

PHYSICS FOR ENGINEERING STUDENTS

Measuring instruments

Structure, characteristics and dynamic response

Lecture 2 — Book-style student notes

Learning focus: Measurement chains · Instrument classifications · Static characteristics · Dynamic models

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Measuring Instruments

A measuring instrument transforms a physical phenomenon into a numerical result that can be interpreted, communicated, and used. The transformation is rarely a single step. A practical system normally senses the measurand, converts it into another physical or electrical quantity, conditions the signal, processes the data, and presents an output. Understanding this chain is essential because every element can modify the information and contribute uncertainty.

Learning objectives

- Identify the functional elements of a measurement chain.
- Distinguish direct comparison from calibrated measurement.
- Classify instruments by reference, energy supply, output form, and data handling.
- Interpret range, calibration curve, sensitivity, linearity, threshold, resolution, accuracy, and precision.
- Model zero-, first-, and second-order dynamic responses.
- Select a counter gate time by balancing resolution and update rate.

Central principle: An instrument does not reveal a physical quantity directly. It produces an indication that must be related to the measurand through a model and a calibration.

Further interpretation

A useful way to analyse any instrument is to follow both the physical quantity and the information carried by it. At the sensor, information is encoded in a physical response; at the transducer, that response becomes a convenient signal; during conditioning and processing, unwanted components are suppressed and a numerical estimate is formed. At every stage we should ask what relation is assumed, what information may be lost, and what new uncertainty is introduced. This perspective prevents the display from being treated as an isolated black box and makes fault diagnosis much easier. If an output appears implausible, the chain can be tested element by element with known inputs and intermediate signal observations.

1. From physical quantity to information

The measurand G is the quantity intended to be measured, such as temperature, pressure, displacement, voltage, or concentration. The instrument compares or relates G to a unit standard U and produces an output Z . In the simplest functional representation, the complete instrument is treated as a mapping from input to output.

$$Z = M(G; U)$$

Here M represents the measuring operation and all transformations inside the instrument. In a direct-reading instrument the output may already be expressed in the required unit. In other systems Z is an intermediate signal—voltage, frequency, counts, or pixel intensity—that must be converted to the measured value by a calibration function.

Element	Function	Example
Measurand	Physical quantity of interest	Temperature
Sensor	Interacts directly with measurand	Platinum resistor
Transducer	Converts one signal form to another	Resistance-to-voltage bridge
Conditioning	Amplifies, filters, linearizes	Instrumentation amplifier
Processing	Applies model and corrections	Digital algorithm
Output	Displays or stores the result	23.46 °C
Traceability: A numerical result is meaningful only when its relation to accepted standards can be demonstrated through an unbroken calibration chain.		

Further interpretation

The distinction between sensing and measuring is important. A sensor may respond to several quantities simultaneously, while a measurement system is designed to infer one explicitly defined measurand. For example, a resistance thermometer responds mainly to temperature, but self-heating, lead resistance, strain, and ageing can also affect its resistance. The measurement model identifies which influences are negligible, controlled, corrected, or included in uncertainty. This is why the same sensor can produce different-quality results in different systems. Engineering measurement is therefore not simply choosing a device; it is designing a complete information pathway whose response is sufficiently selective, traceable, stable, and fast for the intended decision.

2. Direct comparison and calibration

In direct comparison, the unknown quantity is compared with a known standard of the same kind. A balance compares an unknown mass with standard masses; a null bridge compares an unknown resistance with known resistances. Such methods can be extremely accurate because the indication is based on equality or balance rather than on the absolute response of a detector.

Most practical instruments require calibration. Known reference inputs G_i are applied, the corresponding outputs Z_i are recorded, and a calibration relation $Z=f(G)$ is established. During measurement, the inverse relation is used to infer the unknown input from the observed output.

$$\hat{G} = f^{-1}(Z)$$

Null methods

A zero-deflection or null method uses feedback to oppose the measurand with an adjustable known quantity. At balance the detector output is zero. Detector gain and some nonlinearities then have little influence, because the detector is used only to identify the balance condition. Potentiometric voltage measurement and Wheatstone-bridge resistance measurement are classic examples.

Practical trade-off: Null methods favour accuracy and sensitivity, but they may be slower than direct-deflection methods and often require a controllable reference and feedback mechanism.

