Hyperovals on Hermitian generalized quadrangles

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Motivation

Definition

Let $\mathcal{S}$ be a $GQ(s,t)$. A *hyperoval* of $\mathcal{S}$ is a non-empty set of points of $\mathcal{S}$ which intersects every line of $\mathcal{S}$ in either 0 or 2 points.

$G = (\mathcal{P}, \mathcal{B})$ connected incidence system. Let $a \in \mathcal{P}$. The residue of G at a is a geometry $G_a = (\mathcal{P}_a, \mathcal{B}_a)$,

- $\mathcal{P}_a$ points of $\mathcal{P}$ collinear with a,
- $\mathcal{B}_a := \{B \setminus \{a\} : a \in B \in \mathcal{B}\}$.

G is called an *extended* $GQ(s,t)$ (or $EGQ(s,t)$) if and only if each residue is a $GQ(s,t)$

An $EGQ(s,t)$ is called *triangular* if and only if for each antiflag (a, B) there are exactly two points on B collinear with a.

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Let G be a triangular $EGQ(s,t)$, Γ the point-graph of G, Γ_a the subgraph of Γ induced by G_a.
If the distance between $a, b \in \Gamma$ is 2, then $\Gamma_a \cap \Gamma_b$ is the subgraph of a point $GQ(s,t)$-graph induced by a hyperoval of $GQ(s,t)$.

This result makes interesting the classification of all hyperovals of generalized quadrangles.

- Cameron, Hughes, Pasini, Extended generalized quadrangles, *Geom. Ded.* 35 (1990), 193-228.
- Buekenhout, Hubaut, Locally polar spaces and related rank 3 groups, *J.Algebra* 45 (1977), 391-434.
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Known results

Let $\mathcal{S}$ be a $GQ(s,t)$ and let $\mathcal{H}$ be a hyperoval of $\mathcal{S}$. Then

i) 2 is a divisor of $|\mathcal{H}|$;

ii) $|\mathcal{H}| \geq 2(t+1)$ and equality holds if and only if there exists a regular pair $\{x,y\}$ of non-collinear points of $\mathcal{S}$ such that $\mathcal{H} = \{x,y\}^{\perp} \cup \{x,y\}^{\perp\perp}$;

iii) $|\mathcal{H}| \geq (t-s+2)(s+1)$ and if equality holds then every point outside $\mathcal{H}$ is incident with precisely $1+(t-s)/2$ lines which meet $\mathcal{H}$ (hence $s \equiv t \pmod 2$);

iv) $|\mathcal{H}| \leq 2(st+1)$ and equality holds if and only if every line of $\mathcal{S}$ intersects $\mathcal{H}$ in exactly 2 points.

Makhnev, Extensions of $GQ(4,2)$, the description of hyperovals, *Discrete Math. Appl.* 7 (1997), 419-435.

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Known results on $\mathcal{H}(3, q^2)$

The first two known infinite families on $\mathcal{H}(3, q^2)$:

- $|\mathcal{H}| = 2q^3$: union of two hermitian curves with a common tangent deleted,
- $|\mathcal{H}| = 2(q^3 - q)$: union of two hermitian curves with a common chord deleted.

Other infinite families:

- Cossidente, Hyperovals on $\mathcal{H}(3, q^2)$, *J. Combin. Theory A* 118 (2011), 1190-1195.
- Cossidente, Marino, Singer action on $\mathcal{H}(3, q^2)$, q even, (preprint).
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The intersection of $\mathcal{H}(3, q^2)$ and $Q^-(3, q^2)$, q even

tangent lines to $Q^-(3, q^2) \to$ lines of a symplectic generalized quadrangle $\mathcal{W}(3, q^2)$

Plücker map

lines of $PG(3, q^2) \to$ points on $Q^+(5, q^2)$

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- Cossidente, King, Marino, Hyperovals of $\mathcal{H}(3, q^2)$, when q is even, *J. Combin. Theory A* 120 (2013), 1131-1140.
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lines of $PG(3, q^2)$ $\rightarrow$ points on $\mathcal{Q}^+(5, q^2)$

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$\mathcal{W}(3, q^2) \to \mathcal{Q}(4, q^2)$ in a hyperplane $\mathcal{I}$ of $PG(5, q^2)$

$q^4 + 1$ pencils of tangent lines to $\mathcal{Q}^-(3, q^2) \to$ spread of lines of $\mathcal{Q}(4, q^2)$.

