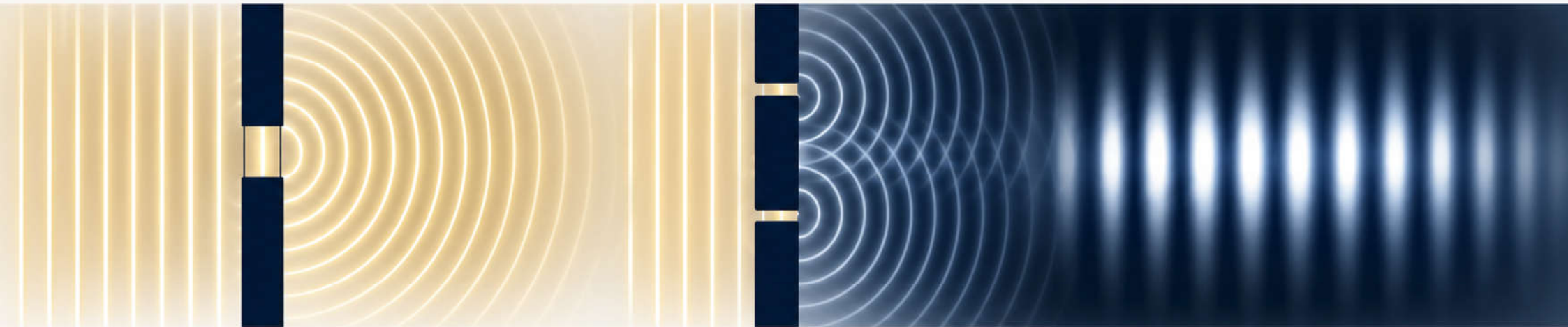


LECTURE 6

BASIC OPTICS I – WAVE OPTICS

DIFFRACTION AND INTERFERENCE



Review of Lecture 5 – Waves Essentials

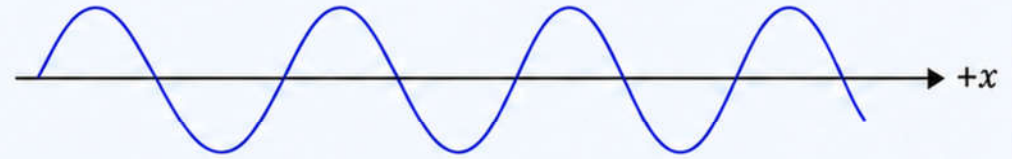
Three ideas connect wave motion to wave optics.

1 TRAVELING WAVE

$$y(x, t) = A \sin(kx - \omega t + \phi_0)$$

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f$$

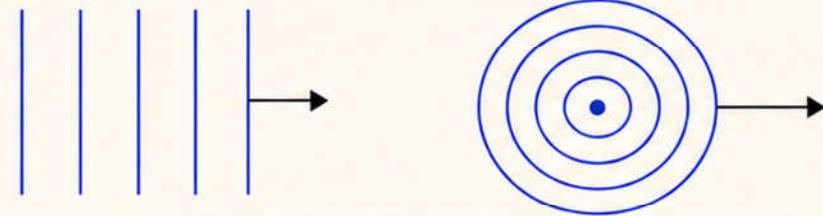
The phase propagates; particles generally oscillate locally.



2 PHASE AND WAVEFRONTS

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta r$$

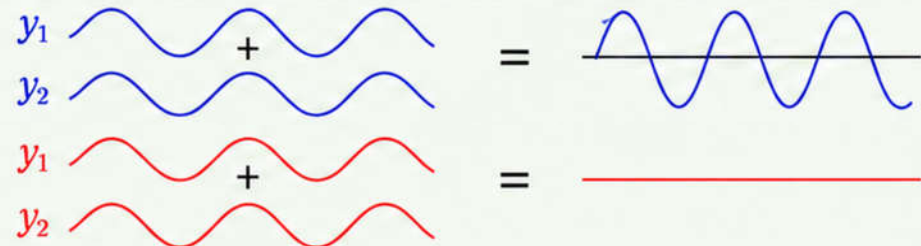
A wavefront joins points with equal phase.



3 SUPERPOSITION

$$y = y_1 + y_2$$

Constructive and destructive addition creates intensity patterns.



Wave optics begins when **phase differences** make **superposition** position-dependent.

Roadmap of Lecture 6

How phase differences create diffraction and interference patterns

1

FOUNDATION

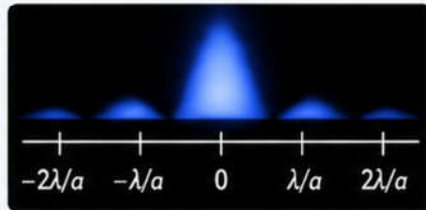
Wavefronts • Huygens' principle

2

SINGLE-SLIT DIFFRACTION

Path difference → minima →
intensity pattern

$$a \sin \theta_m = m\lambda$$



3

DOUBLE-SLIT INTERFERENCE

Path difference → phase →
bright and dark fringes

Bright: $d \sin \theta_m = m\lambda$

Dark: $d \sin \theta_m = (m + \frac{1}{2})\lambda$



4

MEASUREMENT AND DESIGN

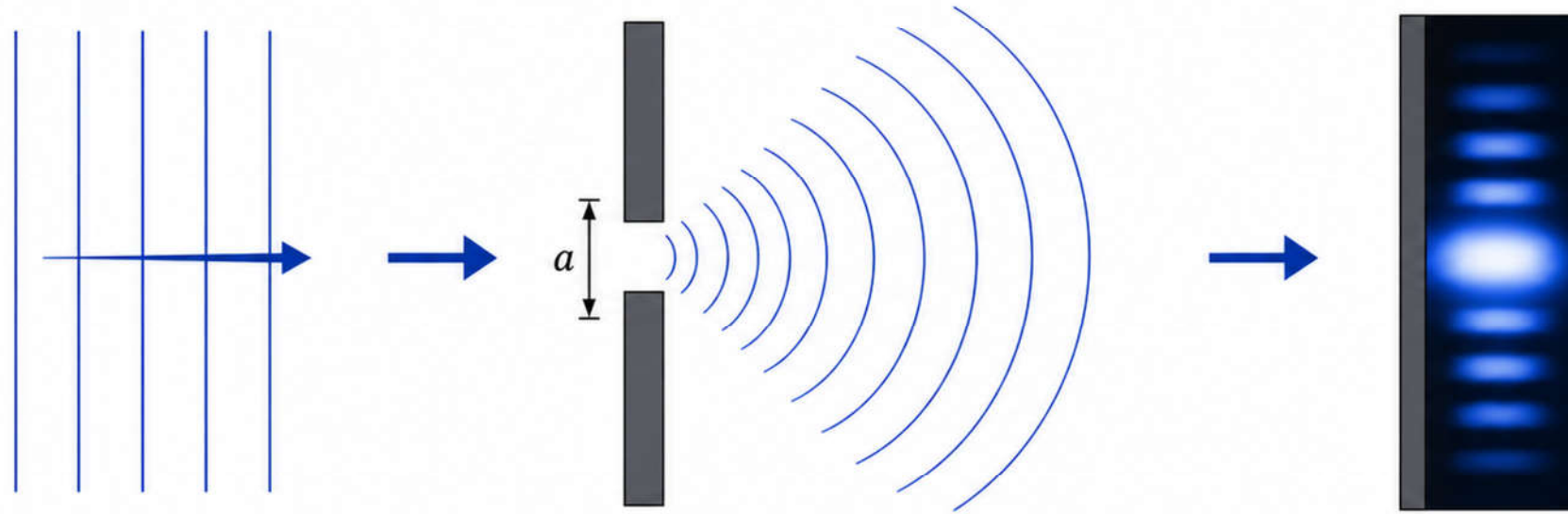
Fringe spacing • worked
examples • engineering limits



Goal: connect the geometry of an aperture to the intensity measured on a screen.

What Is Diffraction?

A wave spreads into regions that geometrical rays would leave in shadow.



