Modified gravity theories and dark matter models tested by galactic rotation curves

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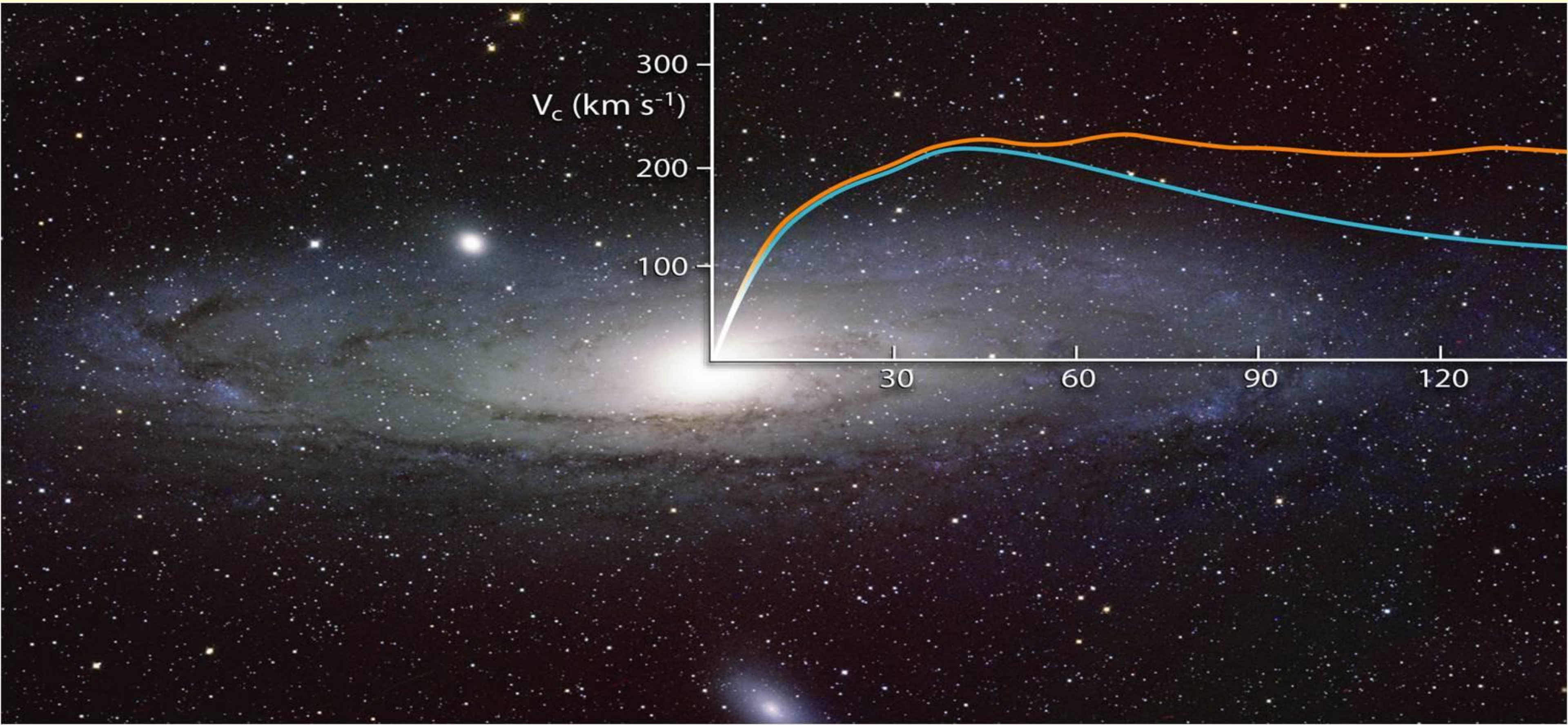
In a spiral galaxy, the radial profile of the gravitating matter M(r) and that of the sum of all luminous components $M_L(r)$ do not match.