$\mathcal{I} \cap \Sigma$ is a $PG(4, q) \to \mathcal{Q}(4, q^2) \cap \mathcal{Q}^-(5, q) = \mathcal{Q}(4, q)$

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Lemma

Let $\mathcal{H}(3, q^2)$ be a Hermitian surface and $\mathcal{Q}^-(3, q^2)$ an elliptic quadric in $PG(3, q^2)$, q even. Then, the generators of $\mathcal{H}(3, q^2)$ that are tangents to $\mathcal{Q}^-(3, q^2)$ are extended lines of a $\mathcal{W}(3, q)$, an elliptic congruence, a hyperbolic congruence or a parabolic congruence of a $PG(3, q)$.

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Generators of $\mathcal{H}(3,q^2)$ tangents to $\mathcal{Q}^-(3,q^2)$ are extended lines of a $\mathcal{W}(3,q)$

$PG(3,q^2)$,

$$\mathcal{H}(3,q^2):\ X_1X_4^q + X_4X_1^q + X_2X_3^q + X_3X_2^q = 0.$$

$\mathrm{PGU}(4,q^2)$ acts transitively on the symplectic subquadrangles embedded in $\mathcal{H}(3,q^2)$,

$\mathcal{W}(3,q)$:

$$H((X_1,X_2,X_3,X_4),(Y_1,Y_2,Y_3,Y_4)) = X_1Y_4+X_4Y_1+X_2Y_3+X_3Y_2,$$

where $X_i, Y_i \in GF(q)$.

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Th.: $\mathcal{Q}^-(3, q^2)$ intersects $\mathcal{W}(3, q)$ in a Baer conic

$\mathrm{PSp}(4, q^2)$ acts transitively on the set of $q^4(q^4 - 1)/2$ elliptic quadrics of $PG(3, q^2)$ whose tangent lines are the lines of $\mathcal{W}(3, q^2)$,

$\mathcal{P} \rightarrow$ pencil of $q^2/2$ elliptic quadrics of $PG(3, q^2)$, passing through the conic

$$\bar{\mathcal{C}} : X_1^2 + X_2X_3 = 0.$$

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for some $w \in GF(q^2)$ such that w has $GF(2)$–trace 1.

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$G \rightarrow$ the $q^2/2$ elliptic quadrics in $\mathcal{P}$ are partitioned into q orbits, say $\bar{\mathcal{O}}_1, \ldots, \bar{\mathcal{O}}_q$, where $|\bar{\mathcal{O}}_i| = q/2$, for $1 \leq i \leq q$.

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$\mathrm{PSp}(4, q) \rightarrow q$ distinct orbits $\mathcal{O}_1, \ldots, \mathcal{O}_q$, where
$\mathcal{O}_i = Orb_{\mathrm{PSp}(4,q)}(\mathcal{E}_i)$, $|\mathcal{O}_i| = q^3(q^4 - 1)/2$, for $1 \leq i \leq q$.

On the intersection of $\mathcal{H}(3, q^2)$ and $Q^-(3, q^2)$, q even

On the other hand $q^4(q^4 - 1)/2$ is the number of all elliptic quadrics of $PG(3, q^2)$ whose tangent lines are the lines of $\mathcal{W}(3, q^2)$ $\rightarrow$ each of those elliptic quadrics intersects $\mathcal{W}(3, q)$ in a Baer conic.

g a generator of $\mathcal{H}(3, q^2)$ $\rightarrow$ $g \cap \mathcal{W}(3, q)$ is a Baer line of $\mathcal{W}(3, q)$ or $g \cap \mathcal{W}(3, q)$ is empty,

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Each extended Baer line of $\mathcal{W}(3, q)$ is tangent to $\mathcal{Q}^-(3, q^2)$.