Before the slit: plane wave

After the slit: many propagation directions

On the screen: an intensity pattern



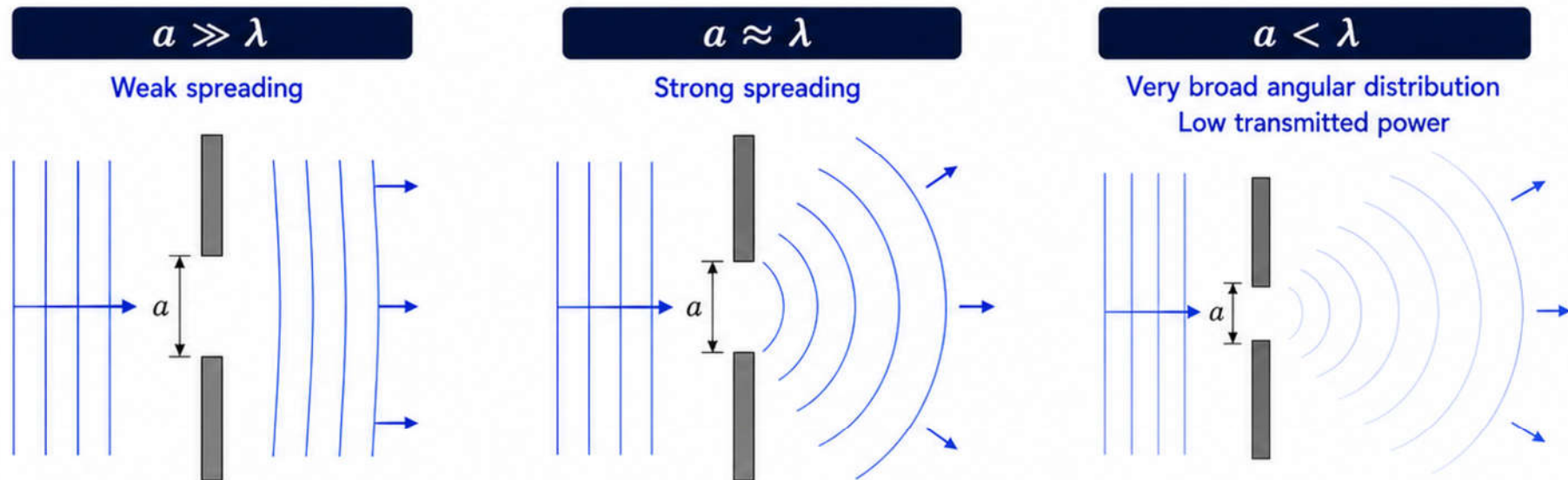
Diffraction is the spreading of a wave caused by an aperture or obstacle.



All waves diffract; the effect becomes most visible when the relevant size is not much larger than the wavelength.

When Is Diffraction Significant?

The angular spread is controlled mainly by the ratio a/λ .



First minimum: $a \sin \theta_1 = \lambda$

For $a \geq \lambda$. If $a < \lambda$, no first minimum exists because $\lambda/a > 1$.



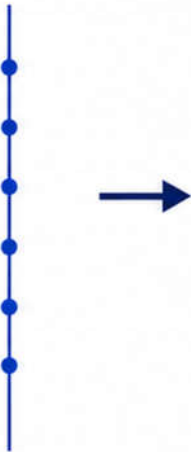
Smaller $a/\lambda \rightarrow$ broader diffraction; larger $a/\lambda \rightarrow$ narrower diffraction.

Huygens' Principle



Every point on a wavefront acts as a source of secondary spherical wavelets. A later wavefront is the envelope tangent to these wavelets.

1 Wavefront at time t



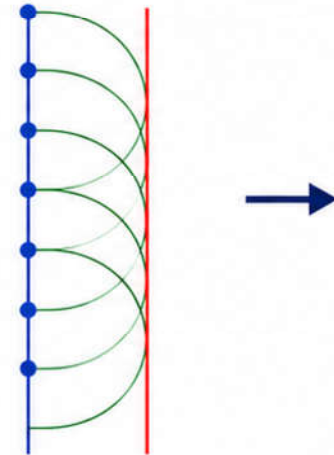
equal-phase points

2 Secondary wavelets after Δt



wavelets grow at speed v

3 New wavefront at $t + \Delta t$



common tangent = new wavefront



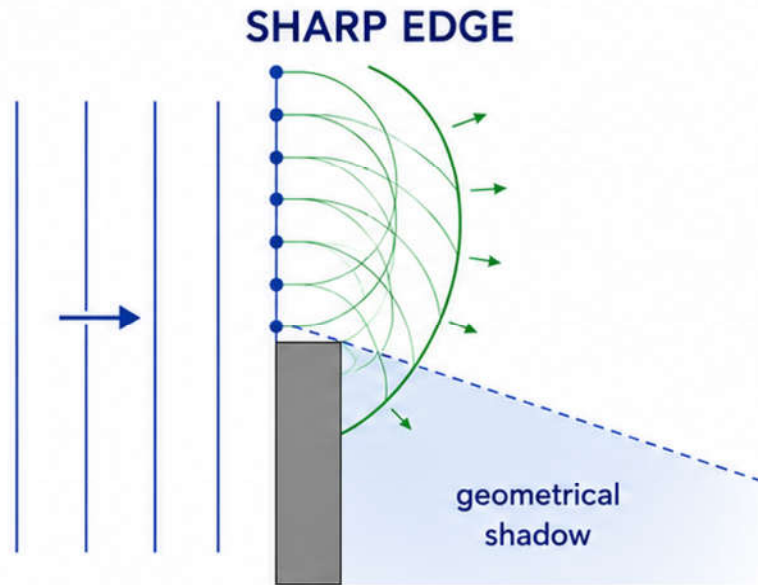
In a uniform medium, a plane wavefront remains plane and advances by $v\Delta t$.



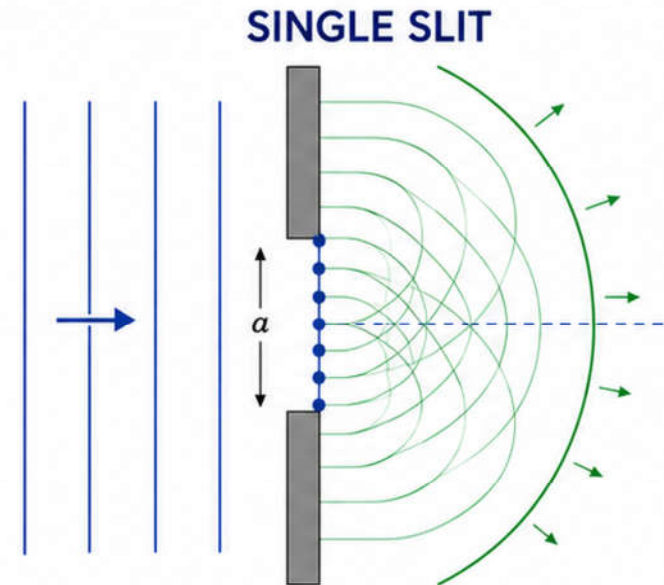
Huygens' principle constructs the position and shape of a wavefront; superposition determines its amplitude.

Huygens' Principle at Boundaries

Only the unobstructed part of the wavefront contributes beyond an opaque barrier.



Contributions from the unobstructed wavefront extend into the geometrical shadow.



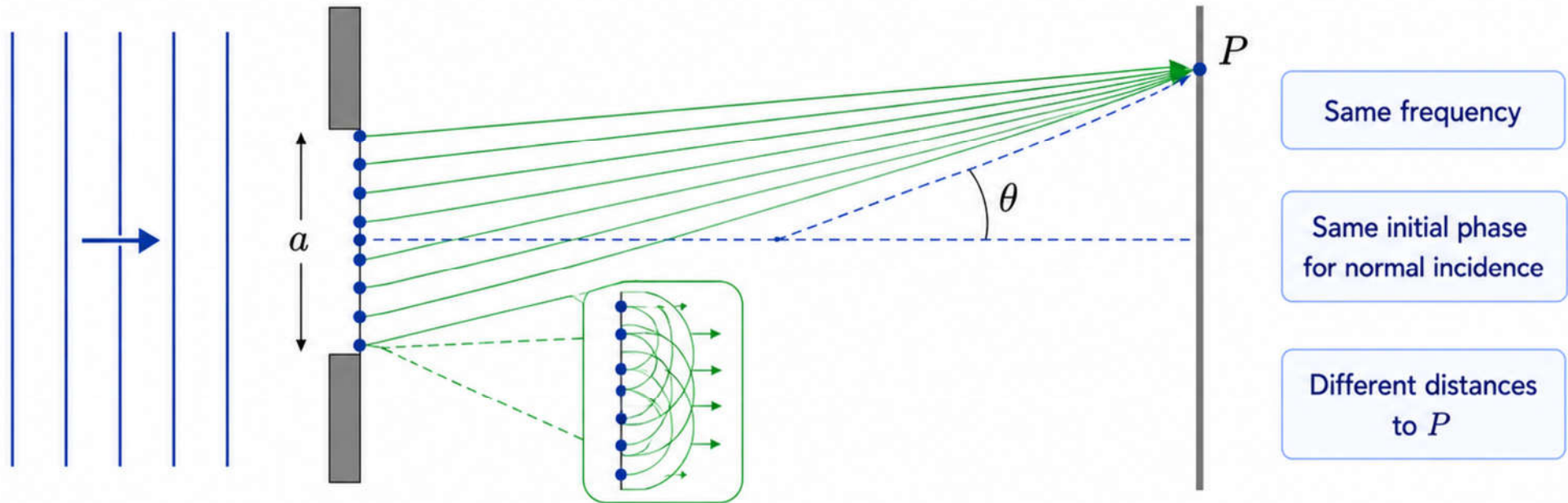
The opening transmits a finite section of the wavefront, producing angular spread.



