“Broadening the knowledge base and supporting the long term professional sustainability of the Research University Centre of Excellence at the University of Szeged by ensuring the rising generation of excellent scientists.”

Doctoral School of Mathematics and Computer Science
Stochastic Days in Szeged
27.07.2012.
Online-to-Confidence-Set Conversions and Application to Sparse Stochastic Bandits

Csaba Szepesvári
(University of Alberta)
Online-to-Confidence-Set Conversions and Application to Sparse Stochastic Bandits

Csaba Szepesvári

Department of Computing Science
University of Alberta
csaba.szepesvari@ualberta.ca

July 27, 2013

Stochastic days – honoring András Krámlı 70th birthday
joint work with Yasin Abbasi-Yadkori and Dávid Pál
Contents

- Linear prediction and (honest) confidence sets
Contents

- Linear prediction and (honest) confidence sets
  - Definition

- Some previous results
- Sparsity
- Online-to-confidence-set conversion

- Online linear prediction
- The conversion
- Why does it work?

- Application to linear bandits
- Sparse linear bandits
- Results
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity

- Online-to-confidence-set conversion
- Online linear prediction
- The conversion
- Why does it work?

- Application to linear bandits
- Sparse linear bandits
- Results
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
  - The conversion
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
  - The conversion
  - Why does it work?
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
  - The conversion
  - Why does it work?
- Application to linear bandits
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
  - The conversion
  - Why does it work?
- Application to linear bandits
  - Sparse linear bandits
Contents

- Linear prediction and (honest) confidence sets
  - Definition
  - Some previous results
  - Sparsity
- Online-to-confidence-set conversion
  - Online linear prediction
  - The conversion
  - Why does it work?
- Application to linear bandits
  - Sparse linear bandits
  - Results
Linear Prediction and
(Honest) Confidence Sets

Getting directions from András: MDPs, ILT, Dregely
The Data

- $X_1, \ldots, X_n \in \mathbb{R}^d$, $Y_1, \ldots, Y_n \in \mathbb{R}$
The Data

- $X_1, \ldots, X_n \in \mathbb{R}^d, Y_1, \ldots, Y_n \in \mathbb{R}$
- $\exists \theta_* \in \mathbb{R}^d$ s.t.

$$Y_t = \langle X_t, \theta_* \rangle + \eta_t, \quad t = 1, \ldots, n$$
The Data

- $X_1, \ldots, X_n \in \mathbb{R}^d$, $Y_1, \ldots, Y_n \in \mathbb{R}$
- $\exists \theta_\ast \in \mathbb{R}^d$ s.t.
  \[ Y_t = \langle X_t, \theta_\ast \rangle + \eta_t, \quad t = 1, \ldots, n \]

- The “noise”, $\eta_t$ is conditionally $R$-sub-Gaussian with some $R > 0$, i.e.,
  \[ \forall \lambda \in \mathbb{R} : \quad \mathbb{E}[e^{\lambda \eta_t} | X_1, \ldots, X_t, \eta_1, \ldots, \eta_{t-1}] \leq \exp \left( \frac{\lambda^2 R^2}{2} \right) . \]
The Data

- \( X_1, \ldots, X_n \in \mathbb{R}^d, Y_1, \ldots, Y_n \in \mathbb{R} \)
- \( \exists \theta_* \in \mathbb{R}^d \text{ s.t.} \)
  \[
  Y_t = \langle X_t, \theta_* \rangle + \eta_t, \quad t = 1, \ldots, n
  \]

- The "noise", \( \eta_t \) is conditionally \( R \)-sub-Gaussian with some \( R > 0 \), i.e.,
  \[
  \forall \lambda \in \mathbb{R} : \quad \mathbb{E}[e^{\lambda \eta_t}|X_1, \ldots, X_t, \eta_1, \ldots, \eta_{t-1}] \leq \exp\left(\frac{\lambda^2 R^2}{2}\right).
  \]

- Often \( X_t \) is chosen based on \((X_1, \ldots, X_{t-1})\) and \((Y_1, \ldots, Y_{t-1})\)
The Data

- $X_1, \ldots, X_n \in \mathbb{R}^d$, $Y_1, \ldots, Y_n \in \mathbb{R}$
- $\exists \theta_\ast \in \mathbb{R}^d$ s.t.
  \[ Y_t = \langle X_t, \theta_\ast \rangle + \eta_t, \quad t = 1, \ldots, n \]

- The “noise”, $\eta_t$ is conditionally $R$-sub-Gaussian with some $R > 0$, i.e.,
  \[ \forall \lambda \in \mathbb{R} : \quad \mathbb{E}[e^{\lambda \eta_t} | X_1, \ldots, X_t, \eta_1, \ldots, \eta_{t-1}] \leq \exp \left( \frac{\lambda^2 R^2}{2} \right) . \]

- Often $X_t$ is chosen based on $(X_1, \ldots, X_{t-1})$ and $(Y_1, \ldots, Y_{t-1})$

Estimation Problems:
The Data

- $X_1, \ldots, X_n \in \mathbb{R}^d, Y_1, \ldots, Y_n \in \mathbb{R}$
- $\exists \theta_* \in \mathbb{R}^d$ s.t.

