Homework No. 7 – October 31, 2017

Exercise 1. A body is moving on a coordinate line, with s in meters and t in seconds. Find the body's speed and acceleration at the endpoints of the interval. When, if ever, during the interval does the body change direction?

(a) $s = t^2 - 3t + 2$, $0 \le t \le 2$

(b) $s = 6t - t^2$, $0 \le t \le 6$

(c) $s = \dfrac{25}{t+5}$, $-4 \le t \le 0$

(d) $s = \dfrac{25}{t^2} - \dfrac{5}{t}$, $1 \le t \le 5$

Exercise 2. At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$ m.

(a) Find the body's acceleration each time the velocity is zero.

(b) Find the body's speed each time the acceleration is zero.

(c) Fin the total distance traveled by the body from $t = 0$ to $t = 2$.

Exercise 3. At time $t \ge 0$, the velocity of a body moving along the s-axis is $v = t^2 - 4t + 3$.

(a) Find the body's acceleration each time the velocity is zero.

(b) When is the body moving forward? Backward?

(c) When is the body's velocity increasing? Decreasing?

Exercise 4. What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?

Exercise 5. The height of a body moving vertically is given by $s = -\dfrac{1}{2}gt^2 + v_0 t + s_0$, $g > 0$, with s in meters and t in seconds. Find the body's maximum height.

Exercise 6. The function $V(x) = x(10 - 2x)(16 - 2x)$, $0 < x < 5$, models the volume of a box.

(a) Find the extreme values of V.

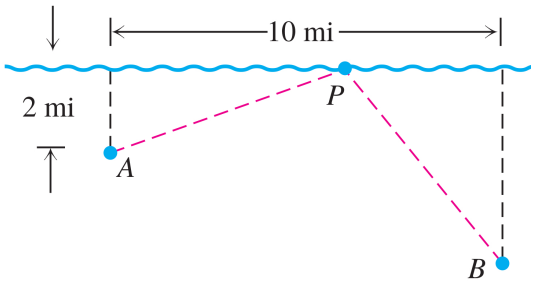
(b) Interpret any values found in part (a) in terms of volume of the box.

Exercise 7. Find the value or values of c that satisfy the equation $\dfrac{f(b) - f(a)}{b - a} = f'(c)$ in the Mean Value Theorem for the following functions and intervals.

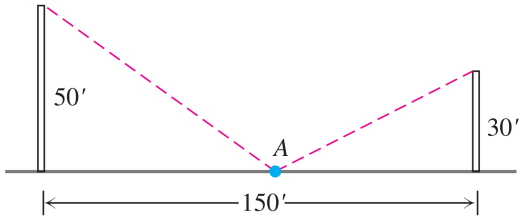
(a) $f(x) = x^2 + 2x - 1$, $[0, 1]$ (b) $f(x) = x + \dfrac{1}{x}$, $\left[\dfrac{1}{2}, 2\right]$ (c) $f(x) = \sqrt{x - 1}$, $[1, 3]$

Exercise 8. A trucker handed in a ticket at a toll booth showing that in 2 hours she had covered 159 mi on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

Exercise 9. Two towns lie on the south side of a river. A pumping station is to be located to serve the two towns. A pipeline will be constructed from the pumping station to each of the towns along the line connecting the town and the pumping station. Locate the pumping station to minimize the amount of pipeline that must be constructed.

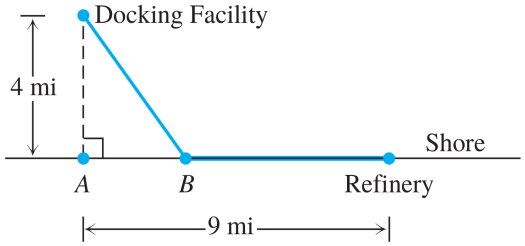


Exercise 10. One tower is 50 ft high and another tower is 30 ft high. The towers are 150 ft apart. A guy wire is to run from point A to the top of each tower.



Locate point A so that the total length of guy wire is minimal.

Exercise 11. Supertankers off-load oil at a docking facility 4 mi offshore. The nearest refinery is 9 mi east of the shore point nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs \$300,000 per mile if constructed underwater and \$200,000 per mile if overland.



(a) Locate point B to minimize the cost of the construction.

(b) The cost of underwater construction is expected to increase, whereas the cost of overland construction is expected to stay constant. At what cost does it become optimal to construct the pipeline directly to point A.

Exercise 12. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?