Diffraction is a superposition effect: the aperture or obstacle changes which parts of the wavefront can contribute.

A Single Slit Is a Continuous Set of Sources

Every point across the opening contributes to the field at the observation point.



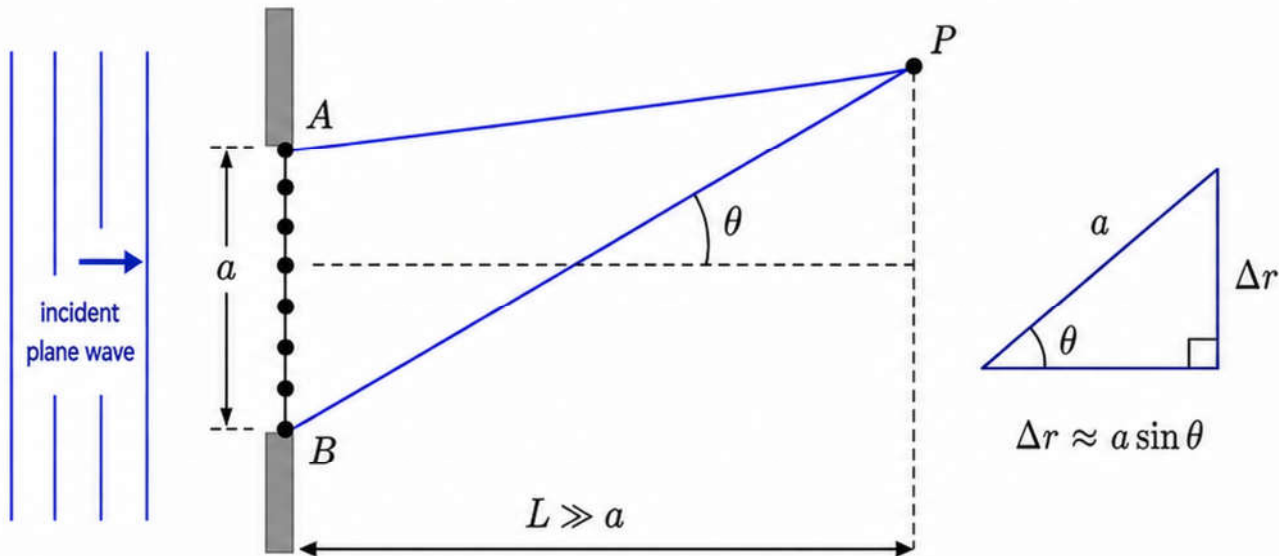
$$E(P) = \text{sum of the contributions from all points across the slit}$$



The diffraction pattern comes from interference across the full aperture — not only from its edges.

Path Difference Across a Single Slit

Different points across the aperture travel different distances to the same observation direction.



1 Separation across the full slit: a

2 Path difference magnitude: $\Delta r \approx a \sin \theta$

3 Phase difference: $\Delta \phi = \frac{2\pi}{\lambda} \Delta r \approx \frac{2\pi a}{\lambda} \sin \theta$

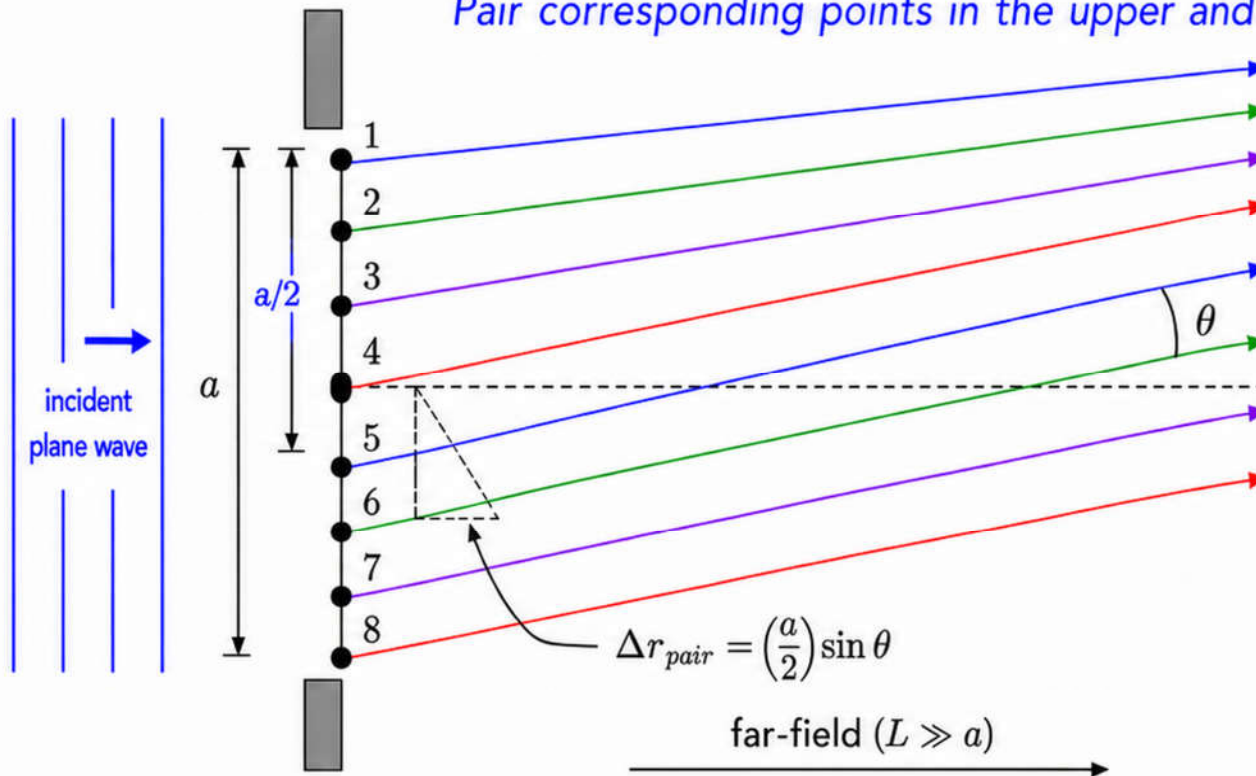
At $\theta = 0$: $\Delta r = 0 \rightarrow$ all points arrive in phase $\rightarrow$ central maximum



The relation $\Delta r \approx a \sin \theta$ uses the far-field approximation; the exact difference is $r_B - r_A$.

First Minimum: Pairwise Cancellation

Pair corresponding points in the upper and lower halves of the slit.



At the first minimum:

$$a \sin \theta_1 = \lambda$$

$$\Delta r_{pair} = \frac{a}{2} \sin \theta_1 = \frac{\lambda}{2}$$

$$\Delta \phi_{pair} = \frac{2\pi \Delta r_{pair}}{\lambda} = \pi$$



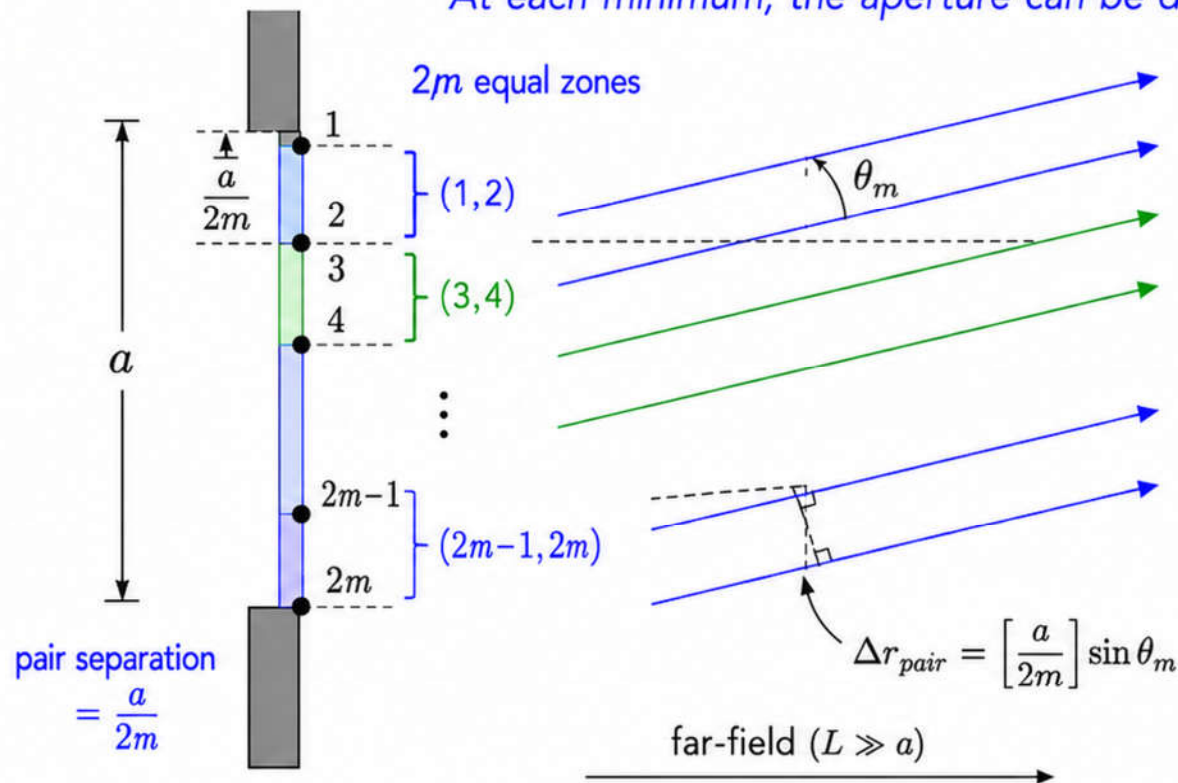
**equal amplitudes + phase difference π
→ cancellation**



Every contribution from one half has a partner from the other half that arrives π out of phase.