Further interpretation

Calibration does not make an instrument permanently correct. It provides evidence about its response at particular times, conditions, ranges, and reference values. Between calibrations, drift, contamination, mechanical change, or electronic ageing may alter the relation. Verification checks with one or more working standards can detect important changes without repeating a full calibration. The calibration interval should reflect required uncertainty, historical stability, use intensity, and consequence of failure. In a null method, balance reduces sensitivity to detector gain, but reference quality and balance criterion remain critical. Thus the method changes the dominant uncertainty contributions rather than eliminating uncertainty altogether.

3. Classification of instruments

Classification helps us predict how an instrument behaves and what limitations must be considered. No single classification is sufficient; the same device can be, for example, differential, active, digital, and recording.

Criterion	First class	Second class
Reference	Absolute: referred to an absolute standard or defining relation	Differential: measures the difference between two quantities
Power	Active: needs an external energy source	Passive: obtains output energy from the measurand
Signal	Analog: continuous in time and amplitude	Digital: discrete samples and quantized levels
Use	Displaying: presents the current value	Recording: stores values for later analysis

“Active” and “passive” refer to the energy required for operation, not to whether the device contains electronics. A thermocouple can generate a voltage from a temperature difference and is passive in this sense, whereas a resistance thermometer needs an excitation current and is active.

Analog and digital are also system-level descriptions. A digital instrument normally begins with an analog physical interaction. Sampling makes time discrete, while quantization maps amplitudes to a finite set of codes.

$$t_n = nT_s, \quad x_q[n] = Q\{x(nT_s)\}$$

Avoid a common mistake: A digital display does not guarantee high accuracy. It provides discrete output and may offer fine resolution, but calibration bias and other errors can remain.

Further interpretation

The classifications in this chapter are complementary design descriptions. A strain-gauge force system is usually active because its bridge requires excitation; it may be differential because opposite gauge arms respond in different directions; it is analog at the bridge output but digital after conversion; and it becomes recording when a data-acquisition system stores the sequence. Mapping these categories clarifies where power, reference, conversion, and storage limitations enter. It also reveals that an instrument may change class depending on how it is used. A thermocouple connected to a voltmeter is passive at the sensing junction, but the complete electronic system still requires power for amplification, conversion, display, and communication.

4. Static characteristics: range and calibration

Static characteristics describe the relation between input and output when the measurand is constant or changes slowly enough for transient effects to be negligible. The measurement range is the interval from X_l to X_u within which specified performance is guaranteed. Values outside this range may saturate the output, damage the sensor, or violate uncertainty specifications.

$$X_{\min} \leq X \leq X_{\max}$$

The calibration curve is the experimentally determined input–output characteristic. It may be represented by a graph, a table, or an equation. A linear approximation is convenient but must be justified over the chosen range.

$$Z = a + SX$$

The intercept a is the output at zero input and S is the sensitivity. Calibration should cover the operating range with traceable standards, repeated readings, and both increasing and decreasing input when hysteresis is possible. Residuals—the differences between measured calibration outputs and fitted values—reveal systematic structure that a single coefficient of determination can hide.

Range selection

A wide range increases versatility but may reduce effective resolution for a fixed number of output codes. Select the narrowest range that safely includes the expected measurand and foreseeable transients.

Specification rule: Performance specifications apply only under stated range and operating conditions. A number displayed outside those conditions is not automatically a valid measurement result.

Further interpretation

Range statements must be interpreted together with overload capacity and resolution. The normal operating range specifies where performance is characterized; the survival range may be wider but does not guarantee valid data. Autoranging instruments can cover several decades by switching gain or reference elements, yet switching boundaries may introduce discontinuities or different uncertainty specifications. During calibration, enough points are needed to reveal curvature and local sensitivity changes. Repeated values at each point separate short-term scatter from the fitted trend, while upward and downward sequences reveal hysteresis. A calibration equation should never be used beyond its validated range merely because software can evaluate it there.

5. Sensitivity, linearity, threshold and resolution

Sensitivity quantifies how strongly the output changes when the input changes. For a nonlinear characteristic, it is a local property equal to the slope of the calibration curve.

$$S(X) = dZ/dX$$

High sensitivity produces a large output change for a small input change, but it is not synonymous with high accuracy. A sensitive instrument may still have offset, scale-factor error, drift, noise, or poor calibration.

