

PHYSICS FOR ENGINEERING STUDENTS

Waves and Oscillations

Wave properties

Lecture 5 — Book-style student notes

Learning focus

Circular motion

Simple harmonic motion

Transverse and longitudinal mechanical waves

Energy associated with wave

University of Szeged • Institute of Physics

1. Why waves and oscillations matter

Waves are among the most useful unifying ideas in physics. Sound in air, vibration in a bridge, seismic motion in the Earth, light in an optical fibre and radio communication appear to be very different phenomena, yet they share the same basic language: amplitude, period, frequency, wavelength, phase and propagation speed. Once this common structure is understood, a large family of engineering problems becomes easier to model and solve.

A wave is a disturbance that propagates through a material medium or through space while carrying energy and information. The essential point is that the disturbance moves, whereas the elements of the medium generally remain near their equilibrium positions. When a pulse passes along a rope, every small piece of rope moves mainly upward and downward; it does not travel to the other end with the pulse. In the same way, an air molecule in a sound wave oscillates back and forth around a local mean position while the acoustic disturbance travels across the room.

Central idea: waves transport energy and information without producing a corresponding long-distance transport of matter. The local motion of the medium and the propagation of the wave must therefore be distinguished carefully.

A strategic problem-solving viewpoint

Wave problems should be approached in layers. First identify what is oscillating and whether a medium is required. Next determine the direction of propagation and the direction of local particle motion. Then list the known wave parameters with units. Finally select the governing relation, check the boundary conditions and interpret the result physically. This sequence prevents the most common errors: confusing amplitude with wavelength, particle speed with wave speed, or a source frequency with a medium-dependent propagation speed.

This chapter begins with the connection between rotation and oscillation, develops the mathematics of simple harmonic motion and travelling waves, and then explains standing waves and normal modes. Worked examples show how to extract physical information from equations and how to use boundary conditions rather than memorised formulas alone.

2. From circular motion to simple harmonic motion

Uniform circular motion provides a geometric model for sinusoidal oscillation. Imagine a point P moving around a circle of radius A with constant angular speed ω . Its angular position may be written $\theta(t) = \omega t + \phi$, where ϕ is the initial phase. The horizontal projection of P has coordinate $x = A \cos \theta$. Substitution gives the standard equation of simple harmonic motion:

$$x(t) = A \cos(\omega t + \phi)$$

The rotating point is only a mathematical construction: a real spring-mounted mass does not travel around a circle. Nevertheless, the construction explains why sine and cosine functions appear naturally and why the angular frequency of an oscillator is measured in radians per second. The radius A becomes the amplitude, while the continuously increasing angle θ becomes the phase of the oscillation.

Period, frequency and angular frequency

One complete revolution corresponds to a phase increase of 2π radians. Therefore, if T is the time for one complete oscillation and f is the number of oscillations per second, the fundamental temporal relations are:

$$f = 1/T, \quad \omega = 2\pi f = 2\pi/T$$

Frequency f is measured in hertz, where $1 \text{ Hz} = 1 \text{ s}^{-1}$. Angular frequency ω is measured in rad s^{-1} . Although radians are dimensionless in strict SI analysis, writing rad/s is valuable because it reminds us that ω describes phase change, not cycles per second.

Velocity and acceleration in SHM

Differentiating the displacement gives the instantaneous velocity and acceleration. These equations also reveal their phase relations:

$$v_x(t) = dx/dt = -A\omega \sin(\omega t + \phi)$$

$$a_x(t) = d^2x/dt^2 = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$$

At the equilibrium position $x = 0$, the speed is maximum and the acceleration is zero. At the turning points $x = \pm A$, the speed is zero and the acceleration has maximum magnitude $\omega^2 A$ and points toward equilibrium. Thus the acceleration is always opposite to the displacement. This is the defining kinematic property of simple harmonic motion.

