta \approx \sin\theta \approx y/L$. The distinction between angle θ and screen coordinate y should be maintained until the approximation is explicitly introduced.

Quantity	Meaning	Relation
a	slit width	aperture dimension
θ	observation angle	measured from the forward axis
L	slit-to-screen distance	far-field geometry
y	screen coordinate	$y = L \tan\theta \approx L \sin\theta$
λ	wavelength	sets phase change per unit path

Unit check: a , λ , L and y must be expressed in compatible length units. The arguments of sine and every phase difference must be dimensionless.

4. Derivation of the single-slit minima

The first minimum

Choose an angle for which $a \sin\theta = \lambda$. Divide the slit into two equal halves. Pair every point in the upper half with the point $a/2$ lower. The path difference within each pair is

$$\Delta r_{\text{pair}} = (a/2)\sin\theta = \lambda/2$$

A half-wavelength path difference corresponds to a phase difference π , so the two fields in every pair are exactly opposite and cancel. Because every point has a partner, the resultant amplitude is zero. This is the first dark fringe.

The second and general minima

For a $\sin\theta = 2\lambda/a$, divide the aperture into four equal strips and pair contributions separated by $a/4$. Again each pair differs by $\lambda/2$ and cancels. For a $\sin\theta = m\lambda/a$, divide the slit into $2m$ equal strips. Pair points separated by $a/(2m)$. Then

$$\Delta r_{\text{pair}} = [a/(2m)]\sin\theta = [m\lambda/(2m)] = \lambda/2$$

Every pair cancels, producing the general condition

$$a \sin\theta_m = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \dots$$

The value $m = 0$ must be excluded from the minimum condition. At $\theta = 0$, the equation would give $0 = 0$, but all contributions are in phase and the central maximum occurs. The integer m labels dark-fringe order on either side of the axis.

Screen positions

For small angles, $\sin\theta_m \approx \tan\theta_m \approx y_m/L$. Therefore

$$y_m \approx m\lambda L/a$$

Successive minima on one side are separated approximately by $\lambda L/a$. The central maximum lies between $m = -1$ and $m = +1$, so its width is twice that value:

$$w_{\text{central}} \approx 2\lambda L/a, \quad \Delta\theta_{\text{central}} \approx 2\lambda/a$$

Interpretation: a narrower slit gives a wider diffraction pattern. Confining the wave more strongly in space necessarily spreads its propagation directions more widely.

5. The single-slit intensity distribution

Pairwise cancellation locates the minima, but the complete intensity curve requires summing the continuously varying phases across the slit. Treating each narrow strip as a phasor and integrating across a uniformly illuminated aperture gives the field amplitude

$$E(\theta) = E(0) [\sin\beta/\beta], \quad \beta = (\pi a/\lambda)\sin\theta$$

Because intensity is proportional to the square of field amplitude,

$$I(\theta) = I_0 [\sin\beta/\beta]^2$$

At $\theta = 0$, both numerator and denominator tend to zero, but $\lim(\beta \rightarrow 0) \sin\beta/\beta = 1$, so $I(0) = I_0$. Minima occur when $\sin\beta = 0$ with $\beta \neq 0$, hence $\beta = m\pi$ and $a \sin\theta = m\lambda$, matching the cancellation argument.

Central and secondary maxima

The central maximum is the brightest and widest feature. Secondary maxima occur between adjacent minima and decrease rapidly in intensity. Their exact positions follow from differentiating $(\sin\beta/\beta)^2$, giving $\tan\beta = \beta$ for nonzero extrema. For many introductory calculations it is enough to know that side maxima lie approximately between minima and are much weaker than the central maximum.

Parameter dependence

Change	Angular effect	Screen effect at fixed L
Increase wavelength λ	pattern widens	minima move farther apart
Increase slit width a	pattern narrows	minima move closer to centre
Increase screen distance L	angles unchanged	linear pattern expands
Increase incident intensity	positions unchanged	all intensities increase

The angular pattern depends on λ/a , not on L . Moving the screen changes the physical spacing in metres but does not change the diffraction angles. This distinction is important when comparing optical systems of different lengths.

