Quantum theory of light-matter interaction: Fundamentals

Lecture 12
Coherent transient phenomena

Mihály Benedict

University of Szeged, Dept. of Theoretical Physics, 2014

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BEFEKTETÉS A JÖVŐBE

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Introduction

In this Lecture we repeat first the interaction of the two-level system with an external (nearly) resonant field in the Bloch-vector formalism, due to F. Bloch, Nobel prize winner for inventing NMR, which is actually the Rabi method applied to nuclear spins in solids.
We consider then two coherent effects: free induction decay and photon echo.
Then we combine coherent interaction with field propagation in an extended pencil shaped medium. Pioneering work in this field was done by S. McCall and E. Hahn (the inventor of spin echo) who developed the theory and performed the first experiments in 1967 on coherent field propagation effects.

Maxwell- Bloch equations

The optical Bloch equations for two-level atoms with relaxation terms have been obtained previously for two-level atoms.:

$$\dot{U} = -\Delta V - U/T_2, \tag{U}$$

$$\dot{V} = \Delta U + \Omega_r W - V/T_2, \tag{V}$$

$$\dot{W} = -\Omega_r V - (W+1)/T_1. \tag{W}$$

These are to be completed with the equation for the slowly-varying field envelope $E_0(z,t)$ determining $\Omega_r = dE_0(t)/\hbar$, as well as with one for the slowly-varying phase of the field, $\phi(z,t)$:

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right) E_0(z,t) = \frac{k}{2\epsilon_0}\mathcal{N}d \int g(\Delta)V(\Delta)d\Delta, \tag{Amp}$$

$$E_0 \left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right) \phi(z,t) = -\frac{k}{2\epsilon_0}\mathcal{N}d \int g(\Delta)U(\Delta)d\Delta. \tag{Phase}$$

Comparison with semiclassical laser theory

The second set of equations are the consequences of Maxwell equations for the field strength in the SVEA. Therefore the above set of equations is frequently called the system of Maxwell-Bloch equations, which are to be solved self-consistently in the sense explained in Lecture 9.
These are essentially the same equations as those used in semiclassical laser theory. We use however a different notation here, because in dealing with coherent interactions this is the customary one.
In laser theory emphasis was on mode decomposition and on the stationary regime, *here we will mainly (but not exclusively) consider processes, during which atomic coherence is maintained, which is the time scale shorter than the relaxation times*: T_1 and T_2.
Note also that in the source term of the equations (Amp) and (Phase) for the field ("Maxwell part") the average over the inhomogeneous broadening $g(\Delta)$ is explicitly included.

Bloch equations without propagation

In the next few slides we study the Bloch equations without propagation effects, so the field strength $\Omega_r(t)$ is taken at a given position of a group of atoms extending much less than the wavelength of the field.

Bloch vector and Bloch sphere

The optical Bloch equations for two-level atoms, in a certain spatial point in the sample, without the relaxation terms are given by

$$\dot{U} = -\Delta V, \tag{U0}$$

$$\dot{V} = \Delta U + \Omega_r W, \tag{V0}$$

$$\dot{W} = -\Omega_r V. \tag{W0}$$

This is the fully coherent case with $T_1 = \infty$, $T_2 = \infty$, conserving

$$U^2 + V^2 + W^2 = const = 1$$

(from the initial condition). The vector $\mathbf{U} = (U, V, W)$ moves along the surface of a sphere called the *Bloch sphere*.
Specific points: $W = -1$: ground state, $W = 1$: fully excited state.

Problem: Invert the definitions $U = b_1 b_2^* + b_1^* b_2$, $V = i(b_1^* b_2 - b_1 b_2^*)$, $W = |b_2|^2 - |b_1|^2$ as given in the end of Lecture 8, where b_1 and b_2 are the modified interaction-picture amplitudes of the levels introduced there. Show that this is possible only up to a common phase factor of the amplitudes.