There are $q + 1$ extended lines of $\mathcal{W}(3, q)$ through each point of $\mathcal{C}$, the remaining $q(q^2 - 1)$ extended lines of $\mathcal{W}(3, q)$ meet $\mathcal{Q}^-(3, q^2)$ in a point on $\mathcal{W}(3, q^2) \setminus \mathcal{W}(3, q)$,

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$$|\mathcal{H}(3, q^2) \cap \mathcal{Q}^-(3, q^2)| = (q^3 - q) + (q + 1) = q^3 + 1.$$

Theorem

In $PG(3, q^2)$, q even, let $\mathcal{H}(3, q^2)$ be a Hermitian surface and $\mathcal{Q}^-(3, q^2)$ be an elliptic quadric such that the generators of $\mathcal{H}(3, q^2)$ that are tangent with respect to $\mathcal{Q}^-(3, q^2)$ are extended lines of a $\mathcal{W}(3, q)$, then

i) if $\mathcal{S}$ is a symplectic subquadrangle embedded in $\mathcal{H}(3, q^2)$, then $\mathcal{S} \cap \mathcal{Q}^-(3, q^2)$ is a Baer conic,

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$\mathrm{PSp}(4, q^2)$ acts transitively on the symplectic subquadrangles embedded in $\mathcal{W}(3, q^2)$,

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Bamberg, Law, Penttila, Tight sets and m-ovoids of finite generalized quadrangles, *Combinatorica* 29 (2009), 1-17.

m-ovoids of a $GQ(s, t)$

Let $\mathcal{S}$ be a $GQ(s, t)$. An m-ovoid $\mathcal{O}$ of $\mathcal{S}$, is a set of points of $\mathcal{S}$ such that

- P is collinear with $(t+1)(m-1)+1$ points of $\mathcal{O}$, if $P \in \mathcal{O}$,
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Hyperovals on $\mathcal{H}(3, q^2)$ from elliptic quadrics

Tight sets of a $GQ(s, t)$

Let $\mathcal{S}$ be a $GQ(s, t)$. A tight set $\mathcal{T}$ of $\mathcal{S}$, is a set of points of $\mathcal{S}$ such that

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mi-Lemma

Let $\mathcal{S}$ be a $GQ(s, t)$, $\mathcal{O}$ an m-ovoid of $\mathcal{S}$ and $\mathcal{T}$ an i-tight set of $\mathcal{S}$, then $|\mathcal{O} \cap \mathcal{T}| = mi$.

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$$\mathcal{H} = ((\mathcal{H}(3, q^2) \cap \mathcal{Q}^-(3, q^2)) \cup \mathcal{E}') \setminus \mathcal{C}.$$

$$\mathcal{H} \subseteq \mathcal{H}(3, q^2), \ |\mathcal{H}| = q^3 + q^2 - 2q.$$

g a generator of $\mathcal{H}(3, q^2)$

g is disjoint from $\mathcal{W}(3, q) \to g$ meets $\mathcal{Q}^-(3, q^2)$ in either 0 or 2 points,

g meets $\mathcal{W}(3, q)$ in a Baer line of $\mathcal{W}(3, q) \to g$ intersects $\mathcal{H}$ in 0 or 2 points according as g meets $\mathcal{C}$ in one point or none.

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On $\mathcal{H}(3, q^2)$, q even, there exists a hyperoval of size $q^3 + q^2 - 2q$.

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g meets $\mathcal{W}(3, q)$ in a Baer line of $\mathcal{W}(3, q) \to g$ intersects $\mathcal{H}$ in 0 or 2 points according as g meets $\mathcal{C}$ in one point or none.

Corollary

On $\mathcal{H}(3, q^2)$, q even, there exists a hyperoval of size $q^3 + q^2 - 2q$.

Let $\mathcal{S}$ be a $GQ(s, t)$, $\mathcal{H}$ a hyperoval of $\mathcal{S}$

$$|\mathcal{H}| \leq 2(st + 1),$$

$$|\mathcal{H}| \geq \begin{cases} 2(t+1) & \text{if} \quad s \geq t \\ (t-s+2)(s+1) & \text{if} \quad s \leq t \end{cases}$$

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The first infinite family on $\mathcal{H}(4, q^2)$:

- Cossidente, Marino, Hyperovals of Hermitian polar space, *Des. Codes Cryptogr.* 64 (2012), 309-314.