$$Y_t = \langle X_t, \theta_* \rangle + \eta_t, \quad t = 1, \ldots, n$$

- The “noise”, $\eta_t$ is conditionally $R$-sub-Gaussian with some $R > 0$, i.e.,

$$\forall \lambda \in \mathbb{R} : \quad \mathbb{E}[e^{\lambda \eta_t}|X_1, \ldots, X_t, \eta_1, \ldots, \eta_{t-1}] \leq \exp \left( \frac{\lambda^2 R^2}{2} \right).$$

- Often $X_t$ is chosen based on $(X_1, \ldots, X_{t-1})$ and $(Y_1, \ldots, Y_{t-1})$

Estimation Problems:

- Estimate $\theta_*$ based on $((X_1, Y_1), \ldots, (X_n, Y_n))$!
The Data

- \( X_1, \ldots, X_n \in \mathbb{R}^d, Y_1, \ldots, Y_n \in \mathbb{R} \)
- \( \exists \theta_* \in \mathbb{R}^d \) s.t.

\[
Y_t = \langle X_t, \theta_* \rangle + \eta_t, \quad t = 1, \ldots, n
\]

- The “noise”, \( \eta_t \) is conditionally \( R \)-sub-Gaussian with some \( R > 0 \), i.e.,

\[
\forall \lambda \in \mathbb{R} : \quad \mathbb{E}[e^{\lambda \eta_t} | X_1, \ldots, X_t, \eta_1, \ldots, \eta_{t-1}] \leq \exp \left( \frac{\lambda^2 R^2}{2} \right)
\]

- Often \( X_t \) is chosen based on \((X_1, \ldots, X_{t-1})\) and \((Y_1, \ldots, Y_{t-1})\)

Estimation Problems:

- Estimate \( \theta_* \) based on \(((X_1, Y_1), \ldots, (X_n, Y_n))\)!
- Construct a confidence set that contains \( \theta_* \) w.h.p.!
Sub-Gaussianity

Definition
Random variable $Z$ is $R$-sub-Gaussian for some $R \geq 0$ if

$$\forall \gamma \in \mathbb{R} \quad \mathbb{E}[e^{\gamma Z}] \leq \exp \left( \frac{\gamma^2 R^2}{2} \right).$$
Sub-Gaussianity

Definition
Random variable $Z$ is $R$-sub-Gaussian for some $R \geq 0$ if

$$\forall \gamma \in \mathbb{R} \quad E[e^{\gamma Z}] \leq \exp \left( \frac{\gamma^2 R^2}{2} \right).$$

The condition implies that

- $E[Z] = 0$
- $\text{Var}[Z] \leq R^2$
Sub-Gaussianity

**Definition**
Random variable $Z$ is $R$-sub-Gaussian for some $R \geq 0$ if

$$
\forall \gamma \in \mathbb{R} \quad \mathbb{E}[e^{\gamma Z}] \leq \exp \left( \frac{\gamma^2 R^2}{2} \right).
$$

The condition implies that

- $\mathbb{E}[Z] = 0$
- $\text{Var}[Z] \leq R^2$

Examples:

- Zero-mean bounded in an interval of length $2R$ (Hoeffding-Azuma)
- Zero-mean Gaussian with variance $\leq R^2$
(Honest) Confidence Sets

Given the data \(((X_1, Y_1), \ldots, (X_n, Y_n))\) and

\[0 \leq \delta \leq 1,\]

construct

\[C_n \subset \mathbb{R}^d\]

such that

\[\Pr(\theta^* \in C_n) \geq 1 - \delta.\]
(Honest) Confidence Sets

Given the data \(((X_1, Y_1), \ldots, (X_n, Y_n))\) and

\[0 \leq \delta \leq 1,\]

construct

\[C_n \subset \mathbb{R}^d\]

such that

\[\Pr(\theta_* \in C_n) \geq 1 - \delta.\]
(Honest) Confidence Sets

Given the data \(((X_1, Y_1), \ldots, (X_n, Y_n))\) and \(0 \leq \delta \leq 1\), construct

\[ C_n \subset \mathbb{R}^d \]

such that

\[ \Pr(\theta^* \in C_n) \geq 1 - \delta. \]
Confidence Sets based on Ridge-regression

Data $(X_1, Y_1), \ldots, (X_n, Y_n)$ such that $Y_t \approx \langle X_t, \theta_* \rangle$
Confidence Sets based on Ridge-regression

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) such that \(Y_t \approx \langle X_t, \theta_* \rangle\)
- Stack them into matrices: \(X_{1:n}\) is \(n \times d\) and \(Y_{1:n}\) is \(n \times 1\)
Confidence Sets based on Ridge-regression

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) such that \(Y_t \approx \langle X_t, \theta_* \rangle\)
- Stack them into matrices: \(X_{1:n}\) is \(n \times d\) and \(Y_{1:n}\) is \(n \times 1\)
- Ridge regression estimator:

\[
\hat{\theta}_n = (X_{1:n}X_{1:n}^T + \lambda I)^{-1}X_{1:n}^T Y_{1:n}
\]
Confidence Sets based on Ridge-regression