General Condition for Single-Slit Minima

At each minimum, the aperture can be divided into cancelling pairs.



Pairwise cancellation requires

$$\left[\frac{a}{2m} \right] \sin \theta_m = \frac{\lambda}{2}$$

$$a \sin \theta_m = m \lambda, \quad m = 1, 2, 3, \dots$$

$\theta = 0$ is not a minimum — it is the central maximum.

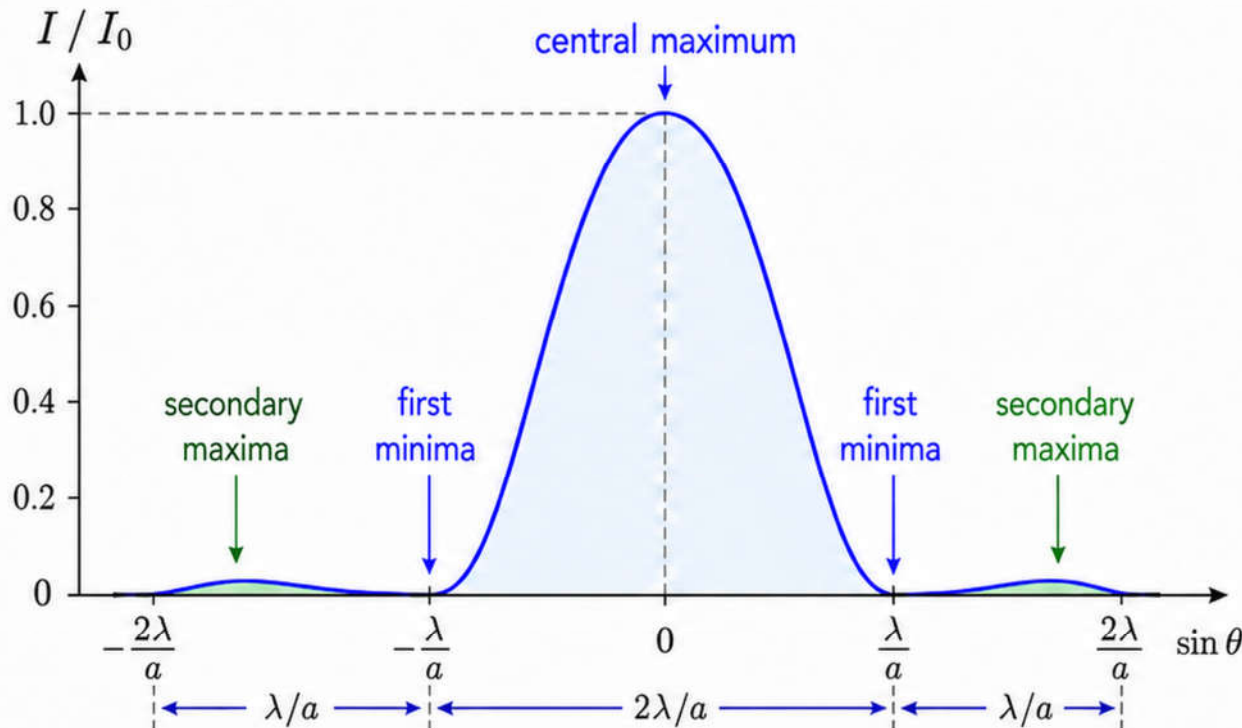
- $m = 1 \rightarrow$ first minimum
- $m = 2 \rightarrow$ second minimum
- $m = 3 \rightarrow$ third minimum



The minima occur when the path difference across the full slit equals an integer number of wavelengths.

Single-Slit Intensity Distribution

A broad central maximum is accompanied by much weaker secondary maxima.



$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2$$

$$\beta = \frac{\pi a}{\lambda} \sin \theta$$

Minima

$$\beta = m\pi \Leftrightarrow a \sin \theta_m = m\lambda, \quad m = \pm 1, \pm 2, \dots$$

Secondary maxima

$$\tan \beta = \beta$$

$$\text{first side maximum: } \beta \approx \pm 4.493$$

$$\text{its intensity} \approx 4.7\% \text{ of } I_0$$



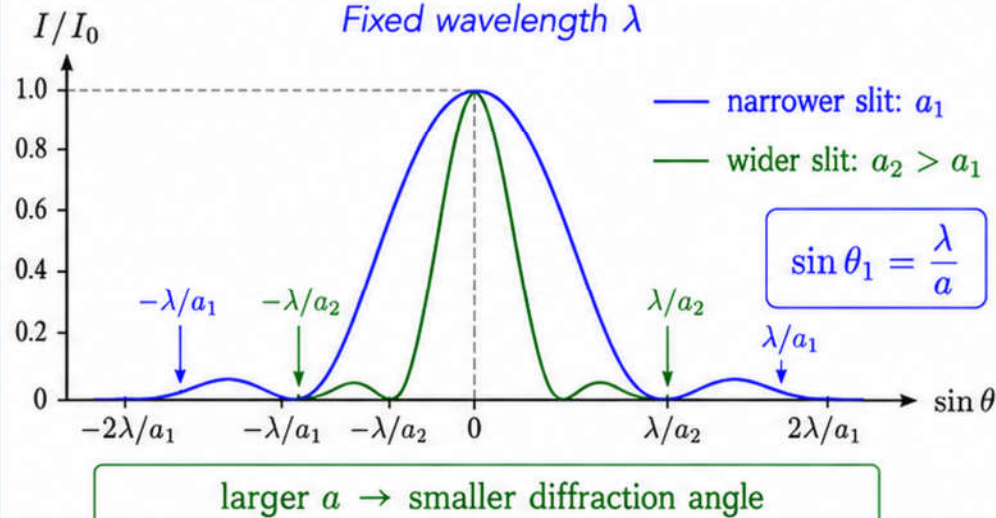
The central maximum contains most of the intensity; the side maxima decrease rapidly away from the axis.

How Slit Width and Wavelength Change the Pattern

The angular scale is controlled by the ratio λ/a .