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$\mathcal{U} := \mathcal{H}(4, q^2) \cap \pi$ Hermitian curve,
$l := \pi^\perp$ secant line.

$$G := Stab_{\mathrm{PGU}(5,q^2)}(\pi) \simeq (\mathrm{GU}(2, q^2) \times \mathrm{GU}(3, q^2))/C_{q+1}$$

Orbits of G on $\mathcal{H}(4, q^2)$

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Hyperovals on $\mathcal{H}(4, q^2)$

$$1 \times \mathcal{S} \leq G$$

$1 \times \mathcal{S}$ cyclic group of order $q^2 - q + 1$

Baker, Ebert, Korchmàros, Szönyi, Orthogonally divergent spreads of Hermitian curve, *LMS. Lecture Note Ser.* 191 (1993), 17-30.

There exists a unique cyclic spread of $\mathcal{U}$, invariant under $1 \times \mathcal{S}$, i.e. a family of $q^2 - q + 1$ secant, no two intersecting in a point of $\mathcal{U}$, no three in a pencil.

$$H = (\mathrm{PGU}(2, q^2) \times \mathcal{S})/C_{q+1}, \quad H \leq G,$$

$\mathcal{O}_3$ splits into q^2 H-orbits, one of which, $\mathcal{O}'$, of size $(q^2 - q + 1)(q^3 - q)$ relative to the cyclic spread,

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$\mathcal{O}'$ is the union of $q^2 - q + 1$ Hermitian curve sharing the points $l \cap \mathcal{H}(4, q^2)$.

$$\mathcal{H} := \mathcal{U} \cup (l \cap \mathcal{H}(4, q^2)) \cup \mathcal{O}'$$

$\mathcal{H}$ is a hyperoval of $\mathcal{H}(4, q^2)$,
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$q^5 - q^4 + q^3 + q^2 + 2 = (q^3 - q^2 + 2)(q^2 + 1)$

Corollary

$\mathcal{H}$ is a $(q^3 - q^2 + 2)$-tight set of $\mathcal{H}(4, q^2)$.

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$\mathcal{B}_i := \{P \in \pi : (\pi_P \cap \mathcal{H}(4, q^2))$ is a non-degenerate Hermitian curve, where $\pi_P :=< P, l_i >\}$,

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For $i = 2$, there exists a hyperoval of the plane π embedded in $\mathcal{B}_2$, say $\mathcal{I}_2$

Donati, Durante, Korchmáros, On the intersection pattern of a unital and an oval in $PG(2, q^2)$, *Finite Fields Appl.* 15 (2009), 785-795.

There exists a non-degenerate conic of $PG(2, q^2)$ disjoint from a non-degenerate Hermitian curve.

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$$\mathcal{H}_i := \bigcup_{P \in \mathcal{I}_i} (\mathcal{H}(4, q^2) \cap \pi_P) \setminus (l_i \cap \mathcal{H}(4, q^2)),$$

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In both cases $\mathcal{H}_i$ is the union of $q^2 + 2$ non-degenerate Hermitian curve minus the points $l_i \cap \mathcal{H}(4, q^2)$.

g a generator of $\mathcal{H}(4, q^2)$,

if $g \cap l_i$ is a point $\rightarrow \pi' :=< l_i, g >$, $\pi' \cap \mathcal{H}(4, q^2)$ is degenerate Hermitian curve,

$\pi' \cap \pi$ is a point $Q \rightarrow Q \notin \mathcal{B}_i$,

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De Clerck, De Feyter, Durante, Two intersection sets with respect to lines on the Klein quadric, *Bull. Belg. Math. Soc. Simon Stevin* 12 (2005), 743-750.

Proposition

On $\mathcal{H}(4, q^2)$, q even, there exist two hyperovals of size $(q^3 - q)(q^2 + 2)$ and $q^3(q^2 + 2)$, respectively.

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THANKS