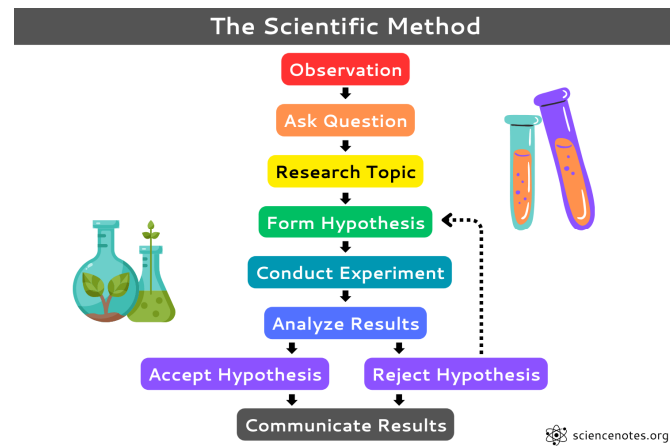
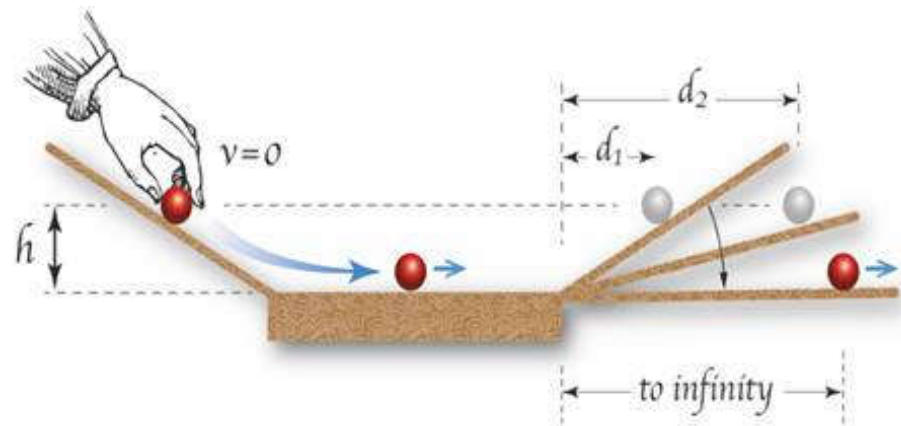


PHYSICAL QUANTITIES, MEASUREMENT AND TRACEABILITY

Lecture 1: From observation to comparable numerical results



The scientific method



Controlled experimental comparison

CENTRAL QUESTION

A result needs a defined measurand, a repeatable comparison, a numerical value, and a stated uncertainty.

ROUTE THROUGH THE LECTURE

Follow one chain: observation → operational rule → measurement model → value with unit and uncertainty → calibrated reference.

Why Measurement Needs a Method

A number becomes a result when the quantity, procedure and uncertainty are stated.

THE SCIENTIFIC METHOD IS A CYCLE



Observation, hypothesis, experiment and evaluation

SCIENTIFIC PURPOSE

For a rolling ball, a model predicts position at chosen times. Record the incline angle and release point, then compare measured positions with the prediction.

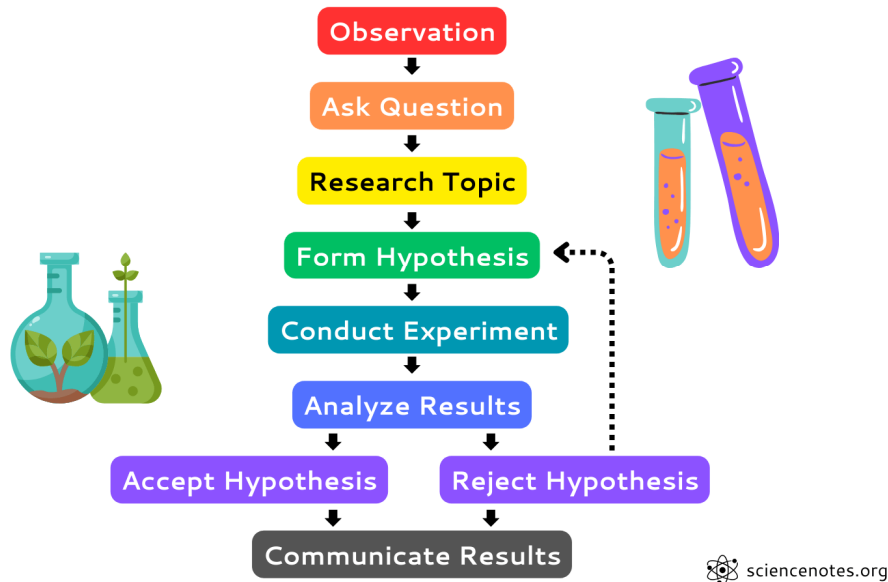
MEASUREMENT PURPOSE

“Length” is incomplete until the object, endpoints and conditions are fixed. A result such as (12.4 ± 0.1) cm states the scale and its uncertainty.