Interpretation: the minus sign in $a = -\omega^2 x$ is not decorative. It expresses the restoring nature of the dynamics: a positive displacement produces a negative acceleration, and a negative displacement produces a positive acceleration.

3. Oscillators: the building blocks of waves

An oscillation is repetitive motion about an equilibrium position. Two ingredients are required: inertia, which carries a system through equilibrium, and a restoring mechanism, which accelerates it back toward equilibrium after displacement. Their continuing interaction produces periodic motion when energy losses are negligible.

The mass–spring oscillator

For an ideal spring, Hooke's law states that the restoring force is proportional to displacement and opposite in direction:

$$F_x = -kx$$

Newton's second law then gives $m \, d^2x/dt^2 = -kx$. Rearranging produces the differential equation of the harmonic oscillator:

$$d^2x/dt^2 + (k/m)x = 0$$

Comparison with $a = -\omega^2x$ identifies the natural angular frequency, period and frequency:

$$\omega_0 = \sqrt{(k/m)}, \quad T = 2\pi\sqrt{(m/k)}, \quad f = (1/2\pi)\sqrt{(k/m)}$$

A stiffer spring increases the restoring acceleration and therefore increases the frequency. A larger mass increases inertia and lowers the frequency. The amplitude does not appear in these expressions because the ideal Hooke-law oscillator is linear: changing the initial displacement changes the energy but not the natural frequency.

Other oscillatory systems

For a simple pendulum of length ℓ at small angular displacement, $\sin \theta \approx \theta$ and the motion is approximately harmonic, with $\omega_0 = \sqrt{(g/\ell)}$. Tuning forks, lattice atoms and ideal LC circuits are further examples: different physical quantities oscillate, but the mathematical structure is closely related.

From one oscillator to a wave

A single oscillator remains local; a wave appears when many oscillators are coupled. Displacing one element produces forces on its neighbours, which then act on further elements, so the disturbance propagates. A string is a continuous chain of coupled mass elements, while sound consists of coupled pressure and density variations in a fluid.

Energy viewpoint: an oscillator exchanges kinetic and potential energy locally; coupling allows that energy exchange to propagate through the medium.

4. What is a wave? Classification and physical interpretation

Mechanical waves require a material medium. What oscillates is matter: string elements, fluid parcels or parts of an elastic solid. Their speed depends on the mechanical properties of that medium. Electromagnetic waves do not require matter; electric and magnetic fields oscillate and can propagate through vacuum with speed $c = 3.00 \times 10^8 \text{ m/s}$.

Transverse and longitudinal mechanical waves

In a transverse wave, particle displacement is perpendicular to the propagation direction. A pulse travelling horizontally along a taut string while the string elements move vertically is the standard example. The wave profile contains crests and troughs. Electromagnetic waves are also transverse: the electric field, magnetic field and propagation direction are mutually perpendicular.

In a longitudinal wave, particle displacement is parallel to the propagation direction. A sound wave in air consists of alternating compressions and rarefactions. At a compression, pressure and density are higher than equilibrium; at a rarefaction, they are lower. A slinky pushed and pulled along its own axis gives a visible model of this behaviour.

Wave class	What oscillates?	Medium required?	Typical examples
Mechanical transverse	Material elements perpendicular to propagation	Yes	String waves, seismic S-waves
Mechanical longitudinal	Material elements parallel to propagation	Yes	Sound, slinky waves, seismic P-waves
Electromagnetic	Electric and magnetic fields	No	Radio, light, X-rays

Water-surface waves require care

A surface water wave is often introduced as transverse because the surface moves up and down while the pattern travels horizontally. In reality, fluid particles can follow approximately circular or elliptical paths, so the motion contains both transverse and longitudinal components. The simplified transverse picture is useful, but the limitations of the model should be recognised.

Diagnostic question: ask ‘in which direction does the local element move?’ and compare it with the direction in which the disturbance propagates. This immediately distinguishes transverse from longitudinal behaviour.