The phenomenon of the mass discrepancy in galaxies emerges from the *radial derivative* of the mass distribution, more precisely from that of the circular velocity

A massive dark component (i.e dark matter) or equivalent modfications of gravity is introduced to account for the disagreement.



I. Brane world model

We solved the field equations for static, spherically symmetric vacuum branes, and obtain the velocity of the test particles in stable circular orbits around the galactic center.

The effective 4d gravitational equation on the brane takes the form (Sasaki et al. 2000):

$$G_{ab} = k_4^2 T_{ab} + k_5^4 S_{ab} - E_{ab}$$

$$\left[\begin{array}{l} k_4^2 = 8\pi G \\ k_5^2 = 8\pi G_5 \end{array} \right]$$

S_{ab} is the local quadratic energy-momentum correction and

E_{ab} is the non-local effect from the free bulk gravitational field.

- E_{ab} can be decomposed irreducibly with respect to a chosen 4-velocity field u^a as
- (Dadhich et al. 2000):

$$E_{ab} = -k_4^4 \left[U u_a u_b + \frac{U-P}{3} h_{ab} + P r_a r_b \right]$$

U: „dark radiation”
P: „dark pressure”
r_a is a unit radial vector

where $h_{ab} = g_{ab} + u_a u_b$ is the induced metric projects orthogonal to u^a.

- In the following we neglect the effect of the cosmological constant
- assume vacuum state (p=ρ=0, $T_{\mu\nu} \equiv 0$ and consequelntly $S_{ab} = 0$)

With these assumptions the field equation takes a much simpler form:

$$R_{ab} = -E_{ab}$$

the
Ricci
tensor

The motion of particles in stable circular orbits on the brane

- In brane world models test particles are confined to the brane.
- However, the bulk has an effect on the motion of the test particles on the brane via the metric.
- The projected Weyl tensor effectively acts as an additional matter source.
- we will restrict our study to the static and spherically symmetric metric given by:

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2 d\Omega^2$$

The Lagrangian $\mathcal{L}$ for a massive test particle traveling on the brane reads

$$\mathcal{L} = \frac{1}{2}\left(-e^{\nu}\dot{t}^2 + e^{\lambda}\dot{r}^2 + r^2\dot{\Omega}^2\right)$$

the dot means differentiation with respect to the affine parameter

conserved quantities:

$$r^2\dot{\Omega} = L$$

related to the particle's angular momentum

$$-e^{\nu(r)}\dot{t} = E$$

energy

- We define the tangential velocity of a test particle on the brane as (Landau & Lifshitz 1975):

$$v_{tg}^2 = e^{-\nu} r^2 \dot{\Omega}^2 \dot{t}^{-2} = e^{-\nu} \frac{L^2}{r^2} \dot{t}^{-2}$$

- after a short calculation we obtain the simpler expression for the tangential velocity of a test particle in a stable circular orbit on the brane as (Matos et al. 2000; Nucamendi et al. 2001):

$$v_{tg}^2 = \frac{r\nu'}{2}$$

where $' = d/dr$

- we assume a simple equation of state relating the „dark radiation” and „dark pressure”

$$P = (a-2)\,U - \frac{B}{k_4^4 r^2}$$

where a and B are constants

After a long but straightforward calculation (for details, see Gergely et al. 2011) the rotational velocity can be written as:

These are free parameters of the model

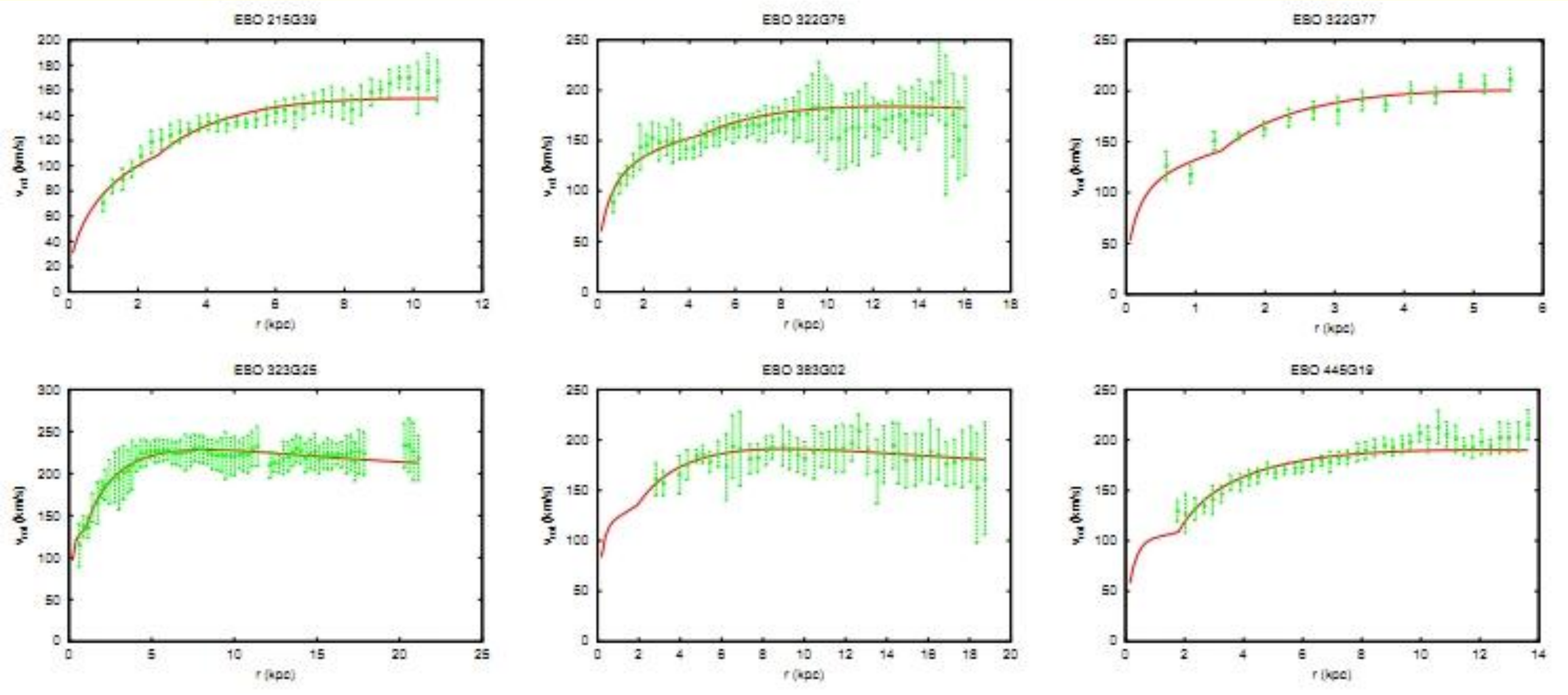
$$\left(\frac{v_{tg}(r)}{c}\right)^2 \approx \underbrace{\frac{G\left(M_b^{tot}\right)}{c^2 r}}_{\text{Baryonic contribution}} + \underbrace{\frac{G\left(M_0^{tot}\right)}{c^2 r} + \beta + C\left(\frac{r_b}{r}\right)^{1-\alpha}}_{\text{these terms come from the projection of the bulk Weyl tensor}} \quad , \; r > r_b$$

This solution is valid for any $r > r_b$ where r_b represents the radius beyond which the baryonic matter can be treated as a perturbation.

r_b has a physical meaning: it is the radius of the bulge, the central baryonic component of the spiral galaxy.