The Scientific Method

A testable explanation connects observations to predictions.

The Scientific Method



The original lecture's scientific-method infographic

MODEL AND PREDICTION

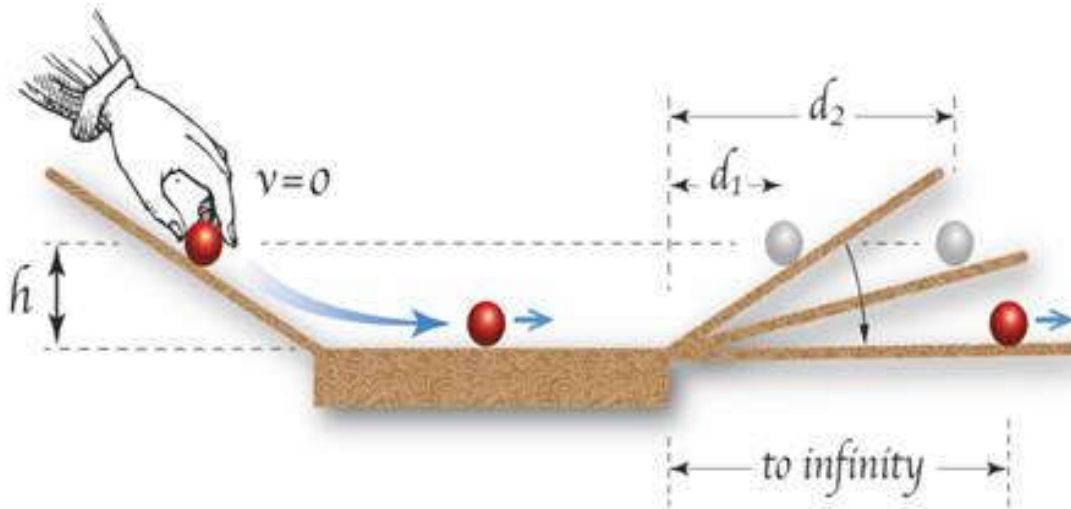
A model of constant acceleration predicts $s \propto t^2$ from rest. Choose times before seeing the data, then decide whether the observed trend agrees within uncertainty.

EXPERIMENT AND REVISION

Vary one input systematically; repeat the measurement; record scatter and calibration limits. A consistent discrepancy identifies missing physics or a faulty assumption.

Controlled Experimentation

Change a chosen condition while keeping competing influences under control.



Galileo's inclined-plane arrangement from the source lecture

EXPERIMENTAL VARIABLE

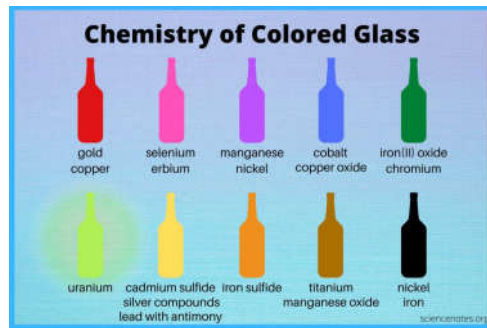
Change the incline angle while keeping the same ball and starting position. Record travel distance at fixed times to isolate the angle's effect.

WHY THE INCLINE HELPS

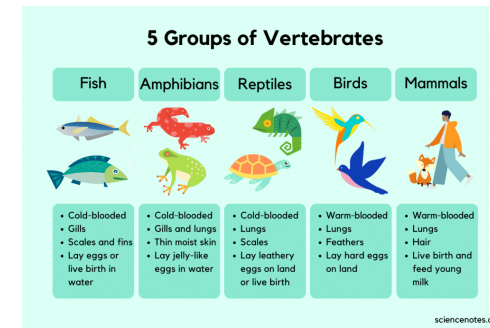
The incline makes the motion slow enough for finite timing resolution. Plot distance versus t^2 , test the trend, and discuss which assumptions permit extrapolation.

Observation Is Not Yet Measurement

Descriptions and classifications are useful, but they do not by themselves assign a reproducible value.