Linearity expresses the closeness of the actual characteristic to a reference straight line. Nonlinearity may be specified as the maximum deviation divided by full-scale output. The chosen reference line—endpoint, best fit, or zero-based—must be stated.

$$\delta_L = \max|Z(X) - Z_{\text{ref}}(X)| / Z_{\text{FS}}$$

The sensitivity threshold is the smallest input that produces a detectable response above background. Resolution is the smallest change in input that can be distinguished. Threshold concerns detection of a quantity; resolution concerns separation of nearby values. Noise, digitization, friction, backlash, and display increments may all set the effective resolution.

Interpretation: Sensitivity is a slope, threshold is a minimum detectable input, and resolution is a minimum distinguishable change. They answer different questions.

Further interpretation

Sensitivity also determines how input-referred noise is calculated. If the output noise standard deviation is u_Z and local sensitivity is S , the equivalent input noise is approximately $u_X = u_Z / |S|$. Increasing sensitivity can therefore improve detection only if it does not introduce saturation, instability, or additional noise. Resolution may be set by quantization when the signal is quiet, but by noise when several output codes fluctuate. In practice, an effective resolution can be estimated from repeated indications and from the smallest controlled input change that produces a reproducible response. Linearity errors can be corrected with a polynomial or lookup table, but the correction itself has uncertainty and must not be confused with perfect linear behaviour.

6. Loading, transparency and operating conditions

A measurement system can disturb the object being measured. This loading effect occurs because the instrument exchanges energy, mass, momentum, or information with the system. A voltmeter with finite input resistance draws current and changes the circuit voltage. A thermometer removes or supplies heat until thermal equilibrium is reached. A pressure probe can alter the local flow field.

Transparency or non-invasiveness describes the ability to obtain information without significantly changing the measurand. Electrical voltage measurement generally requires high input impedance; current measurement requires low impedance. Mechanical sensors should have suitable stiffness and low mass, while optical methods may reduce contact loading but can introduce alignment or surface-interaction effects.

Operating and storage conditions

- Temperature and humidity ranges
- Supply-voltage and electromagnetic environment
- Vibration, shock, pressure, and orientation
- Warm-up time, drift, and recalibration interval
- Storage limits distinct from measurement limits

Environmental influence quantities do not necessarily appear in the reported result, yet they change the instrument response. Their effects should be controlled, corrected, or included in the uncertainty evaluation.

Good practice: Record the instrument, range, settings, calibration status, and relevant environmental conditions together with the measured data. Without this metadata, later interpretation may be impossible.

Further interpretation

Loading should be analysed through the interaction energy or impedance. When a voltmeter of resistance R_V measures a source with Thevenin resistance R_S , the observed voltage is reduced by the divider factor $R_V/(R_S+R_V)$. The error becomes negligible only when R_V is sufficiently larger than R_S for the required uncertainty. Similar reasoning applies mechanically: attaching an accelerometer adds mass and may shift a structure's resonance. Thermally, the sensor's heat capacity and contact conductance affect both equilibrium and response time. A minimally invasive measurement is thus relative to a tolerance; no real instrument is perfectly transparent. The goal is to calculate, measure, or bound the disturbance and include it in the model.

7. Accuracy, precision and sources of inaccuracy

Accuracy describes closeness of agreement between a measured result and the true quantity value. Because the true value is generally unknown, accuracy is a qualitative concept and should not be quoted as a numerical “accuracy value” without defining the specification. Precision describes closeness among repeated results under stated conditions. Resolution is the smallest detectable change; it does not describe closeness to truth or repeatability.

Observation	Likely interpretation	Response
Tight cluster, displaced mean	High precision, systematic bias	Calibrate or correct
Wide scatter around reference	Low precision, small average bias	Repeat and reduce noise
Coarse identical readings	Resolution hides variation	Use finer resolution if needed
Drift with time	Changing systematic influence	Stabilize, monitor, model

Systematic effects shift results in a repeatable way: zero offset, scale error, loading, or an incorrect model. Random effects cause unpredictable scatter: electrical noise, environmental fluctuation, or reading variation. Repetition characterizes random variability but does not remove an unknown bias.

Measurement strategy: Improve validity by combining calibration, environmental control, an appropriate range, repeated observations, independent checks, and transparent uncertainty reporting.

Further interpretation

Accuracy and precision are best understood through repeated measurements of a reference. The mean displacement from the reference suggests bias, whereas the scatter describes repeatability under the stated conditions. However, the reference itself has uncertainty, so the observed difference cannot automatically be assigned entirely to the test instrument. Reproducibility studies deliberately vary operators, days, laboratories, or setups and normally show greater variability than short-term repeatability. A specification such as $\pm 0.5\%$ of reading plus two digits combines scale-dependent and resolution-dependent terms. It should be converted to an uncertainty model rather than quoted as though every value inside the band were equally likely without justification.