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) such that \(Y_t \approx \langle X_t, \theta_* \rangle\)
- Stack them into matrices: \(X_{1:n}\) is \(n \times d\) and \(Y_{1:n}\) is \(n \times 1\)
- Ridge regression estimator:
  \[
  \hat{\theta}_n = (X_{1:n}X_{1:n}^T + \lambda I)^{-1}X_{1:n}^T Y_{1:n}
  \]
- Let \(V_n = X_{1:n}X_{1:n}^T + \lambda I\)
Confidence Sets based on Ridge-regression

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) such that \(Y_t \approx \langle X_t, \theta_* \rangle\)
- Stack them into matrices: \(X_{1:n}\) is \(n \times d\) and \(Y_{1:n}\) is \(n \times 1\)
- Ridge regression estimator:
  \[
  \hat{\theta}_n = (X_{1:n}X_{1:n}^T + \lambda I)^{-1}X_{1:n}^T Y_{1:n}
  \]

- Let \(V_n = X_{1:n}X_{1:n}^T + \lambda I\)

**Theorem ([AYPS11])**

If ||\(\theta_*\)||_2 \leq S, then with probability at least 1 − \(\delta\), for all \(t\), \(\theta_*\) lies in

\[
C_t = \left\{ \theta : ||\hat{\theta}_t - \theta||_{V_t} \leq R \sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right)} + S \sqrt{\lambda} \right\}
\]

where ||\(v||_A = \sqrt{v^T Av}\) is the matrix \(A\)-norm.
Confidence Sets based on Ridge-regression

- Data $((X_1, Y_1), \ldots, (X_n, Y_n))$ such that $Y_t \approx \langle X_t, \theta_* \rangle$
- Stack them into matrices: $X_{1:n}$ is $n \times d$ and $Y_{1:n}$ is $n \times 1$
- Ridge regression estimator:

$$\hat{\theta}_n = (X_{1:n}X_{1:n}^T + \lambda I)^{-1}X_{1:n}^T Y_{1:n}$$

- Let $V_n = X_{1:n}X_{1:n}^T + \lambda I$

**Theorem ([AYPS11])**

If $\|\theta_*\|_2 \leq S$, then with probability at least $1 - \delta$, for all $t$, $\theta_*$ lies in

$$C_t = \left\{ \theta : \|\hat{\theta}_t - \theta\|_{V_t} \leq R \sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right)} + S \sqrt{\lambda} \right\}$$

where $\|v\|_A = \sqrt{v^T A v}$ is the matrix $A$-norm.

Proof technique: [RS70, dLS09].
Confidence Sets based on Ridge-regression

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) such that \(Y_t \approx \langle X_t, \theta_* \rangle\)
- Stack them into matrices: \(X_{1:n}\) is \(n \times d\) and \(Y_{1:n}\) is \(n \times 1\)
- Ridge regression estimator:

\[
\hat{\theta}_n = (X_{1:n}X_{1:n}^T + \lambda I)^{-1}X_{1:n}^T Y_{1:n}
\]

- Let \(V_n = X_{1:n}X_{1:n}^T + \lambda I\)

Theorem ([AYPS11])

If \(\|\theta_*\|_2 \leq S\), then with probability at least \(1 - \delta\), for all \(t\), \(\theta_*\) lies in

\[
C_t = \left\{ \theta : \|\hat{\theta}_t - \theta\|_{V_t} \leq R\sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right)} + S \sqrt{\lambda} \right\}
\]

where \(\|v\|_A = \sqrt{v^T Av}\) is the matrix \(A\)-norm.

Proof technique: [RS70, dLS09]. Extends to separable Hilbert spaces.
Comparison with Previous Confidence Sets

- Bound of [AYPS11]:

\[
\|\hat{\theta}_t - \theta_*\|_{V_t} \leq R \sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right) + S \sqrt{\lambda}}
\]
Comparison with Previous Confidence Sets

- Bound of [AYPS11]:

\[
\|\hat{\theta}_t - \theta_*\|_{V_t} \leq R \sqrt{2 \ln \left(\frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}}\right)} + S\sqrt{\lambda}
\]

- [DHK08]: If \(\|\theta_*\|_2, \|X_t\|_2 \leq 1\) then for a specific \(\lambda\)

\[
\|\hat{\theta}_t - \theta_*\|_{V_t} \leq R \max \left\{ \sqrt{128d \ln(t) \ln(t^2/\delta)}, \frac{8}{3} \ln(t^2/\delta) \right\}
\]
Comparison with Previous Confidence Sets

- **Bound of [AYPS11]:**

  \[
  \|\hat{\theta}_t - \theta_*\|_{V_t} \leq R \sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right)} + S\sqrt{\lambda}
  \]

- **[DHK08]:** If \(\|\theta_*\|_2, \|X_t\|_2 \leq 1\) then for a specific \(\lambda\)

  \[
  \|\hat{\theta}_t - \theta_*\|_{V_t} \leq R \max \left\{ \sqrt{128d \ln(t) \ln(t^2/\delta)}, \frac{8}{3} \ln(t^2/\delta) \right\}
  \]

- **[RT10]:** If \(\|X_t\|_2 \leq 1\)

  \[
  \|\hat{\theta}_t - \theta_*\|_{V_t} \leq 2R\kappa\sqrt{\ln t} \sqrt{d \ln t + \ln(t^2/\delta)} + S\sqrt{\lambda}
  \]

  where \(\kappa = 3 + 2 \ln((1 + \lambda d)/\lambda)\).
Comparison with Previous Confidence Sets