A second diagnostic question is whether the phenomenon can exist in vacuum. Sound cannot, because it needs particles that can undergo pressure and density oscillations. Light can, because the oscillating quantities are electromagnetic fields rather than material particles.

5. The fundamental quantities of a periodic wave

A periodic wave repeats both in space and in time. Its spatial repetition is described by wavelength λ , while its temporal repetition is described by period T . These quantities refer to different measurements and must not be interchanged.

Amplitude, wavelength and period

Amplitude A is the maximum displacement from equilibrium. On a transverse displacement graph, it is measured from the midline to a crest or to a trough. The crest-to-trough distance is $2A$, not A . Wavelength λ is the shortest distance between two points in the same phase, such as successive crests, successive troughs or successive upward zero crossings. Period T is the time required for one full cycle at a fixed position.

Frequency and angular frequency

Frequency f counts cycles per second. The relation $f = 1/T$ is reciprocal: a short period means a high frequency. Angular frequency ω expresses the same temporal repetition in radians per second. Because one cycle contains 2π radians, $\omega = 2\pi f$.

Wave number: spatial angular frequency

Wave number k describes how rapidly phase changes with position. One wavelength corresponds to a phase advance of 2π , hence:

$$k = 2\pi/\lambda$$

The unit is rad/m. The number of complete wavelengths per metre is $1/\lambda = k/(2\pi)$, not k . A shorter wavelength gives a larger wave number and a more rapidly changing spatial phase.

Quantity	Symbol	Definition or relation	SI unit
Amplitude	A	Maximum displacement	m (or field unit)
Wavelength	λ	Spatial repeat length	m
Period	T	Time per cycle	s
Frequency	f	$f = 1/T$	Hz = s ⁻¹
Angular frequency	ω	$\omega = 2\pi f$	rad/s
Wave number	k	$k = 2\pi/\lambda$	rad/m
Phase constant	ϕ	Initial phase shift	rad

Unit check: ω and f describe the same oscillation but are not numerically equal. Similarly, k and $1/\lambda$ are related by the factor 2π . Missing these factors is one of the most frequent errors in wave calculations.

6. Wave speed and the connection between space and time

Wave speed v is the speed with which a point of constant phase, such as a crest, moves. During one period T , the wave pattern advances one wavelength λ . Therefore:

$$v = \lambda/T = f\lambda$$

This equation links the temporal description f with the spatial description λ . It does not state that medium particles travel at speed v . Their velocity is the time derivative of their local displacement and generally changes continuously during an oscillation.

What determines wave speed?

For linear waves in a uniform medium, propagation speed is set mainly by properties of the medium, not by the source frequency or amplitude. The source determines the frequency. Once the wave enters a medium, the allowed wavelength follows from $\lambda = v/f$. When a wave crosses into another medium, frequency remains continuous across the boundary, while speed and wavelength may change.

For a stretched string:
$$v = \sqrt{(F_t/\mu)}$$

Here F_t is the tension and μ is the linear mass density in kg/m. Greater tension provides a stronger restoring effect and increases speed; greater mass per unit length increases inertia and reduces speed. If tension is quadrupled while μ remains constant, speed doubles because of the square-root dependence.

For longitudinal sound in a fluid:
$$v = \sqrt{(B/\rho)}$$

B is the bulk modulus and ρ is density. A large bulk modulus means the medium strongly resists compression and supports faster propagation; a large density increases inertia and tends to lower the speed. In shallow water, an approximate relation is $v = \sqrt{gh}$, where h is water depth.

Dispersion and the scope of the simple rule

The lecture uses the important linear, non-dispersive approximation in which waves of different frequency travel at the same speed in a given uniform medium. Real systems can be dispersive: speed may depend on frequency, as in deep-water waves, waveguides or many optical materials. The relation $v = f\lambda$ still holds for a single sinusoidal component, but different components may possess different phase velocities.

