Lesson 7 - Basic of angular kinematics

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This lesson contains 10 screens teaching text, and 6 videos. This lesson requires approximately 2 - 4 hours of study but can vary depending on the student.

Introduction to angular kinematics

Angular and linear kinematics give scientists important tools to describe and understand exactly how movement occur. **Angular kinematics** is the description of angular motion. Angular kinematics is particularly appropriate for the study of human movement because the motion of most human joints can be described using one, two or three rotations. Generally, the human movement is a complex motion, which contains linear and angular motion components. As you remember, the difference between the translational and the rotational movement was defined in the first lesson. The **linear motion** (translation) may described as a motion along a straight line (rectilinear motion) or a curve line (curvilinear motion).

An object will always rotate—that is, spin or move in a circle—around an axis of rotation. When a body moves around a central imaginary line the motion is known as an **angular rotation**. The axis of rotation is oriented perpendicular to the plane in which the rotation occurs (2, 3).

Volitional human movements are built from rotational movements of the joints when a body part (movement segment) moving relative to another part of the body. In other words, articular motions in the human body are basically angular rotations

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when the moving segment swings around the joint axis. For example, during the flexing and extending, every point on the lower leg moved in a circular path about knee joint. This is a clear swing movement in that plane where the movement itself occurs (2).

There are combined swing movements where the simple swing motion is combined with an additional rotation around the longitudinal axis of the movement segment.

Uniform circular motion can be described as the motion of an object in a circle at a constant speed.

Let's see the **properties of the uniform circular motion** (2, 4):

- As an object moves in a circle, it is constantly changing its direction and moving tangent to the circle. Since the direction of the velocity vector is the same as the direction of the object's motion, the velocity vector is directed tangent to the circle as well. An object moving in a circle is accelerating.
- Accelerating objects are objects which are changing their velocity - either the speed or the direction. An object undergoing uniform circular motion is moving with a constant speed. Nonetheless, it is accelerating due to its change in direction. The direction of the acceleration is inwards.
- The final motion characteristic for an object undergoing uniform circular motion is the centripetal force. The centripetal force acting upon such an object is directed towards the center of the circle. Without such an inward force, an object would continue in a straight line, never deviating from its direction. Yet, with the inward net force directed perpendicular to the velocity vector, the object is always changing its direction and undergoing an inward acceleration.

Please now watch the following video:

https://www.youtube.com/watch?v=h-85rpR-mRM

Kinesiological concepts of angular kinematics

Angular kinematic quantities

Angular displacement (θ: theta) is the vector quantity representing the change in angular position of an object. It is the angle formed between some initial and final angle and noting direction. Angular displacement measured with a goniometer is one way to measure the range of motion of joints (3).

We could also measure the distance around the circle in terms of position, but angle is handy if we are more concerned about the details of the rotation than the translation. Since angle and position give us the same information, we can convert between the two if we know the radius of the circle. An object that rotates through an angle θ will move along an arc of length:

$s = r\theta$

Here s is the displacement, r is the radius of the circle the object travels along (1).

An angle of rotation is just a way to measure how much exactly an object has spun, or how far around the circle an object has gotten. Angles are commonly measured in degrees or radians. A **degree of arc** is a measurement of a plane angle, defined so that a full rotation is 360 degrees.

Please now watch the following video:

https://www.youtube.com/watch?v=JJZDgjFRPXo

The SI unit of angular measure is the **radian.** One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle. It follows that the magnitude in radians of one complete revolution (360 degrees) is the length of the entire circumference divided by the radius, or 2πr / r, or 2π. Thus 2π radians is equal to 360 degrees, meaning that one radian is equal to 180/π degrees (1).

The relation 2πrad = 360°can be derived using the formula for arc length. Taking the formula for arc length, or $l_{arc} = 2\pi r\,(\theta/360°)$. Assuming a unit circle; the radius is therefore one. Knowing that the definition of radian is the measure of an angle that subtends an arc of a length equal to the radius of the circle, we know that 1 = 2π (1 rad/ 360°). This can be further simplified to 1 = 2πrad/ 360°. Multiplying both sides by 360° gives 360° = 2πrad, one radian is just under 57.3 degrees (3).
Please now watch the following video:
https://www.youtube.com/watch?v=cgPYLJ-s5II

If we measure angle, we need to know how angle is changing as objects move. For an object rotating about an axis, every point on the object has the same angular velocity. Angular velocity is measured in rad/s, the same way translational velocity is measured in m/s, and represents the change of angle in a given time.

Angular velocity (ω: omega) is the rate of change of angular position and is usually expressed in degrees per second or radians per second. It can be described by the relationship: $\omega_{average} = \Delta\theta/\Delta t$.