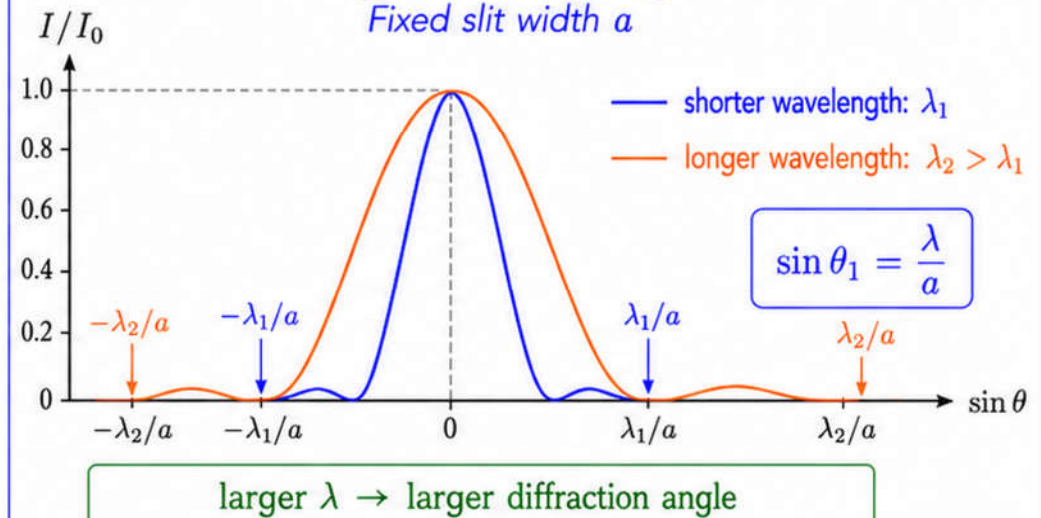
Change the slit width a

Fixed wavelength λ



Change the wavelength λ

Fixed slit width a



Angular width between first minima: $\Delta(\sin \theta) = \frac{2\lambda}{a}$

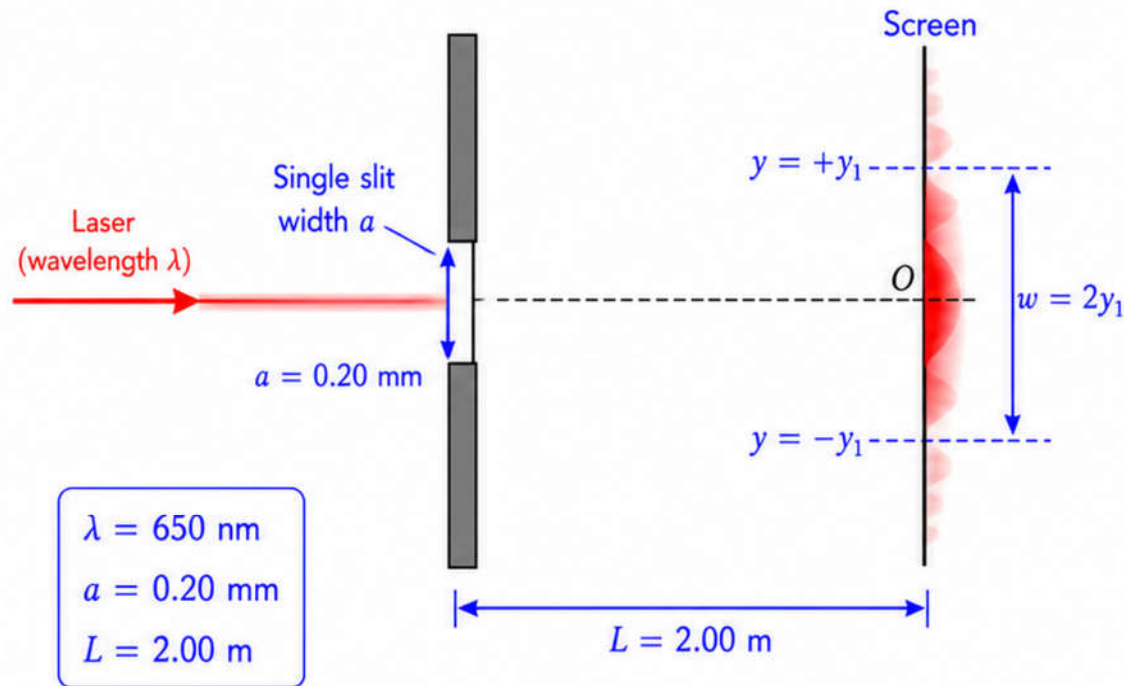
for small angles: central width $\approx \frac{2\lambda L}{a}$



A wider slit narrows the pattern; a longer wavelength broadens it.

Worked Example: Designing a Diffraction Pattern

Predict the central-maximum width produced by a laser and a single slit.



1 First-minimum angle

$$\sin \theta_1 = \frac{\lambda}{a} = \frac{650 \times 10^{-9} \text{ m}}{2.0 \times 10^{-4} \text{ m}} = 3.25 \times 10^{-3}$$

$$\theta_1 = 0.00325 \text{ rad} \approx 0.186^\circ$$

2 Position on the screen

$$y_1 = L \tan \theta_1 \approx L \theta_1$$

$$y_1 \approx (2.00 \text{ m})(3.25 \times 10^{-3}) = 6.50 \text{ mm}$$

3 Central-maximum width

$$w = 2y_1 = 13.0 \text{ mm}$$

Central width: 13.0 mm

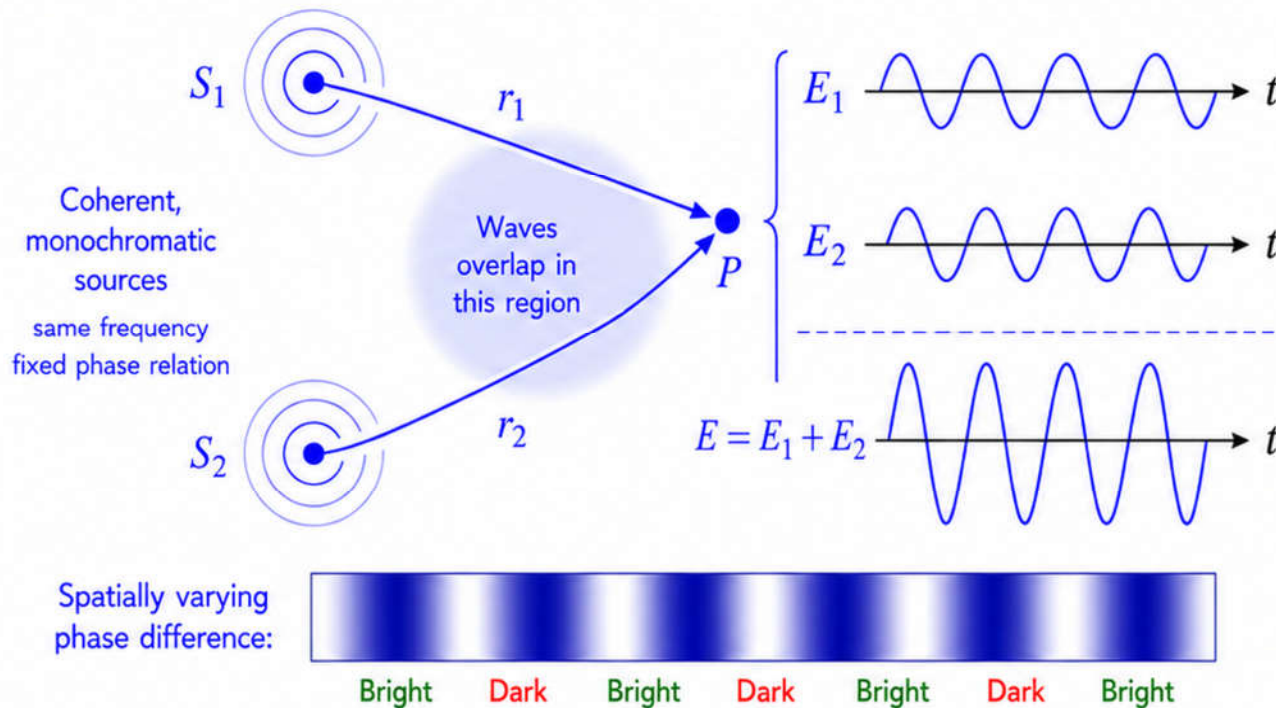
Engineering use: Laser beam shaping • aperture selection • optical sensor design



Diffraction sets a predictable minimum spread: reducing a or increasing λ makes the pattern wider.

What Is Interference of Light?

When coherent waves overlap, their electric fields add.



Superposition

$$E(P, t) = E_1(P, t) + E_2(P, t)$$

Measured intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

Phase difference

$$\delta = \frac{2\pi}{\lambda} \Delta + \delta_0$$

where $\Delta = r_2 - r_1$

Prerequisites for stable interference

- 1 same frequency
- 2 stable phase difference
- 3 overlap in space

Coherent sources produce a stable fringe pattern.

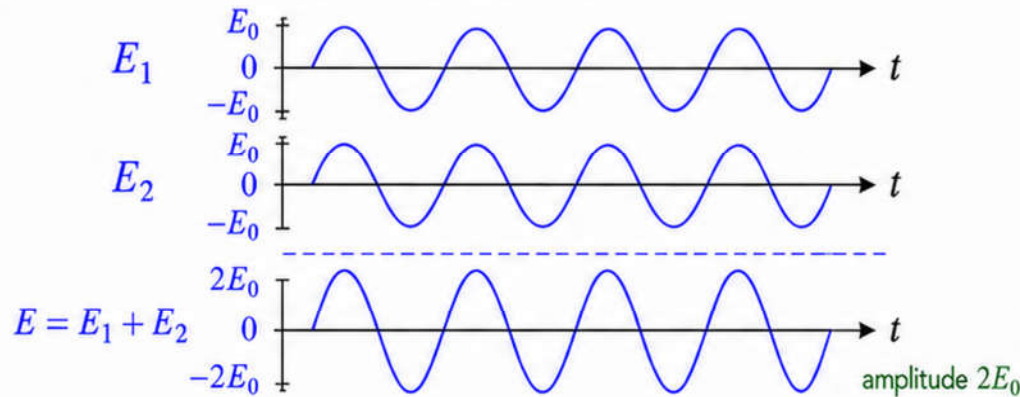


Interference redistributes intensity: phase determines where the waves reinforce or cancel.

Constructive and Destructive Interference

The relative phase determines whether two waves reinforce or cancel.