8. Dynamic behavior and mathematical models

Dynamic characteristics describe an instrument when the measurand varies with time. The output may lag, attenuate, distort, or overshoot even though the static calibration is correct. Three practical

questions are central: How quickly does the output respond? How faithfully does it follow changes? Does it remain stable?

A general linear ordinary differential model relates the input $x(t)$ and output $y(t)$.

$$a_n \frac{d^n y}{dt^n} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + \dots + b_0 x$$

Model order reflects the number of independent energy-storage elements or state variables needed to describe the dominant response. The simplest adequate model is preferred. An unnecessarily high-order model is difficult to identify and may fit noise, while an oversimplified model misses important delay, resonance, or damping.

Standard test inputs

- Step input: reveals rise time, time constant, overshoot, and settling.
- Ramp input: reveals tracking lag and steady dynamic error.
- Sinusoidal input: reveals frequency-dependent gain and phase.
- Impulse input: characterizes the system response over a broad frequency range.

Model validity: A dynamic model is an approximation valid over specified amplitudes, frequencies, and operating conditions. Always state the intended range of use.

Further interpretation

Dynamic testing translates qualitative terms into measurable quantities. Step response gives delay, rise time, overshoot, and settling time; sinusoidal testing gives amplitude ratio and phase shift as functions of frequency. Fidelity requires both to remain within acceptable limits for the waveform components that matter. Filtering introduces its own dynamics: it can reduce noise but also attenuate rapid events and shift their timing. Sampling must be fast enough to represent the bandwidth remaining after analog filtering. Consequently, the sensor, conditioner, filter, converter, and numerical algorithm should be analysed as one dynamic chain rather than as independent catalogue components.

9. Zero- and first-order instruments

A zero-order instrument has negligible energy storage within the frequency range of interest. Its output is an instantaneous scaled copy of the input.

$$y(t) = Kx(t)$$

The static sensitivity K is the only model parameter. Real instruments are never instantaneous at all frequencies; “zero order” means that their response time is negligible compared with the time scale of the measurand.

A first-order instrument has one dominant storage mechanism and is described by a static gain K and time constant τ .

$$\tau \, dy/dt + y = Kx(t)$$

For a step from zero to x_0 , with $y(0)=0$, the response is

$$y(t) = Kx_0(1 - e^{-t/\tau})$$

After one time constant the output reaches 63.2% of its final value; after about 3τ it reaches 95%, and after 5τ it is within approximately 0.7%. A larger τ means a slower instrument and greater attenuation of rapid variations. A thermometer exchanging heat with its environment is often approximated as first order.

Dynamic correction: Waiting for steady state reduces dynamic error, but when rapid measurement is required the model can be used to estimate or compensate for lag—provided noise amplification is controlled.

Further interpretation

For a first-order instrument, the time constant can be estimated from the time required to reach 63.2% of a stable step. It can also be obtained by fitting the complete exponential response, which uses more data and permits residual inspection. If the input changes continuously, the lag depends on its rate as well as on τ . For a ramp input $x=at$, the output eventually follows with an approximately constant delay τ , giving a dynamic error proportional to $a\tau$. Deconvolution can compensate for this lag, but differentiation amplifies high-frequency noise. Practical dynamic correction therefore combines a validated model with regularization or filtering and is restricted to a defensible bandwidth.

10. Second-order instruments

A second-order instrument contains two dominant storage effects and can show overshoot or oscillation. Its standard normalized model is

$$d^2y/dt^2 + 2\zeta\omega_n dy/dt + \omega_n^2 y = K\omega_n^2 x(t)$$

The natural angular frequency ω_n sets the characteristic speed, and the damping ratio ζ determines the form of the transient. For $0 < \zeta < 1$ the system is underdamped and oscillatory; $\zeta=1$ gives critical damping; $\zeta > 1$ gives an overdamped, non-oscillatory but slower response. With $\zeta=0$ there is no damping and an idealized system oscillates indefinitely.

