Homework No. 8 – November 4, 2017

Exercise 1. Find the following indefinite integrals.

(a) $\displaystyle\int \left(3t^2 + \frac{t}{2}\right) dt$

(b) $\displaystyle\int \left(\frac{1}{x^2} - x^2 - \frac{1}{3}\right) dx$

(c) $\displaystyle\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$

(d) $\displaystyle\int \left(8y - \frac{2}{y^{1/4}}\right) dy$

(e) $\displaystyle\int 2x(1 - x^{-3})dx$

(f) $\displaystyle\int \frac{t\sqrt{t} + \sqrt{t}}{t^2} dt$

(g) $\displaystyle\int 28(7x - 2)^{-5}dx$

(h) $\displaystyle\int \sqrt{4x - 2}dx$

(i) $\displaystyle\int \sqrt[5]{1 - y}dy$

(j) $\displaystyle\int \frac{6}{(3x + 5)^4}dx$

(k) $\displaystyle\int \frac{1}{(1 - x)^2}dx$

(l) $\displaystyle\int 4\sqrt[3]{2 - 3x}dx$

Exercise 2. Solve the following initial value problems.

(a) $y' = 2x - 7, \quad y(2) = 0$

(b) $f'(x) = \frac{1}{x^2} + x, \ x > 0, \quad f(2) = 1$

(c) $y'\frac{1}{2\sqrt{x}}, \quad y(4) = 0$

(d) $g'(t) = 3t^{-2/3}, \quad g(-1) = -5$

(e) $f'(x) = \left(x + \frac{1}{x}\right)^2, \quad f(1) = 1$

(f) $f'(x) = \frac{x^2+1}{x^2}, \quad f(1) = -1$

Exercise 3. Evaluate the following integrals.

(a) $\displaystyle\int_{-2}^{0} (2x + 5)dx$

(b) $\displaystyle\int_{-3}^{4} \left(5 - \frac{x}{2}\right) dx$

(c) $\displaystyle\int_{0}^{4} \left(3x - \frac{x^3}{4}\right) dx$

(d) $\displaystyle\int_{-2}^{2} (x^3 - 2x + 3)dx$

(e) $\displaystyle\int_{0}^{1} (x^2 + \sqrt{x})dx$

(f) $\displaystyle\int_{0}^{5} x^{3/2}dx$

(g) $\displaystyle\int_{1}^{32} x^{-6/5}dx$

(h) $\displaystyle\int_{-2}^{-1} \frac{2}{x^2}dx$

(i) $\displaystyle\int_{1}^{-1} (r + 1)^2dr$

(j) $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} (t + 1)(t^2 + 4)dt$

(k) $\displaystyle\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du$

(l) $\displaystyle\int_{1/2}^{1} \left(\frac{1}{v^3} - \frac{1}{v^4}\right) dv$

(m) $\displaystyle\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}ds$

(n) $\displaystyle\int_{9}^{4} \frac{1 - \sqrt{u}}{\sqrt{u}}du$

(o) $\displaystyle\int_{-3}^{3} |x|dx$

(p) $\displaystyle\int_{1}^{2} 6 - x + x^2dx$

(q) $\displaystyle\int_{0}^{2} 6x - x^3 + 3x^2dx$

(r) $\displaystyle\int_{1}^{2} -\sqrt{t} - 2t^3dt$

(s) $\displaystyle\int_2^3 \frac{3x^2 - 2x}{x} dx$

(t) $\displaystyle\int_1^4 \frac{2x^3 - x}{x} dx$

(u) $\displaystyle\int_1^3 \frac{x^3 - x}{\sqrt{x}} dx$

(v) $\displaystyle\int_1^2 \frac{x^2 + 1}{\sqrt[3]{x}} dx$

(w) $\displaystyle\int_1^3 \sqrt{x - 1} dx$

(x) $\displaystyle\int_{-1}^2 \sqrt[3]{x + 2} dx$

Exercise 4. Find the total area between the region and the x-axis.