Constructive interference



Phase and path conditions

$$\delta = 2m\pi$$

$$\Delta = m\lambda$$

$$m = 0, \pm 1, \pm 2, \dots$$

Result

maximum intensity

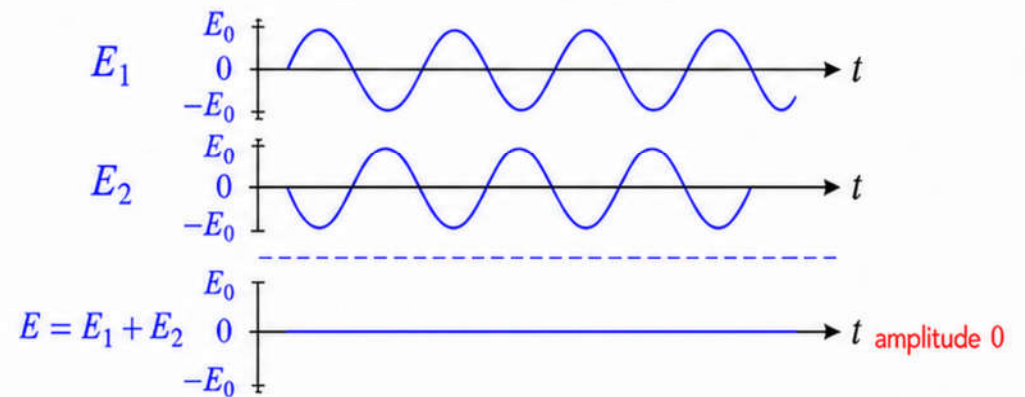
For equal beams:

$$I_{\max} = 4I_{\text{single}}$$

Bright fringe



Destructive interference



Phase and path conditions

$$\delta = (2m + 1)\pi$$

$$\Delta = \left(m + \frac{1}{2}\right)\lambda$$

$$m = 0, \pm 1, \pm 2, \dots$$

Result

minimum intensity

For equal beams:

$$I_{\min} = 0$$

Dark fringe



$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$\cos \delta = +1 \rightarrow \text{maximum}$$

$$\cos \delta = -1 \rightarrow \text{minimum}$$



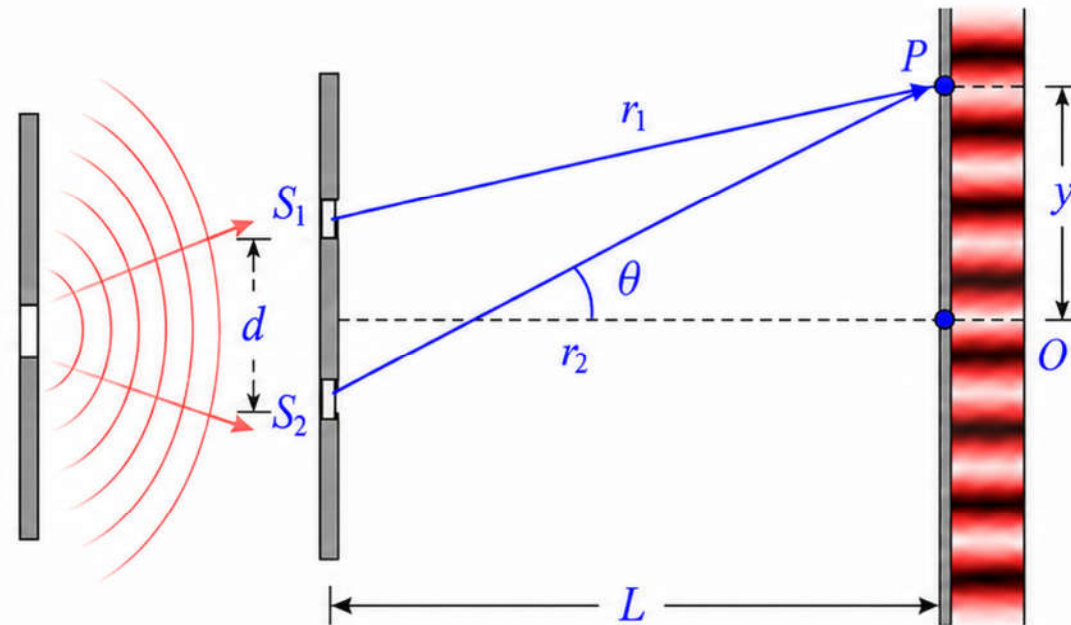
Bright and dark fringes are phase conditions—not separate kinds of light.

Young's Double-Slit Experiment

Two coherent slits convert phase differences into visible fringes.

λ wavelength
 d slit separation
 L screen distance
 y screen position

monochromatic
laser



Observations

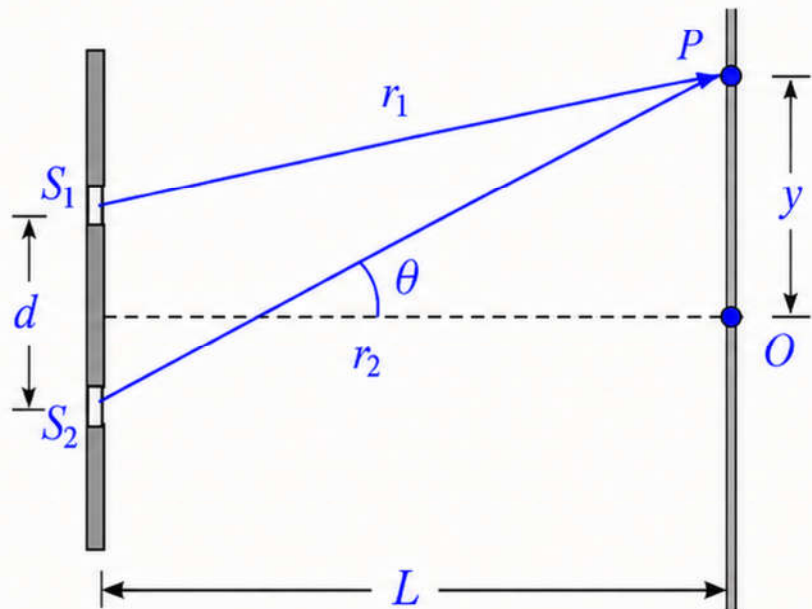
- ✓ central fringe is bright
- ✓ bright and dark fringes alternate
- ✓ pattern is symmetric
- ✓ fringe spacing depends on λ , L , and d



The pattern records how the relative phase of the two slit contributions changes across the screen.

From Geometry to Phase Difference

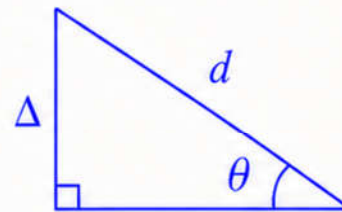
The two paths become unequal away from the central axis.



$$\Delta = r_2 - r_1$$

Far-field geometry (projection)

far field: $L \gg d$



$$\Delta \approx d \sin \theta$$

(approximation, not exact for finite L)

Exact geometry:

$$\Delta = r_2 - r_1$$

Far-field approximation:

$$\Delta \approx d \sin \theta \quad (\text{approx.})$$

Small-angle screen form:

$$\sin \theta \approx \tan \theta \approx y/L$$

$\Rightarrow$

$$\Delta \approx \frac{dy}{L}$$

Phase difference:

$$\delta = \frac{2\pi}{\lambda} \Delta + \delta_0$$

For Young's experiment with identical illumination:

$$\delta_0 = 0$$

✓ At $y = 0$: $\Delta = 0$, $\delta = 0 \rightarrow$ central bright fringe



Geometry sets the path difference; the path difference sets the phase difference.

Where Do the Bright and Dark Fringes Appear?

Interference conditions become measurable positions on the screen.