Problem-solving rule: when frequency changes but the medium and its wave speed do not, wavelength must change inversely: doubling f halves λ . Never assume that a higher source frequency automatically makes the wave travel faster.

7. Phase and the equation of a travelling wave

A sinusoidal wave travelling in the positive x-direction may be written:

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

Every symbol has a distinct role. A fixes the maximum displacement; k controls spatial repetition; ω controls temporal repetition; and ϕ fixes the phase at $x = 0$ and $t = 0$. The complete argument $kx - \omega t + \phi$ is the phase angle. It must be dimensionless, which provides an immediate unit check: kx , ωt and ϕ are all angles in radians.

How the sign determines direction

To follow a particular crest, hold the phase constant. For $kx - \omega t + \phi = \text{constant}$, differentiation gives $k \, dx/dt - \omega = 0$, so $dx/dt = \omega/k > 0$. Thus the minus sign before ωt corresponds to propagation in $+x$. For $kx + \omega t + \phi = \text{constant}$, $dx/dt = -\omega/k$ and the wave travels in $-x$.

$$+x \text{ direction: } y = A \sin(kx - \omega t + \phi) \quad | \quad -x \text{ direction: } y = A \sin(kx + \omega t + \phi)$$

Phase velocity

The speed of a point of constant phase is:

$$v = \omega/k = f\lambda$$

This follows immediately from $\omega = 2\pi f$ and $k = 2\pi/\lambda$. It is a useful consistency check because the factor 2π cancels.

Phase differences

Two points or two waves are in phase when their phase difference is $2\pi n$, where n is an integer. A phase difference π corresponds to complete opposition: a crest of one aligns with a trough of the other. A phase difference $\pi/2$ is a quarter-cycle offset. Phase is defined modulo 2π , so ϕ and $\phi + 2\pi n$ describe the same physical state.

A positive phase constant shifts the spatial snapshot to the left because $A \sin(kx + \phi)$ reaches the same phase at a smaller x . In a time trace $A \sin(-\omega t + \phi)$, the interpretation depends on the sign convention; the safest method is always to solve for the position or time at which a chosen phase occurs.

Two complementary views: at fixed t , $y(x,t)$ is a spatial snapshot of the wave. At fixed x , $y(x,t)$ is the simple harmonic motion of one medium element. A travelling wave contains both descriptions simultaneously.

8. Worked example: extracting information from a wave equation

Consider the transverse wave on a string

$$y(x, t) = 0.050 \sin(6.00x - 40.0t + \pi/4)$$

The safest procedure is to compare it term by term with $y = A \sin(kx - \omega t + \phi)$. This gives $A = 0.050$ m, $k = 6.00$ rad/m, $\omega = 40.0$ rad/s and $\phi = \pi/4$ rad. Because the time term has a minus sign, propagation is in the +x-direction.

Step 1: wavelength

$$\lambda = 2\pi/k = 2\pi/(6.00 \text{ rad/m}) = 1.047 \text{ m}$$

Step 2: frequency and period

$$f = \omega/(2\pi) = 40.0/(2\pi) = 6.37 \text{ Hz}$$

$$T = 1/f = 0.157 \text{ s}$$

Step 3: wave speed

$$v = \omega/k = 40.0/6.00 = 6.67 \text{ m/s}$$

The same result follows from $f\lambda = (6.37 \text{ Hz})(1.047 \text{ m}) \approx 6.67 \text{ m/s}$. Agreement between independent calculation routes is an excellent internal check.

Step 4: form a snapshot

At $t = 0.200$ s, substitute the time into the original equation:

$$y(x, 0.200) = 0.050 \sin(6.00x - 8.00 + \pi/4)$$

$$y(x, 0.200) = 0.050 \sin(6.00x - 7.215)$$

Angles that differ by 2π are equivalent, so the phase constant in the snapshot may also be reduced modulo 2π if desired. The unreduced form is often preferable because it shows exactly how the substitution was made.