- Bound of [AYPS11]:

\[
\|\hat{\theta}_t - \theta^*\|_{V_t} \leq R \sqrt{2 \ln \left( \frac{\det(V_t)^{1/2}}{\delta \det(\lambda I)^{1/2}} \right)} + S \sqrt{\lambda}
\]

- [DHK08]: If \(\|\theta^*\|_2, \|X_t\|_2 \leq 1\) then for a specific \(\lambda\)

\[
\|\hat{\theta}_t - \theta^*\|_{V_t} \leq R \max \left\{ \sqrt{128d \ln(t) \ln(t^2/\delta)}, \frac{8}{3} \ln(t^2/\delta) \right\}
\]

- [RT10]: If \(\|X_t\|_2 \leq 1\)

\[
\|\hat{\theta}_t - \theta^*\|_{V_t} \leq 2R\kappa \sqrt{\ln t} \sqrt{d \ln t + \ln(t^2/\delta)} + S\sqrt{\lambda}
\]

where \(\kappa = 3 + 2 \ln((1 + \lambda d)/\lambda)\).

The bound of [AYPS11] doesn’t depend on \(t\).
Questions

- Are there other ways to construct confidence sets?
Questions

- Are there other ways to construct confidence sets?
- Can we get tighter confidence sets when some special conditions are met?
Questions

▶ Are there other ways to construct confidence sets?
▶ Can we get tighter confidence sets when some special conditions are met?
▶ SPARSITY:

Only $p$ coordinates of $\theta_*$ are nonzero.
Questions

▶ Are there other ways to construct confidence sets?
▶ Can we get tighter confidence sets when some special conditions are met?
▶ SPARSITY:

**Only** \( p \) **coordinates of** \( \theta_* \) **are nonzero.**

▶ Can we construct tighter confidence sets based on the knowledge of \( p \)?
Questions

- Are there other ways to construct confidence sets?
- Can we get tighter confidence sets when some special conditions are met?
- SPARSITY:
  
  Only $p$ coordinates of $\theta_*$ are nonzero.

- Can we construct tighter confidence sets based on the knowledge of $p$?
- Least-squares (or ridge) estimators are not a good idea!
Online-to-Confidence-Set Conversion

Idea: Create a confidence set based on how well an online linear prediction algorithm works.

This is a reduction!

If a new prediction algorithm is discovered, or a better performance bounds for an algorithm becomes available, we get tighter confidence sets.

Hopefully it will work for the sparse case.

Encouragement: Working on my thesis.
Online-to-Confidence-Set Conversion

- Idea: Create a confidence set based on how well an online linear prediction algorithm works.
Online-to-Confidence-Set Conversion

- Idea: Create a confidence set based on how well an online linear prediction algorithm works.
- This is a reduction!
Online-to-Confidence-Set Conversion

- Idea: Create a confidence set based on how well an online linear prediction algorithm works.
- This is a reduction!
- If a new prediction algorithm is discovered, or a better performance bounds for an algorithm becomes available, we get tighter confidence sets.
Online-to-Confidence-Set Conversion

- Idea: Create a confidence set based on how well an online linear prediction algorithm works.
- This is a reduction!
- If a new prediction algorithm is discovered, or a better performance bounds for an algorithm becomes available, we get tighter confidence sets.
- Hopefully it will work for the sparse case.
Online-to-Confidence-Set Conversion

- Idea: Create a confidence set based on how well an online linear prediction algorithm works.

- This is a reduction!

- If a new prediction algorithm is discovered, or a better performance bounds for an algorithm becomes available, we get tighter confidence sets.

- Hopefully it will work for the sparse case.

Encouragement: Working on my thesis.
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$

There are heaps of algorithms for this problem:
- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
- exponentiated gradient algorithm [KW97]
- online LASSO (??)
- SeqSEW [Ger11, DT07]
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
- exponentiated gradient algorithm [KW97]
- online LASSO (??)
- SeqSEW [Ger11, DT07]
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!

There are heaps of algorithms for this problem:
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
Online Linear Prediction

For \( t = 1, 2, \ldots \):

- Receive \( X_t \in \mathbb{R}^d \)
- Predict \( \hat{Y}_t \in \mathbb{R} \)
- Receive correct label \( Y_t \in \mathbb{R} \)
- Suffer loss \( (Y_t - \hat{Y}_t)^2 \)

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on \((X_1, Y_1), (X_2, Y_2), \ldots \)!

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
- exponentiated gradient algorithm [KW97]
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
- exponentiated gradient algorithm [KW97]
- online LASSO (??)
Online Linear Prediction

For $t = 1, 2, \ldots$:

- Receive $X_t \in \mathbb{R}^d$
- Predict $\hat{Y}_t \in \mathbb{R}$
- Receive correct label $Y_t \in \mathbb{R}$
- Suffer loss $(Y_t - \hat{Y}_t)^2$

Goal: Compete with the best linear predictor in hindsight

No assumptions whatsoever on $(X_1, Y_1), (X_2, Y_2), \ldots$!