Bright fringes – constructive

$$\Delta = m\lambda \quad \delta = 2m\pi$$

Using $\Delta \approx dy/L$: $y_m^{(\text{bright})} \approx m \frac{\lambda L}{d}$

$$m = 0, \pm 1, \pm 2, \dots$$

$m = 0$ is the central bright fringe

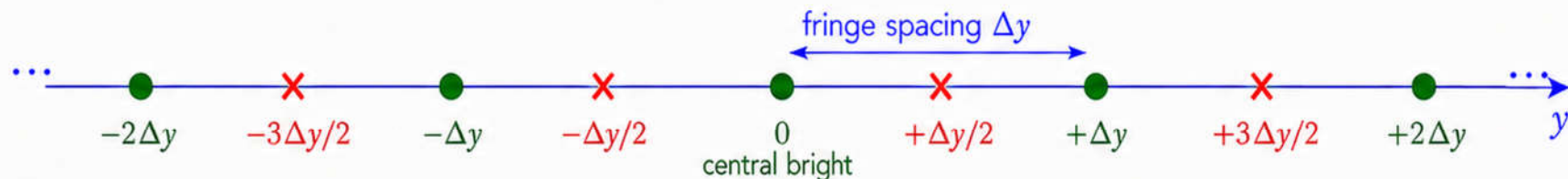
Dark fringes – destructive

$$\Delta = \left(m + \frac{1}{2}\right)\lambda \quad \delta = (2m + 1)\pi$$

Using $\Delta \approx dy/L$: $y_m^{(\text{dark})} \approx \left(m + \frac{1}{2}\right) \frac{\lambda L}{d}$

$$m = 0, \pm 1, \pm 2, \dots$$

dark fringes lie halfway between neighboring bright fringes



$$\Delta y = y_{m+1}^{(\text{bright})} - y_m^{(\text{bright})} \approx \frac{\lambda L}{d}$$

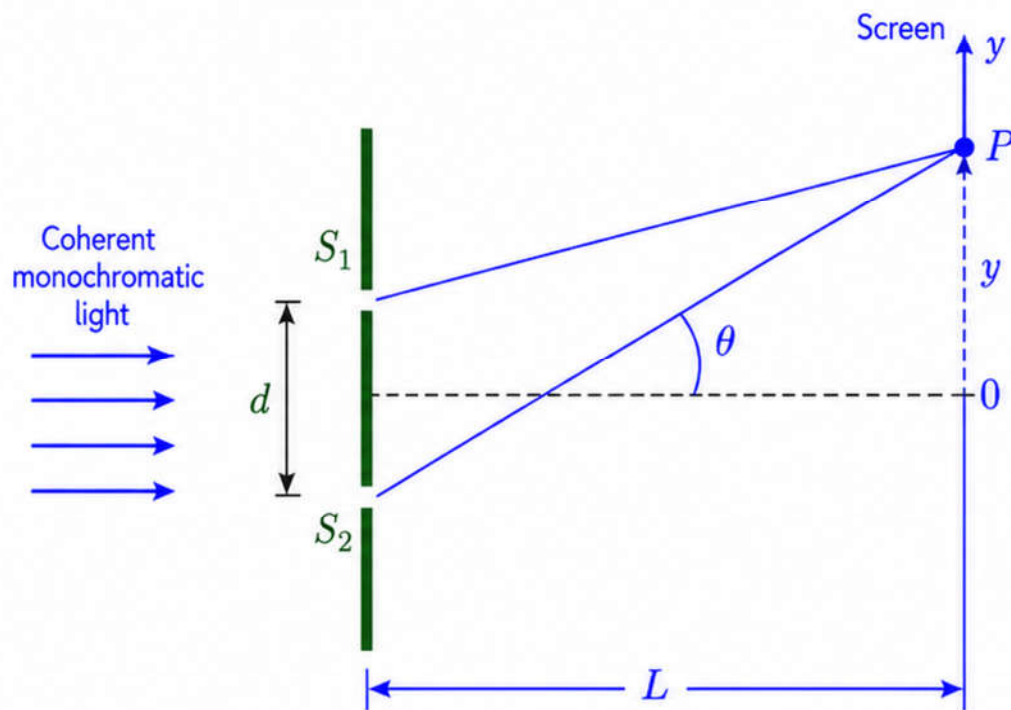
larger $\lambda \rightarrow$ wider spacing | larger $L \rightarrow$ wider spacing | larger $d \rightarrow$ narrower spacing

Fraunhofer / small-angle approximation



Fringe spacing is uniform and scales as $\lambda L/d$.

Ideal double-slit intensity distribution



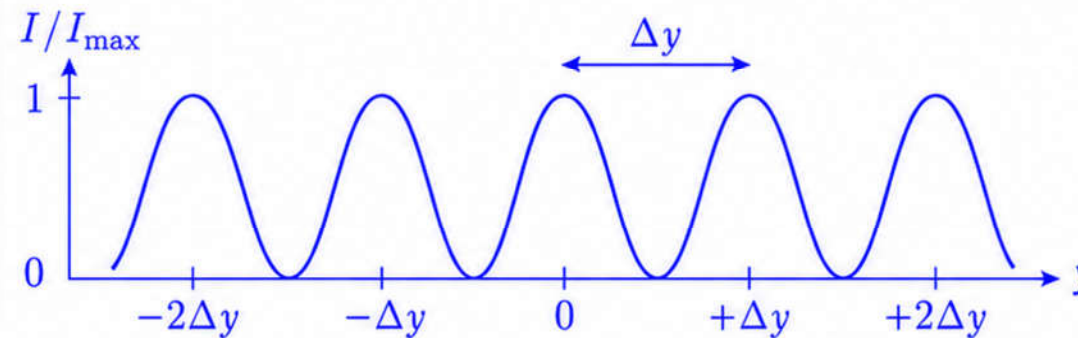
$$\delta = \frac{2\pi}{\lambda} d \sin \theta$$

For small angles: $\sin \theta \approx \frac{y}{L}$

$$I(y) = I_{\max} \cos^2 \left(\frac{\pi d y}{\lambda L} \right)$$

$$\Delta y = \frac{\lambda L}{d}$$

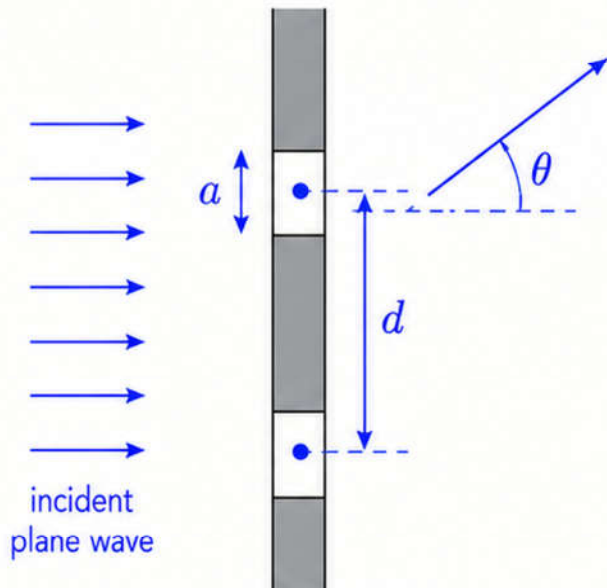
Equal slit amplitudes; Fraunhofer and small-angle approximation.



Ideal narrow slits: equally spaced, equal-height fringes.

Finite slit width is added on the next slide.

Finite-width double slit: diffraction envelope



$$I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$$

$$\beta = \frac{\pi a}{\lambda} \sin \theta$$

$$\alpha = \frac{\pi d}{\lambda} \sin \theta$$

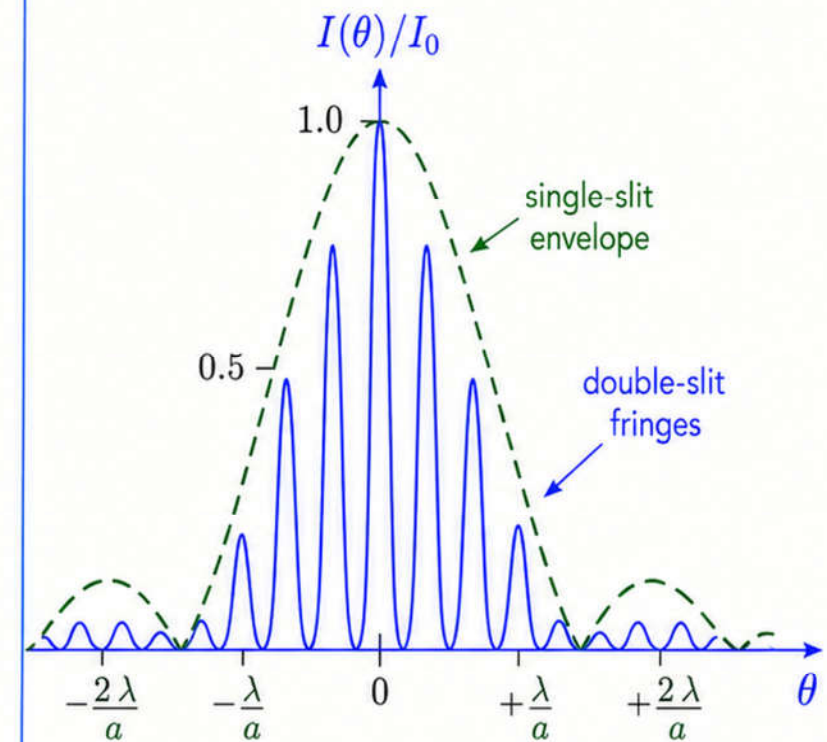
Fringe spacing → set by d

Envelope width → set by a

Interference maxima: $d \sin \theta = m\lambda$

Diffraction minima: $a \sin \theta = n\lambda, n \neq 0$

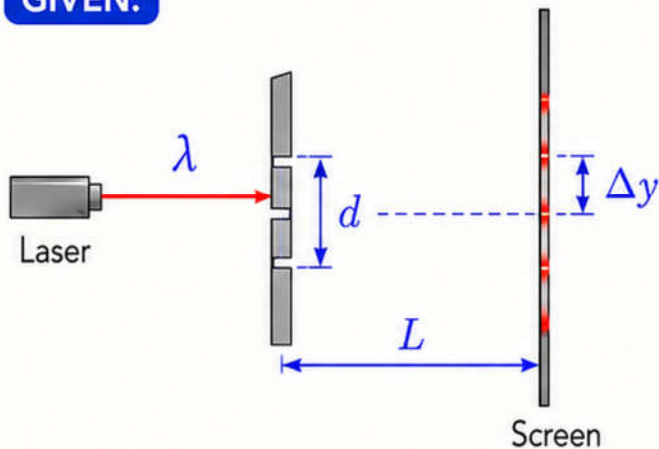
$m = 0, \pm 1, \pm 2, \dots$ $n = \pm 1, \pm 2, \pm 3, \dots$



A real double slit combines two scales: **narrow interference fringes** inside a **broad diffraction envelope**.