(a) $y = -x^2 - 2x, \quad -3 \le x \le 2$

(b) $y = 3x^2 - 3, \quad -2 \le x \le 2$

(c) $y = x^3 - 3x^2 + 2x, \quad 0 \le x \le 2$

(d) $y = x^3 - 4x, \quad -2 \le x \le 2$

(e) $y = x^{1/3}, \quad -1 \le x \le 8$

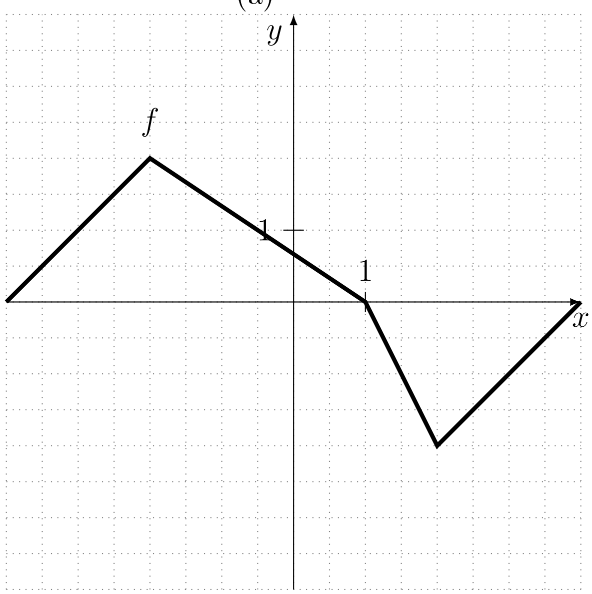
(f) $y = x^{1/3} - x, \quad -1 \le x \le 8$

(g) $y = 1 - \sqrt{x}, \quad 0 \le x \le 4$

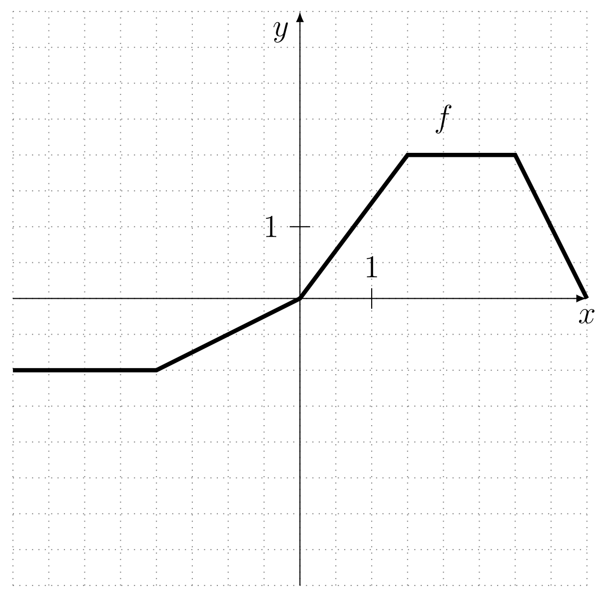
(h) $y = 1 - (x^2/4), \quad -2 \le x \le 3$

Exercise 5. Calculate the *signed* area between the function f and x-axis.

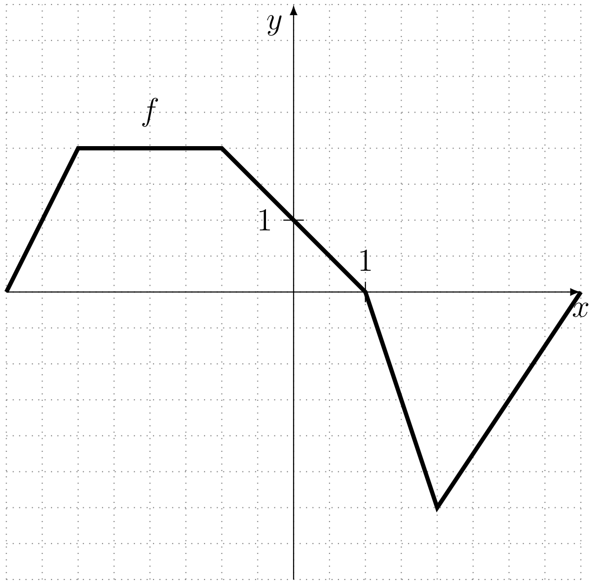
(a)