Graph-reading rule: amplitude A and intensity I are not interchangeable. Intensity is proportional to amplitude squared, so a field amplitude reduced to one half corresponds to one quarter of the intensity.

Fourier viewpoint

In the far field, the angular field distribution is proportional to the Fourier transform of the aperture function. A rectangular slit transforms into a sinc function. This provides a broader engineering insight: narrow structures in one domain correspond to broad structures in the conjugate domain, explaining the inverse relation between slit width and angular spread.

6. Worked example: single-slit diffraction

A laser of wavelength $\lambda = 632.8 \text{ nm}$ illuminates a slit of width $a = 0.120 \text{ mm}$. A screen is $L = 2.00 \text{ m}$ away. Find the first-minimum angle, the first-minimum positions and the width of the central maximum.

Step 1: convert units

$$\lambda = 632.8 \times 10^{-9} \text{ m}, \quad a = 0.120 \times 10^{-3} \text{ m} = 1.20 \times 10^{-4} \text{ m}$$

Step 2: angle of the first minimum

$$\sin\theta_1 = \lambda/a = (632.8 \times 10^{-9})/(1.20 \times 10^{-4}) = 5.273 \times 10^{-3}$$

$$\theta_1 \approx 5.273 \times 10^{-3} \text{ rad} = 0.302^\circ$$

The numerical value is 10^{-3} , not 10^{-5} . This is also consistent with the screen result: an angle of about 0.0053 rad over 2 m gives a displacement of about 0.0106 m .

Step 3: screen positions

$$y_{+1} \approx +L\lambda/a = +1.055 \times 10^{-2} \text{ m} = +1.06 \text{ cm}$$

$$y_{-1} \approx -1.06 \text{ cm}$$

Step 4: central width

$$w = y_{+1} - y_{-1} = 2L\lambda/a = 2.11 \text{ cm}$$

Result	Value	Physical meaning
First-minimum angle	$\pm 5.273 \times 10^{-3} \text{ rad} (\pm 0.302^\circ)$	dark directions
First-minimum positions	$\pm 1.06 \text{ cm}$	edges of central bright band
Central width	2.11 cm	distance between first minima

Discussion and design test

If the slit width were doubled, the first-minimum angle and central width would be halved. If the wavelength were doubled, both would double. Moving the screen to 4.00 m would double the measured width in centimetres but would leave the angular width unchanged.

Consistency check: calculate the same position as $y \approx L\theta$. Agreement with $y = L\lambda/a$ confirms both the arithmetic and the validity of the small-angle approximation.

7. Engineering meaning of diffraction

Diffraction is not an imperfection that can be eliminated by better manufacturing. It is a fundamental consequence of finite apertures. Any system that confines a wave spatially must accept a spread in propagation direction. This places limits on focusing, collimation and angular resolution.

Optical instruments

In cameras, microscopes and telescopes, the lens aperture produces a diffraction pattern rather than a mathematical point image. Increasing aperture diameter narrows the pattern and improves the ability to distinguish nearby objects. For a circular aperture, the first dark ring of the Airy pattern occurs at approximately $\theta = 1.22\lambda/D$. The numerical factor differs from a slit, but the inverse aperture scaling is the same.

Lasers, fibres and antennas

A laser beam emerging through a finite output aperture diverges. A larger beam waist generally yields smaller far-field divergence. In optical fibres, numerical aperture governs acceptance angles and coupling, while modal structure reflects wave boundary conditions. Radio-frequency antennas obey the same wave principle: larger apertures produce narrower beams and higher directivity when wavelength is fixed.

Resolution and design trade-offs

Better angular resolution can be obtained by increasing aperture size or reducing wavelength. Each option has costs. A larger aperture increases size, mass and alignment demands; a shorter wavelength may require different sources, detectors and materials. Engineering design therefore balances diffraction limits against cost, bandwidth, sensitivity and fabrication constraints.