Worked example: fringe spacing

GIVEN:



$$\lambda = 632.8 \text{ nm}$$

$$d = 0.250 \text{ mm}$$

$$L = 2.00 \text{ m}$$

Find: Δy

MODEL AND CALCULATION:

Fraunhofer small-angle relation:

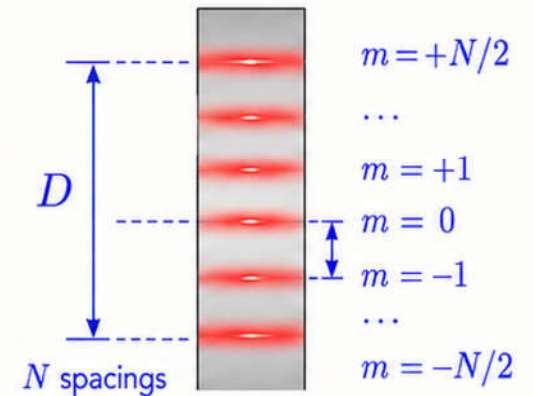
$$\Delta y = \frac{\lambda L}{d}$$

Unit check: $\frac{m \cdot m}{m} = m$ (length)

Substitute values:

$$\begin{aligned} \Delta y &= \frac{(632.8 \times 10^{-9} \text{ m})(2.00 \text{ m})}{2.50 \times 10^{-4} \text{ m}} \\ &= 5.0624 \times 10^{-3} \text{ m} \\ &= \mathbf{5.06 \text{ mm}} \end{aligned}$$

MEASUREMENT PRACTICE:



$$\Delta y = \frac{D}{N}$$

- Measure several spacings, not just one
- Fit y_m versus m : slope = $\lambda L/d$
- Larger L or $\lambda \rightarrow$ wider spacing
- Larger $d \rightarrow$ narrower spacing



Prediction: adjacent bright fringes are **5.06 mm** apart.

Validity: Fraunhofer regime; $|y| \ll L$.

Common mistakes in diffraction and interference

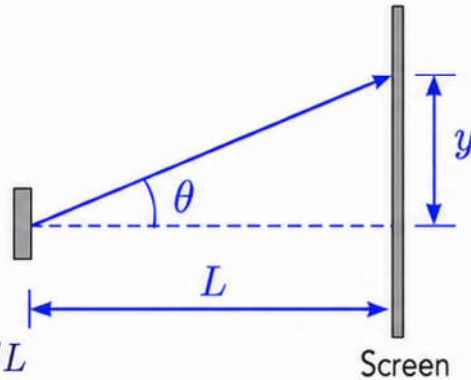
1 GEOMETRY

Mistake:

Using θ and y as if they were identical

Correct:

Use $\sin \theta$ exactly; only for small angles, $\sin \theta \approx \tan \theta \approx y/L$



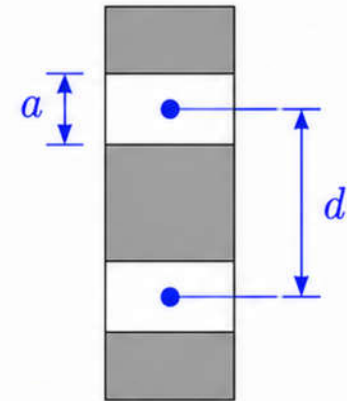
2 SLIT PARAMETERS

Mistake:

Confusing slit width a with slit separation d

Correct:

a sets the diffraction envelope; d sets interference-fringe spacing



3 MAXIMUM VERSUS MINIMUM

Mistake:

Applying the same condition to both

Correct:

Single-slit minima: $a \sin \theta = n\lambda$, $n = \pm 1, \pm 2, \dots$

Double-slit maxima: $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \dots$

The central diffraction maximum is not $n = 0$ minimum.

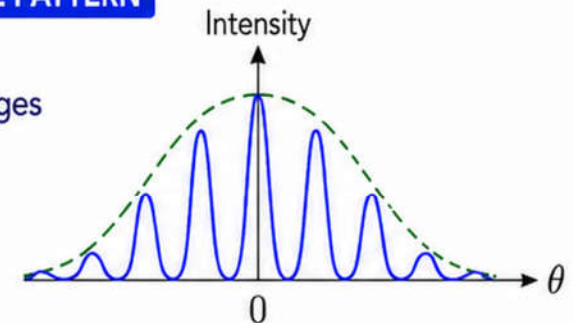
4 IDEAL VERSUS REAL PATTERN

Mistake:

Drawing equal-height fringes for finite-width slits

Correct:

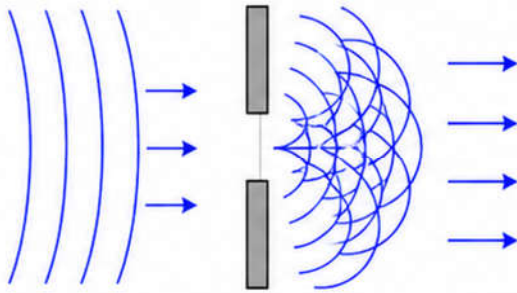
Real fringes lie inside a sinc^2 diffraction envelope



Before calculating: identify the aperture, define a and d , then **check the approximation and units.**

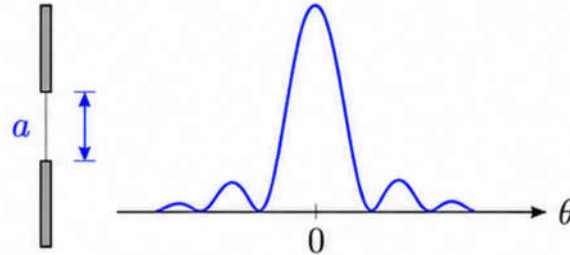
Summary: from aperture to diffraction pattern

1 HUYGENS-FRESNEL IDEA



Every point in the aperture contributes a secondary wave.
The observed field is their superposition.

2 SINGLE SLIT: WIDTH a



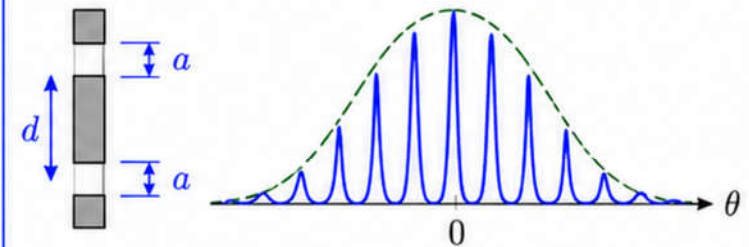
Minima: $a \sin \theta = n\lambda$, $n = \pm 1, \pm 2, \dots$

Small angle: $y_n \approx \frac{n\lambda L}{a}$

Central-maximum width: $w \approx \frac{2\lambda L}{a}$

Larger $a \rightarrow$ narrower pattern

3 DOUBLE SLIT: SEPARATION d



Bright fringes: $d \sin \theta = m\lambda$

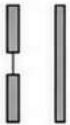
Fringe spacing: $\Delta y = \frac{\lambda L}{d}$

Real pattern: $I(\theta) = I_0 \left(\frac{\sin \beta}{\beta} \right)^2 \cos^2 \alpha$

with $\beta = \frac{\pi a \sin \theta}{\lambda}$ and $\alpha = \frac{\pi d \sin \theta}{\lambda}$

d sets spacing; a sets envelope

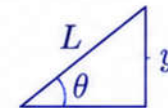
1 Identify aperture



2 Choose condition (minima, maxima, bright fringes, etc.)



3 Convert $\theta \leftrightarrow y$ $y \approx L \tan \theta \approx L\theta$



4 Check units & limits

- ✓ Units consistent?
- ✓ Small-angle limit valid?



Diffraction and interference are the same superposition principle applied to different aperture geometries.