Damping	Step response	Measurement implication
$0 < \zeta < 1$	Overshoot and decaying oscillation	Fast but may ring
$\zeta = 1$	Fastest response without overshoot	Useful design target
$\zeta > 1$	Slow monotonic approach	Stable but sluggish

Resonance can strongly amplify inputs near the natural frequency. Therefore a sensor with adequate static range may still produce misleading peaks or even be damaged by dynamic excitation. The usable bandwidth should be chosen where both amplitude and phase errors remain acceptable.

Selection rule: Match the instrument bandwidth to the phenomenon. A response that is too slow suppresses real variation; an excessively wide bandwidth may admit unnecessary noise.

Further interpretation

Second-order behaviour appears in mechanical sensors, galvanometers, pressure systems, and many structures. Designers often select damping near 0.7 rather than exactly 1 because it provides a useful compromise between rapid rise, limited overshoot, and flat frequency response. The resonance frequency observed in a forced response differs from the undamped natural frequency when damping is appreciable. Mounting and coupling can also change both parameters, so a sensor calibrated on a rigid fixture may behave differently in the actual installation. A ringing output should not immediately be interpreted as an oscillating measurand; it may be the instrument's transient response to a non-oscillatory input.

11. Counters as measuring instruments

A counter measures the number N of input events occurring during a gate interval T_g . It is used for frequency, period, particle, photon, pulse, and event measurements. For direct frequency counting, the estimate is

$$\hat{f} = N/T_g$$

Because N is an integer, the unavoidable ± 1 -count ambiguity gives an approximate relative quantization error of $1/N$. Increasing the gate time increases N and improves frequency resolution, but it slows the update rate. A stable reference clock is also essential because its time-base error transfers directly to the result.

$$|\delta f/f| \approx 1/N + |\delta T_g/T_g|$$

Missed pulses, false triggers, dead time, input threshold noise, finite pulse width, and reference-clock drift are further error sources. At low frequencies, measuring the period with a high-frequency reference clock may provide better resolution than counting cycles in a fixed gate.

Example

If a counter records 10,000 pulses during a 1.000 s gate, the estimate is 10.000 kHz and the ± 1 -count contribution is about 0.01%. Extending the gate to 10 s reduces this contribution by a factor of ten but provides only one update every 10 s.

Design trade-off: Choose gate time from the required uncertainty and time resolution, not merely from the largest available display precision.

Further interpretation

Counters illustrate how a simple digital principle still requires a complete measurement model. Trigger level determines which pulses are accepted, hysteresis prevents repeated triggering on noise, and dead time can cause missed events at high rates. For randomly arriving events, count variability may follow Poisson statistics, adding a $\sqrt{N}$ contribution beyond the deterministic ± 1 -count digitization effect. Reciprocal counters measure the time for one or more input periods using a faster reference clock and are especially effective at low frequencies. Modern instruments may interpolate between reference-clock edges, but their displayed resolution must still be distinguished from time-base accuracy and trigger jitter.

12. Integrated measurement workflow

Reliable measurement begins by defining the measurand and the decision the result must support. The instrument is then selected using range, sensitivity, threshold, resolution, loading, static uncertainty, bandwidth, and environmental limits. Calibration establishes the relation between indication and input; a dynamic model determines whether time variation is followed faithfully.

Recommended workflow

- Define the measurand, unit, expected range, and relevant time scale.
- Select the sensing principle and check loading or interaction with the object.
- Verify calibration status, traceability, operating conditions, and warm-up.
- Choose range, sampling, filtering, and gate or averaging time.
- Perform checks with reference inputs and record full metadata.
- Report the result with appropriate digits and uncertainty.

The numerical display is the end of a physical and mathematical chain, not proof of correctness. A strong measurement design makes every transformation explicit and tests whether the assumptions remain valid.

Key equations

$$S = dZ/dX, \quad \tau dy/dt + y = Kx, \quad \hat{f} = N/T_g$$

Take-home message: Instrument choice is a problem of matching: the measurement system must match the magnitude, dynamics, environment, and required uncertainty of the measurand.

Further interpretation

The final workflow should be documented before data collection whenever possible. A short measurement plan can state the measurand, required uncertainty, operating range, dynamic bandwidth, reference standards, sampling settings, environmental controls, and acceptance checks. Pilot measurements then test whether signal level, noise, loading, and response time meet assumptions. During analysis, raw indications should remain preserved so that revised calibration or corrections can be applied transparently. This disciplined sequence reduces the temptation to adjust methods after seeing the desired outcome. It also makes the result reproducible: another engineer can reconstruct not only the calculation, but the physical route by which the number was obtained.