Morphological observation: visible colour



Classification: groups defined by observable criteria

OBSERVATION

Morphology records visible properties, such as glass colour and shape. Classification groups specimens by stated criteria, as in the vertebrate image.

QUANTIFICATION

Comparative methods order objects, such as softer versus harder. Quantitative measurement adds an operational scale; a spectrometer reports a calibrated spectrum with uncertainty.

Order Relations and Transitivity

A comparison rule must be operational, consistent and meaningful for the property.

Mohs Hardness Scale				
	Mineral	Scale Number	Common Object	
INCREASING HARDNESS ↑ HARDEST		Diamond	----- 10	 Masonry Drill Bit
		Sapphire	9.5 ----- 9	
		Topaz	8.5 ----- 8	
		Amethyst	7.5 ----- 7	
		Opal	6.5 ----- 6	 Steel Nail
		Turquoise	5.5 ----- 5	
		Pearl	4.5 ----- 4	 Knife/Glass Plate
		Coral	3.5 ----- 3	
		Amber	2.5 ----- 2	 Fingernail
		Talc	1.5 ----- 1	
	SOFTEST			

GABRIEL & CO.

Mohs hardness: an ordered, non-additive scale

TRANSITIVITY

Let $A \succcurlyeq B$ mean “A is at least as long as B.” Then $A \succcurlyeq B$ and $B \succcurlyeq C$ imply $A \succcurlyeq C$. Example: $12 \text{ cm} \geq 9 \text{ cm} \geq 6 \text{ cm}$.

ORDERING RULE

Define $A \succ B$ by a repeatable scratch test under stated conditions. Record $A \sim B$ when the test cannot resolve a difference.

INTERPRETATION

A consistent ranking preserves the order of comparisons. Mohs 10 (diamond), 9 (sapphire), 7 (amethyst) expresses order but neither equal steps nor ratios. For length, concatenation and a unit support an additive numerical scale.

Additivity and Operational Definitions

The physical rule for combining systems determines which calculations make sense.

ADDITIVE: LENGTH



3 cm

2 cm



$$L(A \oplus B) = 3 \text{ cm} + 2 \text{ cm} = 5 \text{ cm}$$

NON-ADDITIVE: TEMPERATURE



+



After thermal contact: 20 °C, not 40 °C

COMPOSITION RULE

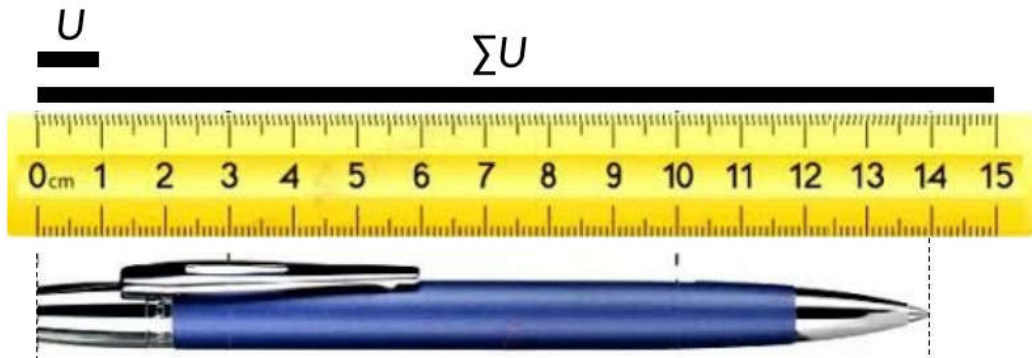
Length is extensive: placing segments end to end defines a reproducible sum. A unit then converts the combined length to a numerical value.

READING RULE

Temperature is intensive. State the object, equilibrium condition, sensor response and calibration scale before reporting T.

Direct Measurement by Comparison

Choose a unit U , compare the measurand G with repeated units, and read the scale.