Physical interpretation

Each string element performs vertical simple harmonic motion with amplitude 5.0 cm and frequency 6.37 Hz. Meanwhile, the shape of the disturbance moves rightward at 6.67 m/s. Successive crests are separated by 1.047 m. The phase constant means the wave is not at a zero-phase reference state at $x = 0$, $t = 0$.

Consistency checklist: identify A , k , ω and ϕ first; obtain λ and f only afterward; infer direction from the sign; then verify v using both ω/k and $f\lambda$. This method is more reliable than trying to remember isolated formulas.

9. The one-dimensional wave equation

A travelling-wave expression describes a particular solution. The wave equation is more general: it specifies the differential relationship that any small-amplitude wave in a uniform, non-dispersive medium must satisfy. In one spatial dimension it is

$$\partial^2 y / \partial x^2 = (1/v^2) \partial^2 y / \partial t^2$$

The left side measures spatial curvature of the displacement profile. The right side measures local acceleration scaled by the square of wave speed. The equation therefore connects how sharply the medium is bent or compressed in space with how rapidly it accelerates in time.

Origin for a stretched string

Take a short string element of length Δx , tension F_t and linear density μ . The tensions at its two ends have slightly different vertical components when the string is curved. For small slopes, the net vertical force is approximately $F_t(\partial^2 y / \partial x^2)\Delta x$. The mass is $\mu\Delta x$ and its transverse acceleration is $\partial^2 y / \partial t^2$. Newton's second law gives

$$F_t(\partial^2 y / \partial x^2)\Delta x = \mu\Delta x(\partial^2 y / \partial t^2)$$

After cancelling Δx , comparison with the standard wave equation yields $v^2 = F_t/\mu$, or $v = \sqrt{F_t/\mu}$. Thus the familiar string-speed formula is not an empirical coincidence; it follows from force balance, inertia and the small-slope approximation.

General solution and verification

$$y(x,t) = F(x - vt) + G(x + vt)$$

F is any suitably smooth shape travelling in $+x$ without changing form, while G travels in $-x$. A sinusoid is only one special case. For $y = A \sin(kx - \omega t + \phi)$, the second derivatives are $\partial^2 y / \partial x^2 = -k^2 y$ and $\partial^2 y / \partial t^2 = -\omega^2 y$. Substitution into the wave equation requires $k^2 = \omega^2/v^2$, hence $v = \omega/k$.

Three-dimensional form

$$\nabla^2 y = (1/v^2) \partial^2 y / \partial t^2$$

Here ∇^2 is the Laplacian, the sum of second spatial derivatives. The three-dimensional equation allows wavefronts to spread in several directions and underlies models of acoustics, elasticity and electromagnetism, although the physical field represented by y differs between systems.

Important limitation: the wave equation determines how valid disturbances evolve, but boundary and initial conditions select the actual solution. Without them, the physical problem is incomplete.

10. Superposition and the formation of standing waves

In a linear medium, the superposition principle states that overlapping waves add algebraically. If y_1 and y_2 are individually valid solutions, then $y = y_1 + y_2$ is also a solution. Interference is the observable consequence of this addition: displacements may reinforce or cancel depending on phase.

Consider two waves of equal amplitude A , wave number k and angular frequency ω travelling in opposite directions:

$$y_1 = A \sin(kx - \omega t), \quad y_2 = A \sin(kx + \omega t)$$

Using $\sin(a - b) + \sin(a + b) = 2 \sin a \cos b$ gives

$$y(x, t) = 2A \sin(kx) \cos(\omega t)$$

This is a standing wave. Its spatial and temporal factors are separated. The pattern does not translate left or right: every point oscillates with the same angular frequency, but the local amplitude $2A|\sin(kx)|$ depends on position.