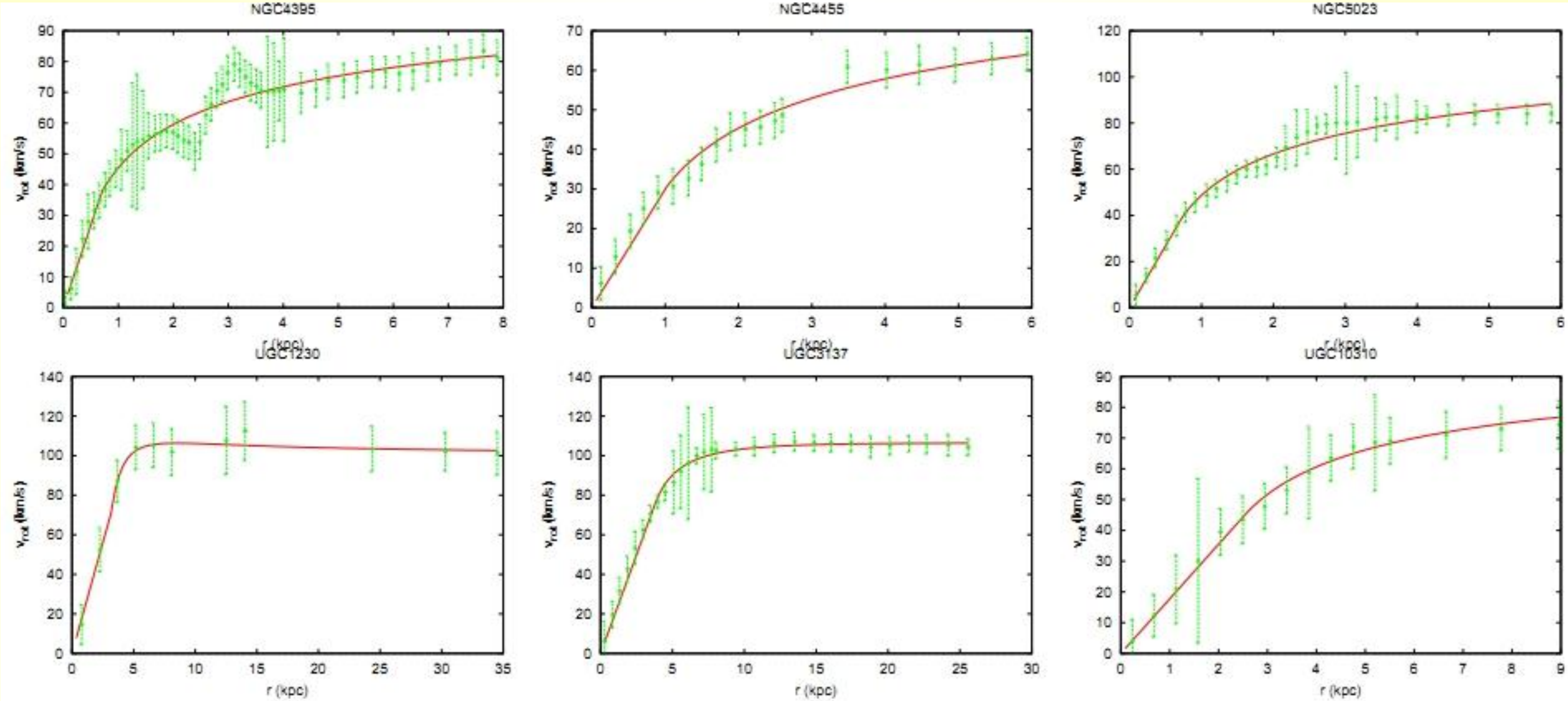
Confronting the Weyl fluid model with observational data

- The model has several free parameters.
- Fixed them in such a way to explain the observed galactic rotation curve behavior.
- Fitting the model to rotation curve data (with chi-square minimization method) allowed us to constrain the Weyl parameters and also determine the baryonic components.



rotation curves of HSB galaxies

rotation curves of LSB galaxies



Conclusions of the fitting:

- The fit was in all cases within 1σ confidence level
- With the parameters determined from the fit the theoretical rotation curves will have an almost flat asymptotic behavior at larger radii, which is consistent with the obsevable curves.