System	Finite aperture effect	Design response
Microscope	diffraction-limited spot	larger numerical aperture, shorter λ
Telescope	limited angular resolution	larger objective diameter
Laser	beam divergence	larger beam waist or aperture
Antenna	broad radiation pattern	larger effective aperture

Big picture: the far-field pattern is a map of the aperture's spatial structure. Diffraction can therefore be used not only as a limitation but also as a measurement and beam-shaping tool.

Engineers exploit designed apertures, gratings and phased arrays to place energy in chosen directions. The same mathematics that predicts unwanted spreading enables controlled spatial filtering, spectroscopy and directional communication.

8. Interference of two coherent light waves

When two light waves overlap, their electric fields add. If the fields have a stable relative phase, the time-averaged intensity depends on that phase. For intensities I_1 and I_2 and phase difference δ ,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

The interference term may be positive or negative. For $\delta = 2\pi m$, $\cos \delta = 1$ and the intensity is maximum. For $\delta = (2m + 1)\pi$, $\cos \delta = -1$ and the intensity is minimum:

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2, \quad I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Complete darkness requires equal intensities. If $I_1 \neq I_2$, destructive interference is incomplete because the larger field cannot be fully cancelled by the smaller one.

Coherence requirements

A stable pattern requires a nearly fixed frequency and relative phase. Two independent ordinary lamps do not maintain a stable phase relation, so their interference averages away. Young's experiment solves this by splitting light from one source into two paths or illuminating two narrow slits with the same incoming wavefront. The slits then act as coherent secondary sources.

Path difference and phase

If the geometrical paths from the two slits to P are r_1 and r_2 and an additional initial phase φ_0 exists,

$$\delta = k(r_2 - r_1) + \varphi_0 = (2\pi/\lambda)\Delta + \varphi_0$$

For identical slits illuminated in phase, $\varphi_0 = 0$. The pattern is then controlled directly by $\Delta = r_2 - r_1$.

Intensity insight: interference redistributes energy; it does not create or destroy it. Bright regions gain intensity at the expense of dark regions, while total energy remains consistent with the incident power.

Visibility

Fringe contrast can be quantified by $V = (I_{\max} - I_{\min}) / (I_{\max} + I_{\min})$. Equal intensities give $V = 1$. Unequal intensities, limited coherence, finite source size and detector averaging reduce visibility. This connects the ideal formulas to real laboratory patterns.

9. Young's double-slit geometry

Two narrow slits S_1 and S_2 are separated by distance d and observed on a screen a distance L away. At a point P making angle θ with the central axis, the far-field path difference is

$$\Delta = d \sin\theta$$

For small angles and screen coordinate y ,

$$\sin\theta \approx \tan\theta = y/L, \quad \text{so} \quad \Delta \approx dy/L$$

This approximation requires $y \ll L$. The central point $y = 0$ has $\Delta = 0$, so waves from equally illuminated slits arrive in phase and the central fringe is bright.

Phase across the screen

$$\delta(y) = (2\pi/\lambda)(dy/L)$$

Moving along the screen increases the phase difference approximately linearly. Each phase increase of 2π reproduces the same intensity, which is why ideal double-slit fringes are equally spaced in the small-angle regime.

Constructive and destructive conditions

Fringe type	Phase condition	Path condition
Bright maximum	$\delta = 2\pi m$	$\Delta = m\lambda$
Dark minimum	$\delta = (2m+1)\pi$	$\Delta = (m+1/2)\lambda$

The integer m may be positive, negative or zero for bright fringes. For dark fringes, conventions differ: one may use $m = 0, 1, 2, \dots$ separately on each side or use signed integers. What matters is the half-integer path difference.

Assumptions behind the simple model

The slits are treated as narrow and mutually coherent, the light is monochromatic, L is much larger than d , and both slit intensities are equal. Finite slit width adds a diffraction envelope; finite spectral bandwidth or source size reduces contrast. Recording these assumptions is part of a complete engineering solution.

Direction versus position: the exact relations use angle θ . Screen formulas are derived only after introducing $y \approx L\theta$. Keeping this order avoids confusing an angular fringe spacing with a linear one.