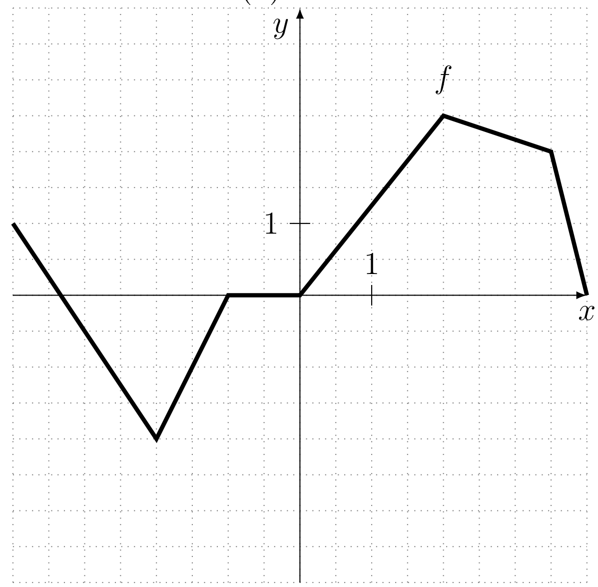
(b)



(c)



(d)



Exercise 6.

(a) Calculate the integral mean of the funcion in the part (a) of the previous problem on the interval $[-4, 2]$.

(b) Calculate the integral mean of the funcion in the part (b) of the previous problem on the interval $[-3, 3]$.

(c) Calculate the integral mean of the funcion in the part (c) of the previous problem on the interval $[-2, 3]$.

(d) Calculate the integral mean of the funcion in the part (d) of the previous problem on the interval $[-3, 2]$.

Exercise 7. Find the average value of the following functions on the given intervals.

(a) $f(x) = x^2 - 1, \quad [0, \sqrt{3}]$

(b) $f(x) = (x-1)^2, \quad [0, 3]$

(c) $f(x) = 3x^2 - 3, \quad [0, 1]$

(d) $f(t) = t^2 - t, \quad [-2, 1]$

(e) $f(x) = |x| - 1, \quad [-1, 1], \quad [1, 3]$

(f) $f(x) = -|x|, \quad [-1, 0], \quad [-1, 1]$

Exercise 8. The velocity of a moving body is described by the function $v(t) = t^2 - t \left(\frac{m}{s}\right)$.

(a) Find the indefinite integral of $v(t)$.

(b) Calculate the average velocity over the interval $[1, 4]$.

(c) Determine the position function $s(t)$, such that $s(0) = 1$.

(d) Find the extremal value of $s(t)$.

Exercise 9. The velocity of a moving body is described by the function $v(t) = t^3 - t^2 \left(\frac{m}{s}\right)$.

(a) Find the antiderivative of $v(t)$.

(b) Calculate the average velocity over the interval $[1, 3]$.

(c) Determine the position function $s(t)$, such that $s(0) = 2$.

(d) Find the extremal value of $s(t)$.

Exercise 10. The rate of change of the mass $m(t)$ of a material over time t is given by the function $m'(t) = v(t) = -\frac{t}{4(1+t^2)^3} \left(\frac{g}{s}\right)$.

(a) Find the antiderivative of $v(t)$.

(b) Calculate the average speed over the interval $[0, 3]$.

(c) Determine the formula for $m(t)$ under the initial condition $m(0) = 1$.