Example: 14 ruler units $\approx$ 14 cm
Value = numerical reading $\times$ unit

The ruler and pencil example from the original lecture

NUMERICAL VALUE

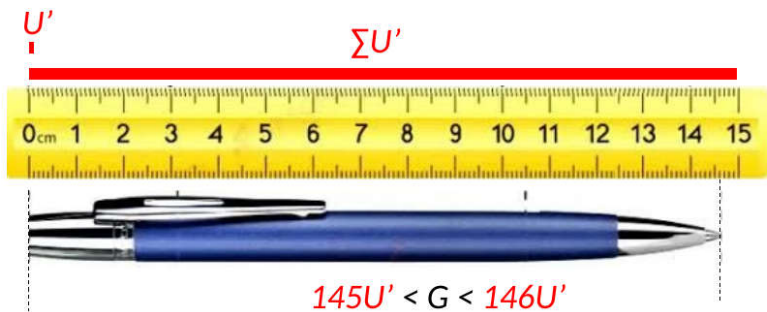
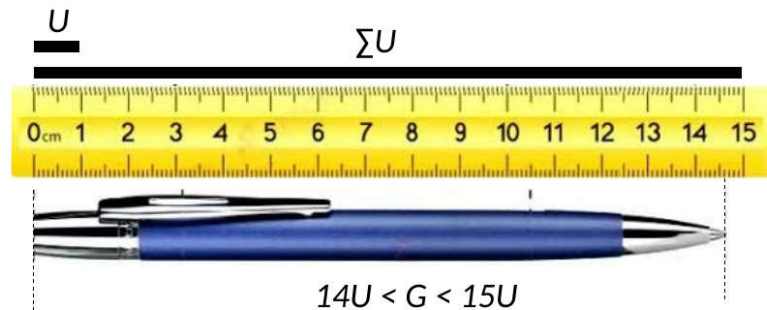
The measurand G equals XU , where U is a chosen unit and $X = G/U$ is dimensionless. The same length is 0.14 m or 14 cm.

COMPOSITION

The zero mark, alignment and endpoint definition are part of the protocol. A real endpoint between marks requires interpolation and a reading uncertainty.

Resolution of a Direct Reading

A scale interval bounds the location of an endpoint but does not determine it exactly.



Finite ruler divisions limit a direct reading

INTERVAL

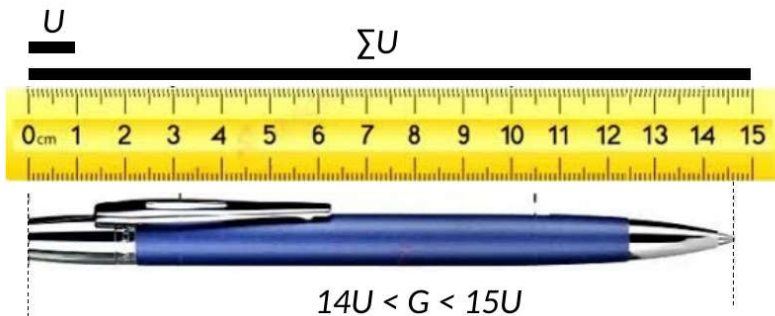
When the endpoint lies between nU and $(n+1)U$, the display alone brackets G within one division. A justified interpolation can narrow the reading.

UNCERTAINTY MODEL

Nearest-mark rounding gives $|e| \leq U/2$. For a uniform error distribution over that interval, $u = U/\sqrt{12}$; this omits calibration and alignment effects.

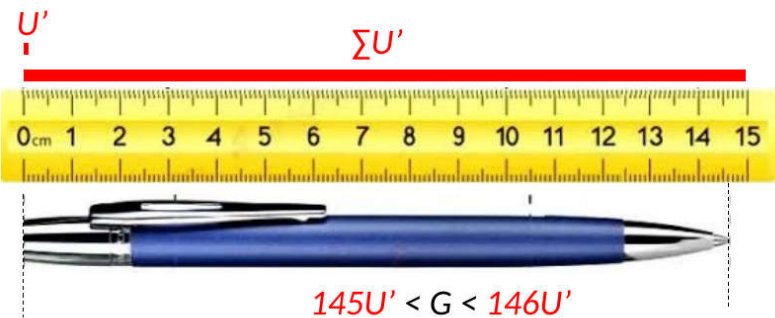
What an Uncertainty Statement Means

The reported bound and the standard uncertainty depend on the reading model.



ROUNDING BOUND

A ruler with 1 mm marks rounded to the nearest mark has a rounding bound of ± 0.5 mm. This bound is conditional on correct zero and alignment.



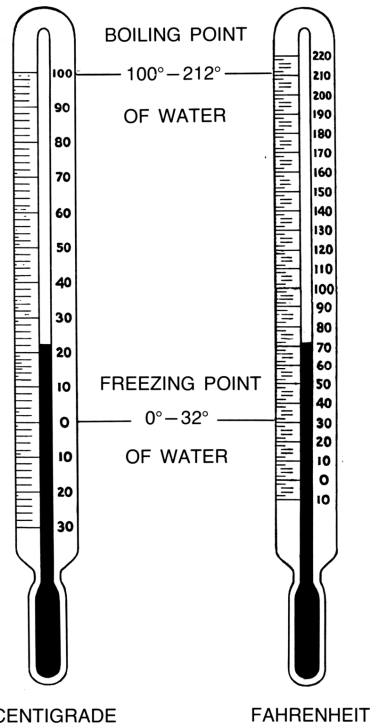
UNIFORM-ERROR MODEL

Uniform rounding error has standard deviation $u = U/\sqrt{12} = 0.29$ mm for $U = 1$ mm. State the distribution model; a $\pm$ bound is not itself a standard deviation.