10. Bright and dark fringe positions

Bright fringes

Constructive interference requires $d \sin \theta_m = m\lambda$. Under the small-angle approximation,

$$y_m = m\lambda L/d, \quad m = 0, \pm 1, \pm 2, \dots$$

The $m = 0$ fringe lies at the centre. Positive and negative orders appear symmetrically above and below it.

Dark fringes

Destructive interference requires $d \sin \theta = (m + 1/2)\lambda$. Therefore

$$y_{m,\text{dark}} = (m + 1/2)\lambda L/d$$

Dark fringes lie halfway between adjacent bright fringes. The central point is not dark for two in-phase slits because its path difference is zero.

Fringe spacing

Subtract the positions of consecutive bright maxima:

$$\Delta y = y_{(m+1)} - y_m = \lambda L/d$$

The same spacing separates consecutive dark fringes. Ideal Young fringes are therefore equally spaced. Increasing wavelength or screen distance expands the pattern; increasing slit separation compresses it.

Parameter change	Effect on Δy	Reason
λ increases	Δy increases	phase accumulates more slowly with path
L increases	Δy increases	same angular pattern projected farther
d increases	Δy decreases	path difference changes faster with angle

Angular fringe spacing

For small angles, $\Delta \theta \approx \lambda/d$. Unlike linear spacing, it does not depend on screen distance. The screen simply converts angle into distance. This is analogous to the distinction between angular and linear width in single-slit diffraction.

Memorisation aid: single slit: minima scale with λ/a . Double slit: fringe spacing scales with λ/d . The symbols a and d refer to different physical dimensions and must not be interchanged.

11. Worked example: double-slit fringe spacing

Monochromatic light of wavelength $\lambda = 632.8 \text{ nm}$ illuminates two narrow slits separated by $d = 0.250 \text{ mm}$. The screen is $L = 1.50 \text{ m}$ away. Find the fringe spacing and the positions of the $m = \pm 2$ bright fringes.

Step 1: SI units

$$\lambda = 6.328 \times 10^{-7} \text{ m}, \quad d = 2.50 \times 10^{-4} \text{ m}, \quad L = 1.50 \text{ m}$$

Step 2: fringe spacing

$$\Delta y = \lambda L / d = (6.328 \times 10^{-7})(1.50) / (2.50 \times 10^{-4})$$

$$\Delta y = 3.797 \times 10^{-3} \text{ m} = 3.80 \text{ mm}$$

Step 3: second-order bright fringes

$$y_{+2} = +2\Delta y = +7.59 \text{ mm}, \quad y_{-2} = -7.59 \text{ mm}$$

Step 4: first dark fringes

$$y_{\text{dark}} = \pm(1/2)\Delta y = \pm 1.90 \text{ mm}$$

The positions are symmetric because the geometry and illumination are symmetric. The central bright fringe is at $y = 0$.

Feature	Order/condition	Position
Central bright	$m = 0$	0
First dark	$\Delta = \pm\lambda/2$	$\pm 1.90 \text{ mm}$
First bright	$m = \pm 1$	$\pm 3.80 \text{ mm}$
Second bright	$m = \pm 2$	$\pm 7.59 \text{ mm}$

Validity check

For the second bright fringe, $y/L \approx 0.00759/1.50 = 0.00506$, much less than one. Thus $\sin\theta \approx \tan\theta \approx y/L$ is highly accurate. The corresponding angle is about 0.290° .

Alternative route: calculate θ from $d \sin\theta = 2\lambda$ and then use $y = L \tan\theta$. The result should agree with the small-angle formula to the reported precision.

Physical interpretation

If d were reduced, the slits would behave more like closely spaced coherent sources and the fringes would spread farther apart. If λ were reduced while all geometry remained fixed, the phase would change more rapidly with position and the fringes would be closer together.

12. Inverse problem: determining slit separation

Interference patterns can measure very small distances. Suppose light of wavelength $\lambda = 633 \text{ nm}$ produces a measured bright-fringe spacing $\Delta y = 4.20 \text{ mm}$ on a screen $L = 1.50 \text{ m}$ away. Determine the slit separation.