There are heaps of algorithms for this problem:

- online gradient descent [Zin03]
- online least-squares [AW01, Vov01]
- exponentiated gradient algorithm [KW97]
- online LASSO (??)
- SeqSEW [Ger11, DT07]
Online Linear Prediction, cnt’d

- Regret with respect to a linear predictor $\theta \in \mathbb{R}^d$

$$\rho_n(\theta) = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 - \sum_{t=1}^{n} (Y_t - \langle X_t, \theta \rangle)^2$$
Online Linear Prediction, cnt’d

- **Regret** with respect to a linear predictor $\theta \in \mathbb{R}^d$

  $$
  \rho_n(\theta) = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 - \sum_{t=1}^{n} (Y_t - \langle X_t, \theta \rangle)^2
  $$

-Prediction algorithms come with “regret bounds” $B_n$:

  $$
  \forall n \quad \rho_n(\theta) \leq B_n
  $$
Online Linear Prediction, cnt’d

- **Regret** with respect to a linear predictor $\theta \in \mathbb{R}^d$

$$\rho_n(\theta) = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 - \sum_{t=1}^{n} (Y_t - \langle X_t, \theta \rangle)^2$$

- Prediction algorithms come with “regret bounds” $B_n$:

$$\forall n \quad \rho_n(\theta) \leq B_n$$

- $B_n$ depends on $n, d, \theta$ and possibly $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$
Online Linear Prediction, cnt’d

- Regret with respect to a linear predictor $\theta \in \mathbb{R}^d$

$$\rho_n(\theta) = \sum_{t=1}^{n} (Y_t - \hat{Y}_t)^2 - \sum_{t=1}^{n} (Y_t - \langle X_t, \theta \rangle)^2$$

- Prediction algorithms come with “regret bounds” $B_n$:

$$\forall n \quad \rho_n(\theta) \leq B_n$$

- $B_n$ depends on $n, d, \theta$ and possibly $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$

- Typically, $B_n = O(\sqrt{n})$ or $B_n = O(\log n)$
Good Regret Implies Small Risk

- Data: \( \{(X_t, Y_t)\}_{t=1}^{n} \) is i.i.d.,
  \[ Y_t = \langle X_t, \theta_* \rangle + \eta_t, \eta_t \text{ R-sub-Gaussian} \]
Good Regret Implies Small Risk

- Data: \{\( (X_t, Y_t) \)\}_{t=1}^{n} is i.i.d.,
  \( Y_t = \langle X_t, \theta_* \rangle + \eta_t \), \( \eta_t \) R-sub-Gaussian

- Online Learning Algorithm:
  \( \mathcal{A} \) produces \( \{\theta_t\}_{t=1}^{n} \) and predicts \( \hat{Y}_t = \langle X_t, \theta_t \rangle \)
Good Regret Implies Small Risk

- Data: $\{(X_t, Y_t)\}_{t=1}^n$ is i.i.d.,
  $Y_t = \langle X_t, \theta_* \rangle + \eta_t$, $\eta_t$ $R$-sub-Gaussian

- Online Learning Algorithm:
  $\mathcal{A}$ produces $\{\theta_t\}_{t=1}^n$ and predicts $\hat{Y}_t = \langle X_t, \theta_t \rangle$

- Regret bound: $\forall n: \rho_n(\theta_*) \leq B_n$
Good Regret Implies Small Risk

- Data: \( \{(X_t, Y_t)\}_{t=1}^n \) is i.i.d.,
  \[ Y_t = \langle X_t, \theta_\star \rangle + \eta_t, \eta_t \text{ } R\text{-sub-Gaussian} \]

- Online Learning Algorithm:
  \( \mathcal{A} \) produces \( \{\theta_t\}_{t=1}^n \) and predicts \( \hat{Y}_t = \langle X_t, \theta_t \rangle \)

- Regret bound: \( \forall n: \rho_n(\theta_\star) \leq B_n \)

- Risk of vector \( \theta \): \( R(\theta) = \mathbb{E}[(Y_1 - \langle X_1, \theta \rangle)^2] \).
Good Regret Implies Small Risk

- Data: \( \{(X_t, Y_t)\}_{t=1}^n \) is i.i.d.,
  \[ Y_t = \langle X_t, \theta_* \rangle + \eta_t, \eta_t \text{ } R\text{-sub-Gaussian} \]

- Online Learning Algorithm:
  \( \mathcal{A} \) produces \( \{\theta_t\}_{t=1}^n \) and predicts \( \hat{Y}_t = \langle X_t, \theta_t \rangle \)

- Regret bound:
  \( \forall n: \rho_n(\theta_*) \leq B_n \)

- Risk of vector \( \theta \):
  \[ R(\theta) = E[(Y_1 - \langle X_1, \theta \rangle)^2]. \]

**Theorem ([CBG08])**

*Let \( \bar{\theta}_n = \frac{1}{n} \sum_{t=1}^n \theta_t \). Then, w.p. \( 1 - \delta \),*

\[
R(\bar{\theta}_n) \leq \frac{B_n}{n} + \frac{36}{n} \ln \left( \frac{B_n + 3}{\delta} \right) + \sqrt{\frac{B_n}{n^2} \ln \left( \frac{B_n + 3}{\delta} \right)}. 
\]
Online-to-Confidence-Set Conversion

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) where \(Y_t = \langle X_t, \theta_* \rangle + \eta_t\)
  and \(\eta_t\) is conditionally \(R\)-sub-Gaussian.
Online-to-Confidence-Set Conversion