II. Bose-Einstein Condensate (BEC) model

The ΛCDM model successfully describes among others the:

- the accelerated expansion of the Universe
- the observed temperature fluctuations in the cosmic microwave background radiation
- the large scale matter distribution

Despite these important achievements, on galactic scales the ΛCDM model meets with severe difficulties in explaining the observed distribution of the invisible matter

- N-body simulations, performed in this scenario, predict a very characteristic density profiles that feature a well pronounced central cusp (Navarro et al. 1996):

$$\rho_{NFW}(r) = \rho_s/(r/r_s)(1 + r/r_s)^2$$

characteristic density

scale radius

- On the observational side, rotation curves show a nearly constant density core

Cold dark matter in a form of a Bose-Einstein condensate fixes the above short-comings.

We performed a complete analysis of a selected sample of dwarf, HSB and LSB galaxies.

- At very low temperatures, all particles in a dilute Bose gas condense to the same quantum ground state, forming a Bose-Einstein condensate.
- Condensation process was first observed experimentally in 1995 in dilute alkali gases.
- This happens below a well defined temperature (Dalfovo et al. 1999):

$$T < 2\pi\hbar^2/\underbrace{mk_B n^{2/3}}$$

m: mass of the particle
kB: the Boltzmann's constant
n: number density

The density distribution of the BEC dark matter halo is given by (Boehmer and Harko 2007):

$$\rho_{DM}(r) = \rho_{DM}^{(c)} \frac{\sin kr}{kr}$$

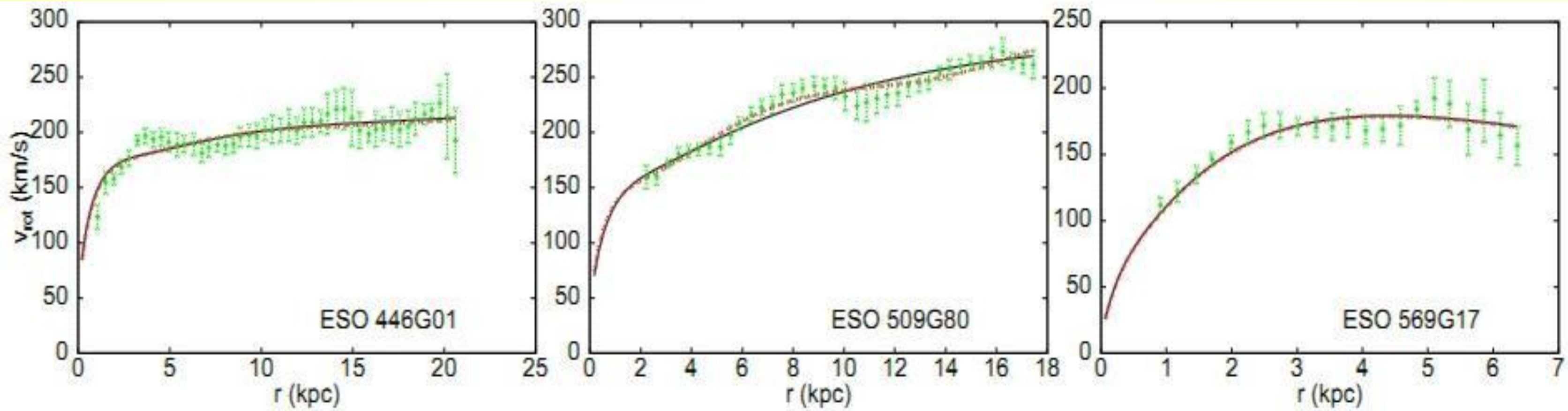
$k = \pi/R_{DM}$ R_DM is the size of the BEC halo

$\rho_{DM}^{(c)}$ the central density of the condensate

From this, the rotational velocity is obtained as :