Nodes and antinodes

Nodes are points that never move. They occur where $\sin(kx) = 0$:

$$kx = n\pi \Rightarrow x_n = n\pi/k = n\lambda/2, \quad n = 0, 1, 2, \dots$$

Antinodes are points with maximum amplitude $2A$. They occur where $|\sin(kx)| = 1$:

$$kx = (n + 1/2)\pi \Rightarrow x = (2n + 1)\lambda/4$$

Adjacent nodes are separated by $\lambda/2$, as are adjacent antinodes. A node and the nearest antinode are separated by $\lambda/4$.

Energy in a standing wave

The two component waves carry energy in opposite directions. Their average energy fluxes cancel, so there is no net energy transport through an ideal standing-wave pattern. Energy still changes locally between kinetic and potential forms. At certain instants the medium is momentarily at rest but strongly deformed; a quarter period later it may pass through equilibrium with maximum local speeds.

Do not confuse ‘stationary pattern’ with ‘stationary medium’: most points in a standing wave move continuously. Only the nodes remain fixed; the word standing refers to the positions of nodes and antinodes.

11. Boundary conditions and normal modes

Standing waves become physically important when reflections confine waves between boundaries. The boundaries restrict which spatial patterns can exist. These allowed patterns are normal modes, and their frequencies are natural or resonant frequencies.

String fixed at both ends

A fixed endpoint must have zero transverse displacement. For a string of length L fixed at $x = 0$ and $x = L$:

$$y(0,t) = 0, \quad y(L,t) = 0$$

The length must contain an integer number of half-wavelengths:

$$L = n\lambda_n/2 \Rightarrow \lambda_n = 2L/n, \quad n = 1, 2, 3, \dots$$

$$k_n = n\pi/L, \quad f_n = v/\lambda_n = nv/(2L)$$

The fundamental mode $n = 1$ has one antinode and nodes at both ends. The n th mode has n antinodes and $n + 1$ nodes including the endpoints. Its frequency is n times the fundamental in an ideal non-dispersive string; these integer multiples are harmonics.

One end fixed and the other free

A fixed end is a node. At an ideal free end, the transverse force must vanish, which for a taut string requires zero slope $\partial y/\partial x = 0$; this produces an antinode. Only an odd number of quarter-wavelengths fits:

$$L = (2n - 1)\lambda_n/4 \Rightarrow \lambda_n = 4L/(2n - 1)$$

$$f_n = (2n - 1)v/(4L), \quad n = 1, 2, 3, \dots$$

The allowed frequencies are odd multiples of the fundamental: f_1 , $3f_1$, $5f_1$ and so on. This distinction is essential in strings, air columns and resonators. Applying the fixed–fixed formula to a fixed–free system gives incorrect results even if every algebraic step is performed correctly.

Boundary type	End behaviour	Allowed wavelengths	Allowed frequencies
Fixed–fixed	Node at both ends	$\lambda_n = 2L/n$	$f_n = nv/(2L)$
Fixed–free	Node then antinode	$\lambda_n = 4L/(2n-1)$	$f_n = (2n-1)v/(4L)$

Core method: sketch the end conditions first, mark required nodes and antinodes, and count half- or quarter-wavelength segments. The formula should emerge from the geometry rather than replace it.

12. Worked example: resonant frequencies of a stretched string

A string of length $L = 1.20$ m is fixed at both ends, and transverse waves travel on it with speed $v = 240$ m/s. Find the first three resonant frequencies and their wavelengths.

Step 1: choose the correct boundary model

Because both ends are fixed, both endpoints are nodes. Therefore the permitted relations are $\lambda_n = 2L/n$ and $f_n = nv/(2L)$, with $n = 1, 2, 3, \dots$

Step 2: calculate the fundamental

$$f_1 = v/(2L) = 240/[2(1.20)] = 100 \text{ Hz}$$

$$\lambda_1 = 2L = 2.40 \text{ m}$$

The wavelength can exceed the physical length of the string because the string contains only half of the fundamental wavelength.