- Data $(X_1, Y_1), \ldots, (X_n, Y_n)$ where $Y_t = \langle X_t, \theta_* \rangle + \eta_t$ and $\eta_t$ is conditionally $R$-sub-Gaussian.
- Predictions $\hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_n$
Online-to-Confidence-Set Conversion

- Data $(X_1, Y_1), \ldots, (X_n, Y_n)$ where $Y_t = \langle X_t, \theta^* \rangle + \eta_t$ and $\eta_t$ is conditionally $R$-sub-Gaussian.
- Predictions $\hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_n$
- Regret bound $\rho(\theta^*) \leq B_n$
Online-to-Confidence-Set Conversion

- Data \((X_1, Y_1), \ldots, (X_n, Y_n)\) where \(Y_t = \langle X_t, \theta_* \rangle + \eta_t\) and \(\eta_t\) is conditionally \(R\)-sub-Gaussian.
- Predictions \(\hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_n\)
- Regret bound \(\rho(\theta_*) \leq B_n\)

Theorem (Conversion, [AYPS12])

*With probability at least \(1 - \delta\), for all \(n\), \(\theta_*\) lies in*

\[
C_n = \left\{ \theta \in \mathbb{R}^d : \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta \rangle)^2 \leq 1 + 2B_n + 32R^2 \ln \left( \frac{R \sqrt{8 + \sqrt{1 + B_n}}}{\delta} \right) \right\}
\]
Proof Sketch

Algebra: With probability 1, due to the regret bound $B_n$,

$$\sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2 \leq B_n + 2 \sum_{t=1}^{n} \eta_t (\hat{Y}_t - \langle X_t, \theta_* \rangle) .$$  \hspace{2cm} (1)
Proof Sketch

Algebra: With probability 1, due to the regret bound \( B_n \),
\[
\sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2 \leq B_n + 2 \sum_{t=1}^{n} \eta_t (\hat{Y}_t - \langle X_t, \theta_* \rangle) .
\]

\((M_n)_{n=1}^{\infty}\) is a martingale. Using the same argument as in [AYPS11], we get that w.p. \( 1 - \delta \), for all \( n \geq 0 \),
\[
|M_n| \leq R \sqrt{2 \left( 1 + \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2 \right)} \times \ln \left( \frac{\sqrt{1 + \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2}}{\delta} \right).
\]
Proof Sketch

Algebra: With probability 1, due to the regret bound $B_n$,

$$\sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2 \leq B_n + 2 \sum_{t=1}^{n} \eta_t (\hat{Y}_t - \langle X_t, \theta_* \rangle) . \quad (1)$$

$(M_n)_{n=1}^\infty$ is a martingale. Using the same argument as in [AYPS11], we get that w.p. $1 - \delta$, for all $n \geq 0$,

$$|M_n| \leq R \sqrt{2 \left(1 + \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2 \right)} \times \ln \left(\sqrt{1 + \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta_* \rangle)^2} \delta \right).$$

Combine with (1) and solve the inequality.
Application to Sparse Linear Prediction

Theorem ([Ger11])

For any $\theta$ such that $\|\theta\|_{\infty} \leq 1$ and $\|\theta\|_0 \leq p$, the regret of $\text{SEQSEW}$ is bounded by

$$\rho_n(\theta) \leq B_n = O(p \log(nd)) .$$
Application to Sparse Linear Prediction

Theorem ([Ger11])

For any $\theta$ such that $\|\theta\|_\infty \leq 1$ and $\|\theta\|_0 \leq p$, the regret of SEQSEW is bounded by

$$\rho_n(\theta) \leq B_n = O(p \log(nd))$$.

Corollary

$\exists A > 0$ s.t. with probability at least $1 - \delta$, for all $n$, $\theta_*$ lies in

$$C_n = \left\{ \theta \in \mathbb{R}^d : \sum_{t=1}^{n} (\hat{Y}_t - \langle X_t, \theta \rangle)^2 \leq 1 + 2Ap \log(nd) + 32R^2 \ln \left( \frac{R\sqrt{8} + \sqrt{1Ap \log(nd)}}{\delta} \right) \right\}.$$
Application to Linear Bandits

Encouragement: Gittin’s mistake
Stamina: András’ theory of hiking
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$. 
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$

- Receive convex set $D_t \subset \mathbb{R}^d$
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$

- Receive convex set $D_t \subset \mathbb{R}^d$
- Choose an action $X_t \in D_t$
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$

- Receive convex set $D_t \subset \mathbb{R}^d$
- Choose an action $X_t \in D_t$
- Receive a reward

$$Y_t = \langle X_t, \theta_* \rangle + \eta_t,$$

where $\eta_t$ is conditionally on the past $R$-sub-Gaussian.
Linear Bandits

Unknown, but fixed weight vector $\theta_* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$

- Receive convex set $D_t \subset \mathbb{R}^d$
- Choose an action $X_t \in D_t$
- Receive a reward

$$Y_t = \langle X_t, \theta_* \rangle + \eta_t,$$

where $\eta_t$ is conditionally on the past $R$-sub-Gaussian.

Goal: Maximize total reward.
Linear Bandits

Unknown, but fixed weight vector $\theta^* \in \mathbb{R}^d$.