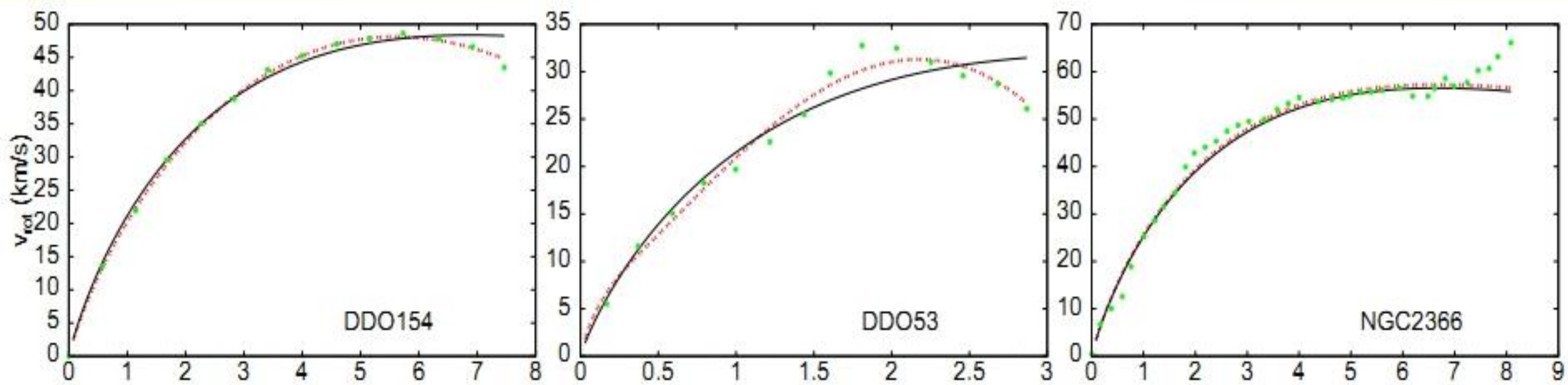
$$v_{DM}^2 = \frac{4\pi G \rho_{DM}^{(c)}}{k^2}\left(\frac{\sin kr}{kr} - \cos kr\right)$$

Confronting the BEC model with observational data (HSB and dwarf galaxies)



HSB

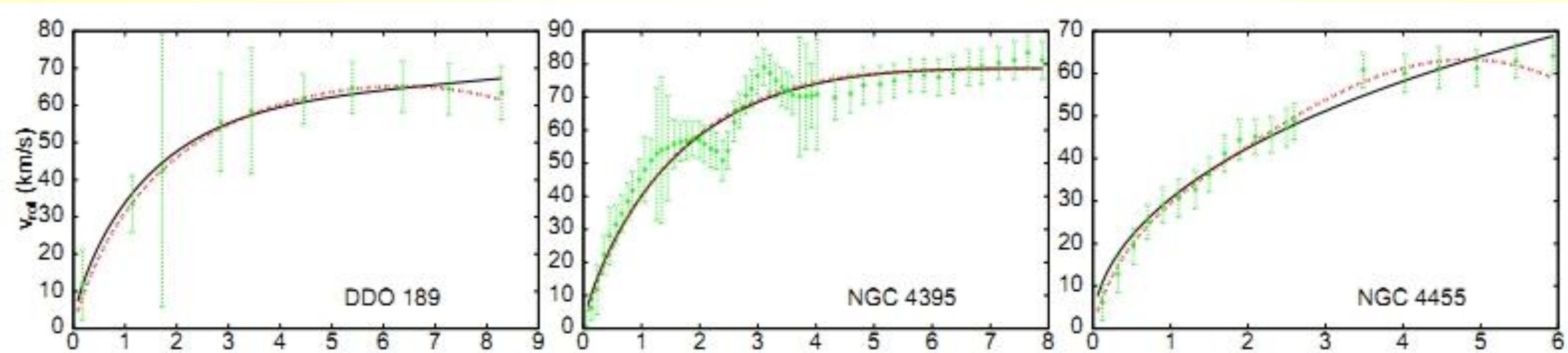
The BEC parameters and the NFW parameters was calculated by fitting the models to the data on rotation curves.