Step 3: calculate higher modes

$$f_2 = 2f_1 = 200 \text{ Hz}, \quad \lambda_2 = 2L/2 = 1.20 \text{ m}$$

$$f_3 = 3f_1 = 300 \text{ Hz}, \quad \lambda_3 = 2L/3 = 0.800 \text{ m}$$

Mode n	Name	Wavelength λ_n	Frequency f_n	Pattern
1	Fundamental	2.40 m	100 Hz	1 antinode, 2 nodes
2	Second harmonic	1.20 m	200 Hz	2 antinodes, 3 nodes
3	Third harmonic	0.800 m	300 Hz	3 antinodes, 4 nodes

Parameter changes

The frequencies scale as $f_n \propto v/L$. Doubling the length while keeping wave speed unchanged halves every resonant frequency. Because $v = \sqrt{(F_t/\mu)}$, quadrupling the tension doubles v and therefore doubles all resonant frequencies. Quadrupling μ halves v and halves the frequencies.

A contrasting fixed–free result

If the same 1.20 m system were fixed at one end and free at the other, its fundamental would be $f_1 = v/(4L) = 50$ Hz. The next allowed frequencies would be 150 Hz and 250 Hz, not 100 Hz increments. This comparison shows why the boundary sketch is the first essential step.

Engineering meaning: resonance is not automatically dangerous or beneficial. Musical instruments and sensors exploit it, while structures and rotating machinery are often designed so that operating excitations avoid natural frequencies or are sufficiently damped.

13. Applications, common mistakes and final problem-solving guide

Engineering applications

Electromagnetic waves carry information in radio, satellite and optical-fibre links. Ultrasound forms medical images; non-destructive testing detects hidden cracks; seismic waves reveal underground structure; and acoustical or structural engineers control reflection and resonance. The same concepts support radar, sonar, photonics, material characterisation and energy harvesting. Frequency and wavelength are selected according to resolution, penetration, attenuation and boundary interaction.

Frequent mistakes and their corrections

Amplitude A is measured from equilibrium to an extreme; wavelength λ is measured between equal-phase points. Period T is seconds per cycle; frequency f is cycles per second. Remember $\omega = 2\pi f$ and $k = 2\pi/\lambda$. The form $(kx - \omega t)$ moves right and $(kx + \omega t)$ moves left. Wave speed is not particle velocity, and every standing-wave formula must match its boundary conditions.

A reliable solution sequence

Step	Action	Purpose
1	Identify the system and what oscillates.	Classify the wave and decide whether a medium is required.
2	Draw propagation, displacement and boundaries.	Fix directions, nodes and antinodes before algebra.
3	List known quantities with SI units.	Prevent unit mixing and expose missing data.
4	Write definitions before derived formulas.	Use $f=1/T$, $\omega=2\pi f$, $k=2\pi/\lambda$ and $v=f\lambda$ consistently.
5	Apply the medium or boundary relation.	Select $v=\sqrt{F/\mu}$, fixed–fixed modes, fixed–free modes, etc.
6	Check dimensions, signs and limiting behaviour.	Test whether the answer is mathematically and physically plausible.

Review problems

1. For a 440 Hz sound wave at 343 m/s, find T , λ , ω and k . 2. A string has $F_t = 180$ N, $\mu = 0.020$ kg/m and $L = 1.50$ m; find its speed and fixed–fixed fundamental. 3. For $y = 0.020 \cos(5.0x + 30t)$, identify all wave parameters and direction. 4. Explain why λ changes at a medium boundary while f remains constant. 5. Derive the first three fixed–free modes from a sketch.

Essential relations: $f=1/T$, $\omega=2\pi f$, $k=2\pi/\lambda$, $v=f\lambda=\omega/k$

Travelling wave: $y=A \sin(kx \mp \omega t + \phi)$ | **Standing wave:** $y=2A \sin(kx)\cos(\omega t)$