In round $t = 1, 2, \ldots$

- Receive convex set $D_t \subset \mathbb{R}^d$
- Choose an action $X_t \in D_t$
- Receive a reward

$$Y_t = \langle X_t, \theta^* \rangle + \eta_t,$$

where $\eta_t$ is conditionally on the past $R$-sub-Gaussian.

**Goal:** Maximize total reward.

Sparse bandits: $\theta^*$ is sparse.
Motivation

- multi-armed bandits, with infinitely many arms
Motivation

- multi-armed bandits, with infinitely many arms
- dependency between the rewards of arms (linear structure)
Motivation

- multi-armed bandits, with infinitely many arms
- dependency between the rewards of arms (linear structure)
- applications: web-advertisement, network routing, ...
Motivation

- multi-armed bandits, with infinitely many arms
- dependency between the rewards of arms (linear structure)
- applications: web-advertisement, network routing, ...
  - action = arm = ad = feature vector
Motivation

- multi-armed bandits, with infinitely many arms
- dependency between the rewards of arms (linear structure)
- applications: web-advertisement, network routing, ...
  - action = arm = ad = feature vector
  - reward = click
Motivation

- multi-armed bandits, with infinitely many arms
- dependency between the rewards of arms (linear structure)
- applications: web-advertisement, network routing, ...

  - action = arm = ad = feature vector
  - reward = click
  - sparsity: high-dimensional parameter spaces/feature vectors
Regret

- If we knew $\theta_*$, then in round $t$ we’d choose action

$$X_t^* = \arg\max_{x \in D_t} \langle x, \theta_* \rangle$$
Regret

- If we knew $\theta_*$, then in round $t$ we’d choose action

$$X^*_t = \arg\max_{x \in D_t} \langle x, \theta_* \rangle$$

- Our regret is how much less total reward we have incurred:

$$\text{Regret}_n = \sum_{t=1}^{n} \langle X^*_t, \theta_* \rangle - \sum_{t=1}^{n} \langle X_t, \theta_* \rangle$$
Regret

- If we knew $\theta_*$, then in round $t$ we’d choose action

$$X_t^* = \arg\max_{x \in D_t} \langle x, \theta_* \rangle$$

- Our regret is how much less total reward we have incurred:

$$\text{Regret}_n = \sum_{t=1}^{n} \langle X_t^*, \theta_* \rangle - \sum_{t=1}^{n} \langle X_t, \theta_* \rangle$$

- We want $\text{Regret}_n / n \rightarrow 0$ as $n \rightarrow \infty$
Optimism in the Face of Uncertainty

- Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta^* \in C_t$ with high probability.
Optimism in the Face of Uncertainty

- Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta_* \in C_t$ with high probability.

- **OFUL Algorithm:** In round $t$, choose

  $$ (X_t, \tilde{\theta}_t) = \arg\max_{(x, \theta) \in D_t \times C_{t-1}} \langle X_t, \theta \rangle $$
Optimism in the Face of Uncertainty

- Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta_* \in C_t$ with high probability.
- **OFUL Algorithm**: In round $t$, choose

$$ (X_t, \tilde{\theta}_t) = \arg \max_{(x, \theta) \in D_t \times C_{t-1}} \langle X_t, \theta_t \rangle $$

- $\tilde{\theta}_t$ is an “optimistic” estimate of $\theta_*$. 

The “OFU” principle goes back to at least [LR85].
Algorithm UCB1 of [Aue03] is a special case.
Widely applied, very active in machine learning (internet giants).
Optimism in the Face of Uncertainty

- Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta_* \in C_t$ with high probability.
- **OFUL Algorithm**: In round $t$, choose

  $$ (X_t, \tilde{\theta}_t) = \operatorname{argmax}_{(x, \theta) \in D_t \times C_{t-1}} \langle X_t, \theta_t \rangle $$

- $\tilde{\theta}_t$ is an “optimistic” estimate of $\theta_*$
- The “OFU” principle goes back to at least [LR85]
Optimism in the Face of Uncertainty

- Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta_* \in C_t$ with high probability.
- **OFUL Algorithm**: In round $t$, choose

$$ (X_t, \tilde{\theta}_t) = \argmax_{(x, \theta) \in D_t \times C_{t-1}} \langle X_t, \theta_t \rangle $$

- $\tilde{\theta}_t$ is an “optimistic” estimate of $\theta_*$
- The “OFU” principle goes back to at least [LR85]
- Algorithm UCB1 of [Aue03] is a special case
Maintain a confidence set $C_t \subseteq \mathbb{R}^d$ such that $\theta_* \in C_t$ with high probability.

**OFUL Algorithm**: In round $t$, choose

$$(X_t, \tilde{\theta}_t) = \arg\max_{(x,\theta) \in D_t \times C_{t-1}} \langle X_t, \theta_t \rangle$$

$\tilde{\theta}_t$ is an “optimistic” estimate of $\theta_*$. 

The “OFU” principle goes back to at least [LR85]. 

Algorithm UCB1 of [Aue03] is a special case.