dwarf

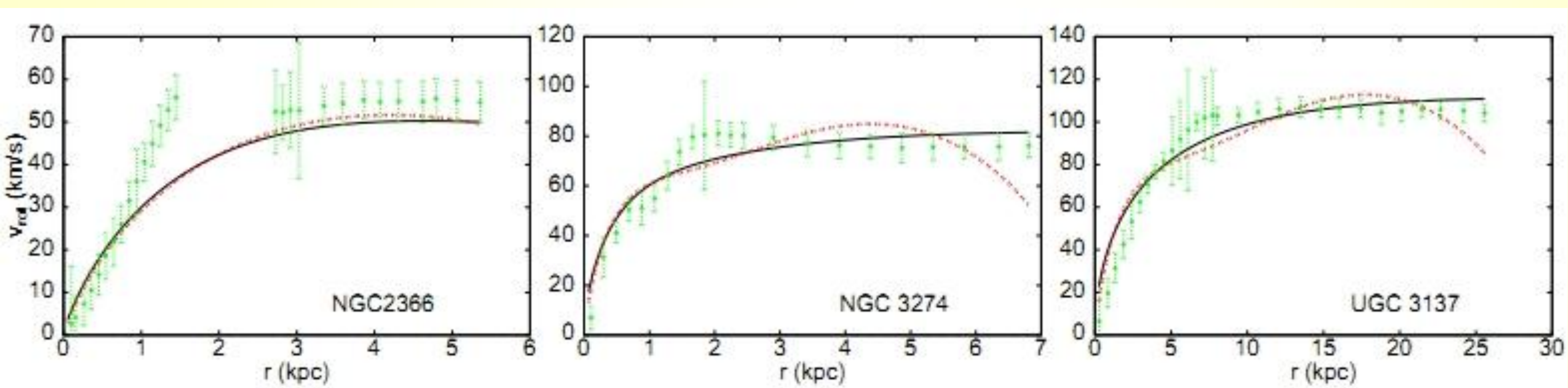
We performed the rotation curve fitting with the BEC and the NFW model respectively. BEC model gave better results than the NFW model, without exception.

Confronting the BEC model with observational data (LSB galaxies)



LSB I.

In the case of LSB I. galaxies, the combined BEC model gives a slightly better fit than the NFW one.



LSB II.

Nevertheless, our model can not be applied to the LSB II. galaxies, where plateau regions do appear. For these galaxies the NFW profile is proved to be a better assumption to fit the rotation curves.

SUMMARY

- Rotation curves provide a tool for studying the distribution and properties of gravitating matter.
- The shapes of the curves show that either gravity should be modified or dark matter is needed on galactic scale.
- We investigated higher-dimensional modifications of general relativity and found, that ***Weyl fluid is compatible with the rotation curves***.
- We assumed a cold dark matter distribution in the form of Bose-Einstein condensate. BEC model is suitable to explain the rotation curves of HSB and dwarf galaxies, but unable to explain flat rotation curves with long plateau regions.

References

- Boehmer, C.G., Harko, T., 2007, JCAP 06, 025
- Dadhich N., Maartens R., Papadopoulos P., Rezania, V., 2000, Phys. Lett. B, 487, 1
- Dalfovo, F., Giorgini, S., Pitaevskii, L.P., Stringari, S., 1999, Rev. Mod. Phys. 71, 463
- de Blok W. J. G., Bosma A., 2002, Astron. Astrophys. 385, 816
- Dwornik, M., Gergely, L. Á., Harko, T., Rotation curves in Bose-Einstein Condensate Dark Matter Halos (in preparation)
- Gergely, L. Á., Harko, T., Dwornik, M., Kupi, G., Keresztes, Z., 2011, MNRAS, 415, 3275
- Landau L. D., Lisfshitz E. M., 1975, The Classical Theory of Fields, Pergamon Press, Oxford
- Navarro J. F., Frenk C. S., White, S. D. M., 1996, Ap. J. 462, 563
- Palunas,P., Williams, T. B., 2000, Astron. Journal, 120, 2884
- Sasaki M., Shiromizu T., Maeda K., 2000, Phys. Rev. D, 62, 024008
- Yegorova I. A., Salucci P., 2007, Month. Not. Roy. Astr Soc. 377, 507

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