A finite division places the endpoint within an interval

Indirect Measurement

Calculate a desired quantity from directly measured inputs.



Liquid-in-glass thermometer: a calibrated indirect reading

DERIVED VALUE

Measure a travel distance d between defined endpoints and elapsed time t between matching events. The mean speed $v = d/t$ is computed from those direct readings.

CALIBRATION MATTERS

In a liquid thermometer, the observed column position is mapped to T by fixed points or a calibration curve. Readability, thermal equilibrium and calibration all limit the inferred value.

Uncertainty Propagates Through a Model

Sensitivity coefficients convert input uncertainties into an uncertainty for the derived result.

DIRECT INPUTS

Distance d

$2.000 \pm 0.005 \text{ m}$

AND

Time t

$1.000 \pm 0.010 \text{ s}$

MODEL: $v = d/t$

SENSITIVITY COEFFICIENTS

For $v = d/t$: $\partial v / \partial d = 1/t$ and $\partial v / \partial t = -d/t^2$. Each derivative converts an input change into a speed change.

INDEPENDENT INPUTS

$$u^2(y) \approx \sum [(\partial f / \partial x_i) u(x_i)]^2.$$

For $v = d/t$:

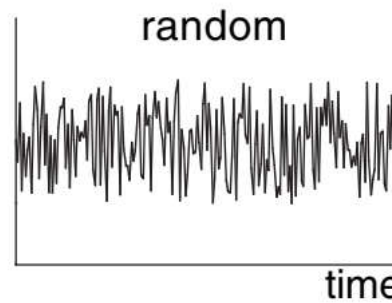
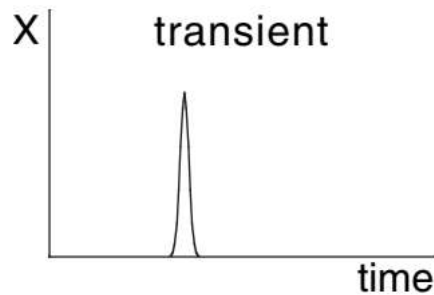
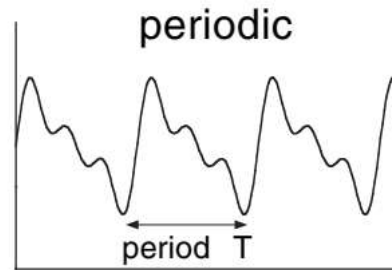
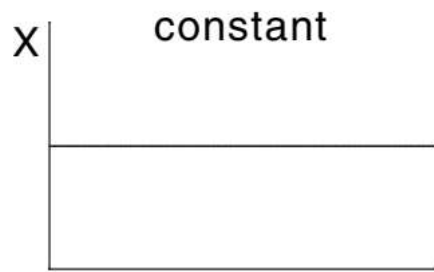
$$u(v)/v \approx \sqrt{[(u(d)/d)^2 + (u(t)/t)^2]}.$$

NUMERICAL RESULT AND LIMITS

Distance: 0.25%; time: 1.00%. Combined 1.031%, so $u(v) \approx 0.021 \text{ m s}^{-1}$ and $v = (2.00 \pm 0.02) \text{ m s}^{-1}$. Correlation adds covariance terms.

Time Dependence Changes the Measurement

Instrument response and sampling must match the dynamics of the quantity.



Four source patterns: constant, periodic, transient and random

TIME SCALES

For a periodic signal of period T , sample often enough to resolve its shape, and observe several cycles. A pulse of width Δt needs faster detector response.

DESIGN IMPLICATION

A slow meter reports a time average and can miss a ns laser pulse. Choose sampling rate, bandwidth and observation time from the signal dynamics.

Random Events: From Arrival Times to Counts

A defined observation window turns irregular arrivals into a count and an estimated rate.

EVENT TIMELINE • 0 TO 10 s



Each tick is one detection. The next arrival time is unknown.

RATE FROM ONE WINDOW

Count $N = 18$ events in $\Delta t = 10$ s.
Estimated rate $\hat{r} = N/\Delta t = 1.8 \text{ s}^{-1}$.
State detector threshold and dead time.