The angular velocities of joints are particularly relevant in biomechanics, because they represent the angular speed of anatomical motions. If relative angles are calculated between anatomical segments, the angular velocities calculated can represent the speed of flexion/extension and other anatomical rotations (3).

Angular acceleration (α: alpha**)** is a vector and quantity and is the rate of change of angular velocity, measured in rad/s2, just as translational acceleration is measured in m/s2 (2, 3).
Please now watch the following video:
https://www.youtube.com/watch?v=1Lx4BbTsNCA

According to Newton's second law, the greater a body's mass, the greater its resistance to linear acceleration. Resistance to angular acceleration is also a function of a body's mass. Inertia is a body's tendency to resist acceleration.

Please brush up your knowledge with the following video about mass and inertia:
https://www.youtube.com/watch?v=YbWjx3LUc0U

Angular kinetics

As you remember, **torque** is the ability of a force to produce rotation around an axis. In rotational motion, torque is required to produce an angular acceleration of an object. Resistance to angular acceleration depends not only on the mass possessed by an object but also on the distribution of that mass in relation to the axis of rotation. The amount of torque required to produce an angular acceleration depends on the distribution of the mass of the object. The moment of inertia is a value that describes the distribution. **Movement of inertia** represents the resistance to angular acceleration, based on both mass and the distance the mass is distributed from the axis of rotation (2).

$I = mr^2$ moment of inertia ($kg \cdot m^2$)

where:

m = mass

r = radius

It can be found by integrating over the mass of all parts of the object and their distances to the center of rotation, but it is also possible to look up the moments of inertia for common shapes. The torque on a given axis is the product of the moment of inertia and the angular acceleration.

If we make an analogy between translational and rotational motion, then this relation between torque and angular acceleration is analogous to the Newton's Second Law. Namely, taking torque to be analogous to force, moment of inertia analogous to mass, and angular acceleration analogous to acceleration, then we have an equation very much like the Second Law (2).

The **angular momentum** (L) of a rigid object is defined as the product of the moment of inertia and the angular velocity. Angular momentum is a vector quantity. It is an important quantity in physics because it is a conserved quantity—the total angular momentum of a system remains constant unless acted on by an external torque (2, 4).

L = mvr

where:

L = angular momentum

m = mass

v = velocity

r = radius

OR

$L = I\omega$

where:

L = angular momentum

I = moment of inertia

ω = angular velocity

The SI unit for angular momentum is kg m2/s.

Please now watch the following video:
https://www.youtube.com/watch?v=iWSu6U0Ujs8

For example, angular momentum is useful in analyzing the spins of figure skaters. According to the law of the conservation of angular momentum, the angular momentum of an object will not change unless external torque is applied to the object. When spinning, a figure skater will bring their limbs closer to his or her body in order to increase their angular velocity and rotate faster. In this case the moment of inertia (I in the equation directly above) is decreased by bringing the arms/limbs closer to the body (and the angular momentum will stay the same according to the law of the conservation of angular momentum), the angular velocity must increase.

Please now watch the following video about the relationship between spinning of figure skater and angular momentum:
https://www.youtube.com/watch?v=FmnkQ2ytlO8

Study questions:

TRUE/FALSE questions

Read each statement below carefully. Choose T if you think a statement is TRUE. Choose F if you think the statement is FALSE.

1. Torque is the ability of a force to produce rotation around an axis.
   T or F
2. The centripetal force acting upon such an object is directed towards the tangent of the circle.
   T or F
3. When a body moves around a central imaginary line the motion is known as a linear motion.
   T or F
4. Uniform circular motion can be described as the motion of an object in a circle at a variable speed.
   T or F
5. Movement of inertia represents the resistance to angular acceleration, based on both angular velocity and the angular distance from the axis of rotation.
   T or F
6. When spinning, a figure skater will bring his or her arms closer to his or her body in order to decrease their angular velocity and rotate slower.
   T or F

Multiple-choice questions

Read each question and answer choice carefully and choose the ONE best answer.

1. In system international (SI), the angular acceleration is measured in:
   A. degree per second
   B. degree per second square
   C. radian per second
   D. radian per second square

2. The rate at which the angular velocity changes with time is called:
   A. angular displacement

B. angular velocity
C. angular acceleration
D. torque

3. Angular displacement is defined by the units:

A. degree
B. radian
C. kg
D. both A and B

4. Resistance to angular acceleration depends on:

A. the mass of an object
B. the rate of change of angular velocity
C. the distribution of the mass in relation to the axis of rotation
D. both A and C

References

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