Rearrange before substituting

$$\Delta y = \lambda L / d \quad \Rightarrow \quad d = \lambda L / \Delta y$$

Insert SI values

$$d = [(633 \times 10^{-9} \text{ m})(1.50 \text{ m})] / (4.20 \times 10^{-3} \text{ m})$$

$$d = 2.26 \times 10^{-4} \text{ m} = 0.226 \text{ mm}$$

Dimensional analysis gives $\text{m} \cdot \text{m} / \text{m} = \text{m}$, as required. The result is a sub-millimetre separation that can be inferred without directly resolving the two slits under a microscope.

Sensitivity and measurement quality

The inferred separation is proportional to λ and L and inversely proportional to Δy . A relative uncertainty estimate for independent small uncertainties is

$$(u_d/d)^2 \approx (u_\lambda/\lambda)^2 + (u_L/L)^2 + (u_{\Delta y}/\Delta y)^2$$

Instead of measuring one fringe interval, measure the distance across N intervals and divide by N . This reduces the relative reading error. A least-squares fit of fringe position y_m versus order m is even better: the slope equals $\lambda L / d$ and uses all recorded maxima.

Engineering interpretation

Optical interference converts tiny path or geometry changes into measurable fringe shifts. This principle underlies interferometers, surface profiling, displacement sensing, refractive-index measurement and wavelength calibration. Sensitivity can be extremely high because one fringe shift corresponds to a path change of approximately one wavelength.

Measured quantity	Known quantities	Inferred result
fringe spacing Δy	λ and L	slit separation $d = \lambda L / \Delta y$
fringe spacing Δy	d and L	wavelength $\lambda = d \Delta y / L$
fringe shift	λ and geometry	small path or displacement change

Data-analysis rule: use many fringes, fit a straight line, report uncertainty and verify the small-angle approximation. The algebra is simple; measurement discipline determines the quality of the result.

13. Diffraction envelope, applications and final guide

Real double slits have finite width

If each slit has width a and the centre-to-centre separation is d , the two-slit interference oscillations are multiplied by the single-slit diffraction envelope:

$$I(\theta) = I_{\max} [\sin\beta/\beta]^2 \cos^2\alpha$$

$$\beta = (\pi a/\lambda)\sin\theta, \quad \alpha = (\pi d/\lambda)\sin\theta$$

The $\cos^2\alpha$ factor produces closely spaced interference fringes; the sinc-squared factor controls their slowly varying envelope. An interference maximum may be missing if it coincides with a diffraction minimum. This combined model is the physically realistic link between the two main parts of the lecture.

Common mistakes

Do not confuse slit width a with slit separation d . Do not include $m = 0$ among single-slit minima. Do not claim that the two edge rays cancel at the first single-slit minimum; cancellation occurs through paired contributions across the aperture. Use $\Delta = m\lambda$ for double-slit bright fringes and $\Delta = (m + 1/2)\lambda$ for dark fringes. Convert nanometres and millimetres to metres consistently, and distinguish angular widths from screen distances.

A reliable problem-solving sequence

Step	Question or action	Outcome
1	Single aperture or two coherent sources?	Choose diffraction or interference model.
2	What are a , d , λ , L and the required output?	Create a labelled geometry and SI data list.
3	Find the path difference.	Use a $\sin\theta$ or $d \sin\theta$ with the correct meaning.
4	Apply phase/cancellation condition.	Locate minima, maxima or fringe order.
5	Introduce small-angle geometry only if justified.	Convert θ to y using $y \approx L\theta$.
6	Check units, symmetry and limiting trends.	Confirm physical plausibility.

Final self-check

Be able to derive the first single-slit minimum by pairwise cancellation, calculate a diffraction width, obtain double-slit fringe spacing, and explain why unequal slit intensities do not give perfectly dark minima.

$$\text{Single slit: } a \sin\theta_m = m\lambda, \quad I = I_0(\sin\beta/\beta)^2$$

$$\text{Double slit: } d \sin\theta = m\lambda \text{ (bright),} \quad \Delta y = \lambda L/d$$