REPEAT EQUAL WINDOWS



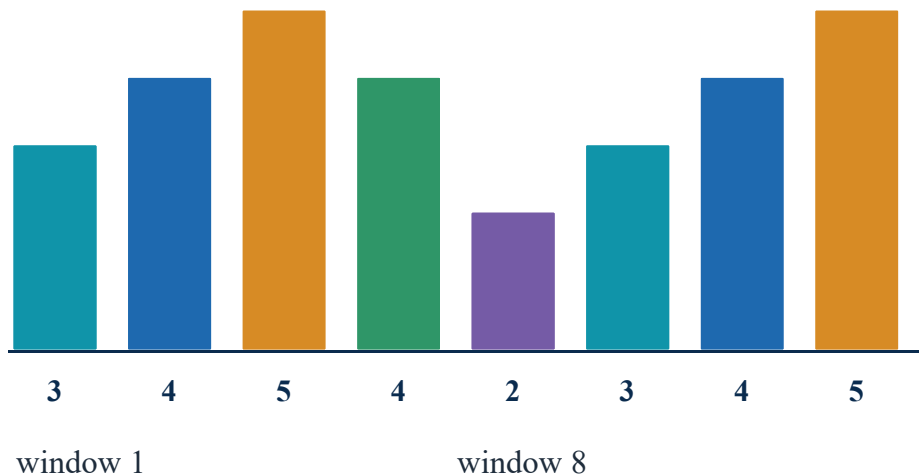
MODEL CHECK

If arrivals are independent at a stable rate, counts in equal windows follow a Poisson model.

Counting Statistics and Repeatability

Repeated windows reveal scatter and help test a counting model.

COUNTS IN EIGHT EQUAL WINDOWS



MEAN AND SPREAD

Counts: 3, 4, 5, 4, 2, 3, 4, 5. Mean = 3.75.

Sample standard deviation $s \approx 1.04$ counts per window.

UNCERTAINTY OF THE MEAN

For independent windows, $u(\text{mean}) \approx s/\sqrt{8} \approx 0.37$ counts per window. This describes precision of the mean, while the individual counts fluctuate more.

Seven SI Base Quantities and Units

Each row connects a physical quantity, its unit name and its symbol.

QUANTITY	UNIT	SYMBOL
TIME	second	s
LENGTH	metre	m
MASS	kilogram	kg
ELECTRIC CURRENT	ampere	A
TEMPERATURE	kelvin	K
AMOUNT OF SUBSTANCE	mole	mol
LUMINOUS INTENSITY	candela	cd

WHAT BASE MEANS

The seven base quantities have independent dimensions by convention. This is a choice of framework, not a claim that only seven properties exist.

DERIVED QUANTITIES

Physical equations combine base quantities: $[v] = L T^{-1}$ and $[F] = M L T^{-2}$. Their units therefore follow as $m s^{-1}$ and $kg m s^{-2}$.

From Constants to Second, Metre and Kilogram

The definitions form a dependency chain; laboratory realizations make them usable.

SECOND s

$$\Delta\nu_{\text{Cs}} = 9\,192\,631\,770\,\text{s}^{-1}$$

The caesium-133 transition fixes the duration of a second.

METRE m

$$c = 299\,792\,458\,\text{m s}^{-1}$$

With the second defined, light travel in vacuum fixes length.

KILOGRAM kg

$$h = 6.626\,070\,15 \times 10^{-34}\,\text{kg m}^2\,\text{s}^{-1}$$

With s and m defined, fixed h defines mass. A Kibble balance can realize it.

Four More Defining Constants

The remaining constants connect electrical, thermal, chemical and photometric quantities to SI.

AMPERE A

$$e = 1.602\,176\,634 \times 10^{-19} \text{ C}$$

C = A s: fixed elementary charge and the second determine current.

KELVIN K

$$k = 1.380\,649 \times 10^{-23} \text{ J K}^{-1}$$

The Boltzmann constant links temperature to energy measured in joules.

MOLE mol

$$N_a = 6.022\,140\,76 \times 10^{23} \text{ mol}^{-1}$$

One mole contains this exact number of specified entities.

CANDELA cd

$$K_{cd} = 683 \text{ lm W}^{-1} \text{ at } 540 \text{ THz}$$

Luminous efficacy links photometric output to radiant power.

Defining values are exact. Every practical realization still has measurement uncertainty.

How Derived Mechanical Units Are Obtained

Start from the physical equation and substitute coherent base units.