Widely applied, very active in machine learning (internet giants)
Confidence Set $C_t$

- $\hat{\theta}_t$: center of $C_t$ (e.g., least-squares estimate)
- $\theta_*$ lies somewhere in $C_t$ w.h.p.
- Next optimistic estimate, $\tilde{\theta}_{t+1}$, is on the boundary of $C_t$
Regret of OFUL with Ridge-Regressor Estimator

Theorem ([DHK08, AYPS11])

If \( \| \theta_* \|_2 \leq 1 \) and \( D_t \)'s are subsets of the unit 2-ball then with probability at least \( 1 - \delta \)

\[
\text{Regret}_n \leq O(Rd\sqrt{n} \cdot \text{polylog}(n, d, 1/\delta))
\]
Empirical Results

OFUL using the confidence set of [AYPS11]
Empirical Results

<table>
<thead>
<tr>
<th>Time</th>
<th>Regret</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

OFUL using the confidence set of [AYPS11]
OFUL using the confidence set of [DHK08]

25 / 31
Empirical Results

OFUL using the confidence set of [DHK08]
Empirical Results

OFUL using the confidence set of [DHK08] – “Old bound”

<table>
<thead>
<tr>
<th>Time</th>
<th>New bound</th>
<th>Old bound</th>
<th>New bound with rare switching</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Empirical Results

OFUL using the confidence set of [DHK08] – “Old bound”

![Graph showing regret over time for different bounds and with rare switching](image-url)
OFUL with Online-to-Confidence-Set Conversion

Theorem
If $|\langle x, \theta^* \rangle| \leq 1$ for all $x \in D_t$ and all $t$ then with probability at least $1 - \delta$, for all $n$, the regret of Optimistic Algorithm is

$$\text{Regret}_n \leq O\left(\sqrt{dnB_n} \cdot \text{polylog}(n, d, 1/\delta, B_n)\right).$$
OFUL Combined with SeqSEW

Theorem ([AYPS12])

Suppose $\|\theta_\ast\|_2 \leq 1$ and $\|\theta_\ast\|_0 \leq p$. Via the conversion, OFUL has regret

$$O(R\sqrt{pdn} \cdot \text{polylog}(n, d, 1/\delta))$$
OFUL Combined with SeqSEW

Theorem ([AYPS12])

Suppose \( \|\theta_*\|_2 \leq 1 \) and \( \|\theta_*\|_0 \leq p \). Via the conversion, OFUL has regret

\[
O(R \sqrt{pdn} \cdot \text{polylog}(n, d, 1/\delta))
\]

... which is better than \( O(Rd \sqrt{n} \cdot \text{polylog}(n, d, 1/\delta)) \).
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm

\[ d = 200, \quad \mathcal{D} = \mathcal{D}_{co}(A_1, \ldots, A_{200}) \]

\[ A_i \in \{-1, +1\}^{200} \]

\[ p = 10 \]

\[ \theta^*, i \in \{0, 0.1\} \]

\[ \eta_t \sim N(0, 0.1^2) \]
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm
OFUL-LS: OFUL with ridge regression
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm
OFUL-LS: OFUL with ridge regression
\( d = 200: D_t = D = \text{co}(A_1, \ldots, A_{200}), A_i \in \{-1, +1\}^{200} \)
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm
OFUL-LS: OFUL with ridge regression

\( d = 200: \ D_t = D = co(A_1, \ldots, A_{200}), \ A_i \in \{-1, +1\}^{200} \)

\( p = 10 \) with \( \theta_{*,i} \in \{0, 0.1\}, \)
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm
OFUL-LS: OFUL with ridge regression

\[ d = 200: D_t = D = \text{co}(A_1, \ldots, A_{200}), A_i \in \{-1, +1\}^{200} \]

\[ p = 10 \text{ with } \theta_{*,i} \in \{0, 0.1\}, \eta_t \sim \mathcal{N}(0, 0.1^2) \]
Results on Sparse Bandits

OFUL-EG: OFUL with the EG algorithm
OFUL-LS: OFUL with ridge regression

\[ d = 200: \ D_t = D = \text{co}(A_1, \ldots, A_{200}), \ A_i \in \{-1, +1\}^{200} \]

\[ p = 10 \text{ with } \theta_{*,i} \in \{0, 0.1\}, \ \eta_t \sim \mathcal{N}(0, 0.1^2) \]
Summary
Summary

- Online-to-Confidence-Set Conversion
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
- Top results on the recent Yahoo article recommendation competition (with no tuning, RR)
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
- Top results on the recent Yahoo article recommendation competition (with no tuning, RR)

Open Problems
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
- Top results on the recent Yahoo article recommendation competition (with no tuning, RR)

Open Problems

- Confidence sets for batch algorithms e.g. offline LASSO.
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
- Top results on the recent Yahoo article recommendation competition (with no tuning, RR)

Open Problems

- Confidence sets for batch algorithms e.g. offline LASSO.
- Adaptive bandit algorithm that doesn’t need $p$ upfront.
Summary

- Online-to-Confidence-Set Conversion
- First confidence set for sparse linear prediction
- Application to bandits
- Top results on the recent Yahoo article recommendation competition (with no tuning, RR)

Open Problems

- Confidence sets for batch algorithms e.g. offline LASSO.
- Adaptive bandit algorithm that doesn’t need $p$ upfront.
- When $D_t$ has few corners, LS wins. Best of both worlds?
Lesson about health
Lesson about health
Wonderful memories
Lesson about health
Wonderful memories
THANK YOU!


[Zin03] Martin Zinkevich. Online convex programming and generalized infinitesimal