SPEED

$$v = \Delta s / \Delta t$$

$$\text{m/s} = \text{m s}^{-1}$$

ACCELERATION

$$a = \Delta v / \Delta t$$

$$\text{m s}^{-2}$$

FORCE

$$F = ma$$

$$\text{N} = \text{kg m s}^{-2}$$

ENERGY

$$E = Fd$$

$$\text{J} = \text{N m} = \text{kg m}^2 \text{s}^{-2}$$

PRESSURE

$$p = F/A$$

$$\text{Pa} = \text{N/m}^2 = \text{kg m}^{-1} \text{s}^{-2}$$

Check: the two sides of a physical equation must have the same dimensions.

Coherent Derived Units and SI Prefixes

Equations set the unit algebra; prefixes change only the size of a unit.

CHARGE FROM CURRENT

$Q = It$. In SI, $1 \text{ C} = 1 \text{ A s}$. For $I = 10 \text{ mA}$ flowing for 2 s , $Q = 20 \text{ mC}$. The unit follows directly from current and time.

POTENTIAL DIFFERENCE

$V = E/Q$. Thus $1 \text{ V} = 1 \text{ J/C} = 1 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-1}$.
Moving 1 C through 1 V transfers 1 J of energy.

RESISTANCE

$R = V/I$. Hence $1 \Omega = 1 \text{ V/A} = 1 \text{ kg m}^2 \text{ s}^{-3} \text{ A}^{-2}$.
Example: 2 V across a component at 0.5 A corresponds to 4Ω .

COHERENCE AND DECIMAL SCALES

$F = ma$ gives $1 \text{ N} = 1 \text{ kg m s}^{-2}$ with no extra factor.
Prefixes rescale units: $1 \text{ km} = 10^3 \text{ m}$ and $1 \text{ ms} = 10^{-3} \text{ s}$. Therefore $1 \text{ mm/ms} = 1 \text{ m/s}$.

Standards Evolved from Artifacts to Constants

The reference can be universally defined while instruments still require calibration.



From trade and artifact standards to laboratory realizations

THEN

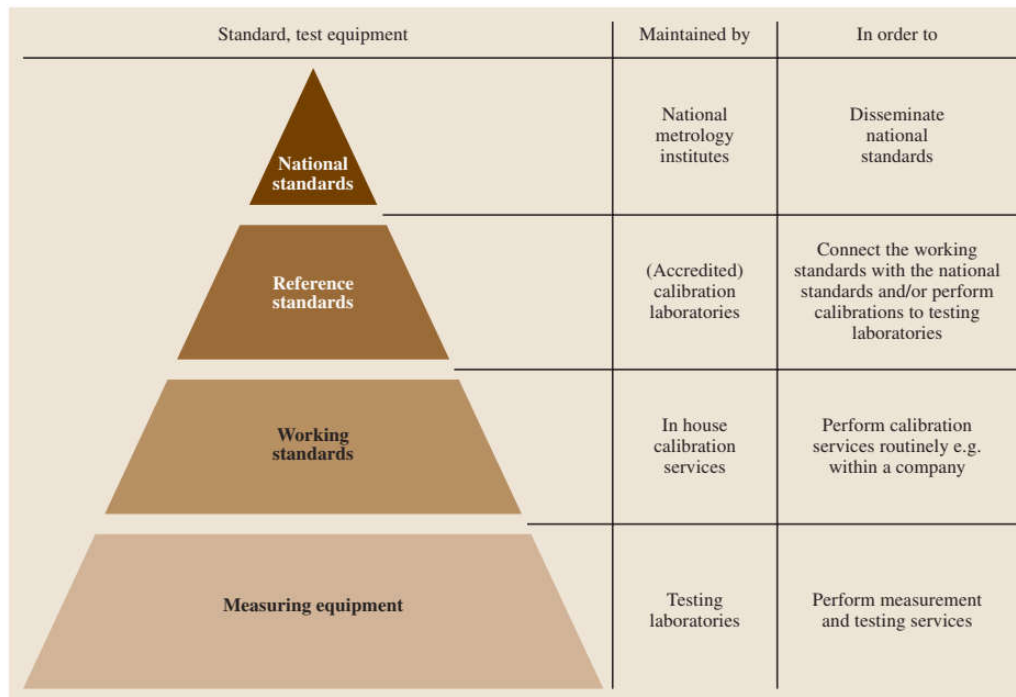
Trade required common references. Artifact yards and kilogram prototypes could be copied, but drift and damage limited long-term agreement.

NOW

Fixed defining constants do not drift. Experiments that realize units, and subsequent comparisons between standards, still have measurable uncertainty.

Metrological Traceability

A documented chain of calibrations connects a result to a recognized reference.



Calibration hierarchy from the source lecture

CHAIN

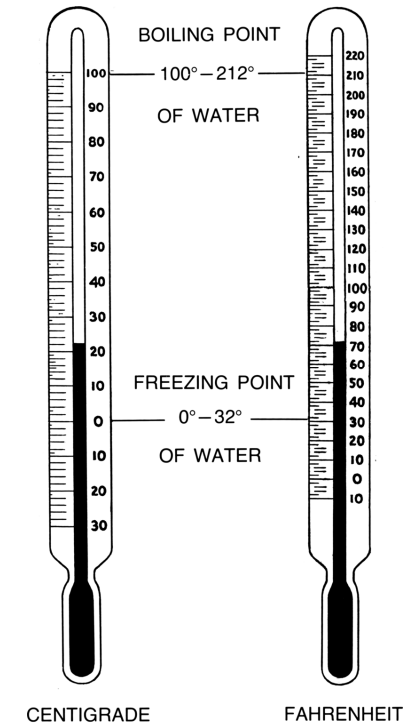
Follow an unbroken documented sequence from instrument to working standard, reference standard and SI realization. Every comparison contributes uncertainty.

COMPARABILITY

Traceability makes the reference path auditable. Check whether the combined uncertainty is small enough for the decision and whether calibration covers the measured range.

Calibration Converts a Sensor Reading

A calibrated relationship maps an accessible response to the desired physical quantity.



Read the thermometer column length L

TWO REFERENCE POINTS

Ice point

$$L_0 = 20 \text{ mm} \leftrightarrow T_0 = 0 \text{ }^{\circ}\text{C}$$

100 °C bath

$$L_1 = 120 \text{ mm} \leftrightarrow T_1 = 100 \text{ }^{\circ}\text{C}$$

Idealized linear response; reference baths require stated conditions.

TRANSFER FUNCTION

If the response is linear over this interval, $T(L) = T_0 + (T_1 - T_0)(L - L_0)/(L_1 - L_0)$.

CONDITIONS AND UNCERTAINTY

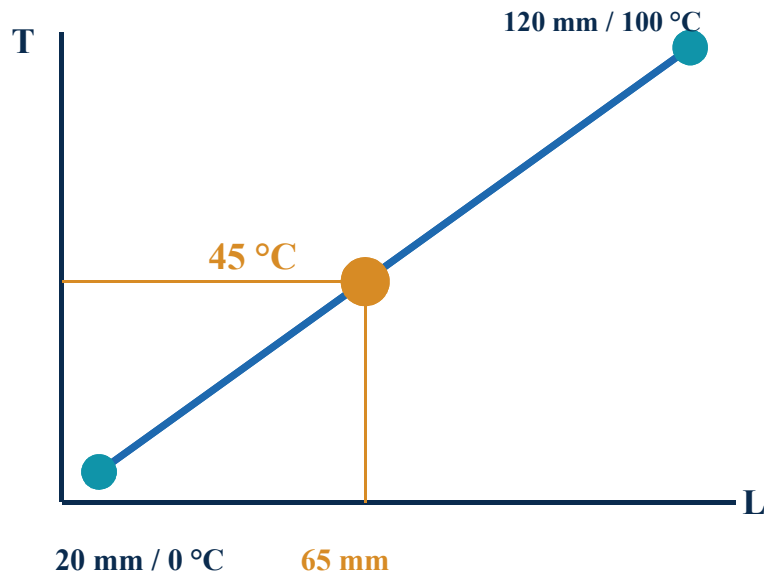
Check linearity and thermal equilibrium. Uncertainty in L and in both calibration points propagates to T ; validity is limited to the calibrated range.

Worked Calibration: Column Length to Temperature

The measured column length becomes temperature only through the calibrated map.

CALIBRATION LINE $T(L)$

Read $L = (65.0 \pm 0.5) \text{ mm}$



CALCULATE THE DESIRED QUANTITY

$T = 0^\circ\text{C} + (100^\circ\text{C})(65-20)/(120-20) = 45.0^\circ\text{C}$. One millimetre of column length corresponds to 1°C here.

UNCERTAINTY AND VALIDITY

For fixed exact calibration points, $u(T) = |dT/dL|u(L) = (1^\circ\text{C}/\text{mm})(0.5 \text{ mm}) = 0.5^\circ\text{C}$. Report $T = (45.0 \pm 0.5)^\circ\text{C}$ from reading uncertainty alone. Real calibration adds terms.

The Complete Measurement Chain

A useful result preserves the path from physical property to a comparable number.