Resonant coherent solution

Fully coherent case: $T_1 = \infty$, $T_2 = \infty$; and on exact resonance $\Delta = 0$
$U(t) \equiv 0$, $V^2(t) + W^2(t) = 1$, implying the following parametrization:

$$V = -\sin\Theta, \quad W = -\cos\Theta, \quad \text{for} \quad W(0) = -1,$$
$$V = \sin\Theta, \quad W = \cos\Theta, \quad \text{for} \quad W(0) = 1,$$

the first used for atoms initially in the ground state, while the second for atoms initially in their excited state

Considering the $W(0) = -1$ case:

$$\dot{V} = \Omega_r W = -\dot{\Theta}\cos\Theta,$$
$$\dot{W} = -\Omega_r V = -\dot{\Theta}\sin\Theta.$$

Which means that $\dot{\Theta} = \Omega_r(t)$, and

$$\Theta = \int_0^t \Omega_r(t')dt',$$

Pulse angle and Rabi oscillations

The expression

$$\Theta = \int_0^t \Omega_r(t')dt',$$

is called the *Bloch angle* and

$$\mathcal{A} := \Theta(\infty)$$

is the *pulse area*. For a given $\Omega_r(t)$, the solution of the Bloch equations is

$$V(t) = -\sin \int_0^t \Omega_r(t')dt', \qquad W = -\cos \int_0^t \Omega_r(t')dt'$$

for the ground state at $t = 0$, and with reversed sign for the excited case. The Bloch vector turns around the U axis by an angle $\Theta = \int_0^t \Omega_r(t')dt'$. These are the Rabi oscillations in their cleanest form.

Rabi solution

Fully coherent case $T_1 = \infty$, $T_2 = \infty$; out of exact resonance: $\Delta \neq 0$. Solution, which can be obtained directly by solving the system :

$$\dot{U} = -\Delta V, \quad \dot{V} = \Delta U + \Omega_r W, \quad \dot{W} = -\Omega_r V,$$

or rewriting the Rabi solution for b_2 and b_1 of Lecture 8 in terms of U, V and W. For the initial conditions $U(0) = V(0) = 0, W(0) = -1$, this gives with $\Omega = \sqrt{\Omega_r^2 + \Delta^2}$:

$$U(t, \Delta) = \frac{\Delta\Omega_r}{\Omega^2}(1 - \cos\Omega t),$$

$$V(t, \Delta) = -\frac{\Omega_r}{\Omega}\sin\Omega t,$$

$$W(t, \Delta) = -1 + \frac{\Omega_r^2}{\Omega^2}(1 - \cos\Omega t).$$

Free induction decay

Assume that the atoms are first excited by a short pulse of area Θ_0 such that its bandwidth is sufficiently large, and all the atoms with different Δ's can be treated as resonant ones. After this the system evolves freely ($\Omega_r = 0$) from

$$U(0,\Delta) = 0, \quad V(0,\Delta) = -\sin\Theta_0, \quad W(0,\Delta) = -\cos\Theta_0,$$

and we assume that $T_1 = \infty$, $T_2 < \infty$.
Then according to the Bloch equations, an individual atom with detuning Δ evolves according to

$$U(t,\Delta) - iV(t,\Delta) = i\sin\Theta_0 e^{-(1/T_2 + i\Delta)t}, \qquad W = W(0).$$

Problem: Prove by substitution the validity of this solution.
Assume for simplicity $1/T_2 = 0$, *and show that an atom with detuning* Δ *precesses in the UV plane with an angular velocity of* Δ.

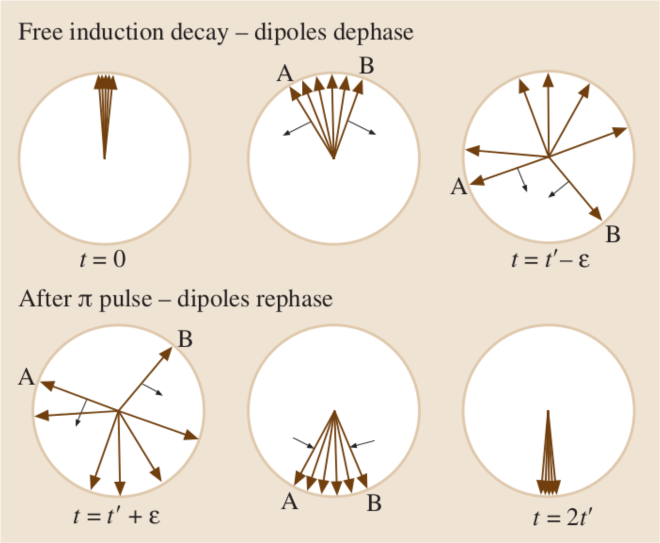
Free induction decay II

As different atoms have different detunings Δ, an oriented collection of Bloch vectors, all pointing in the $\mathbf{U}(0)$ direction initially, suffer a different phase shift Δt and the average value of V, i.e. the macroscopic polarization becomes *zero in a short time.*
This effect is called free induction decay (FID).
Assuming a Gaussian Doppler $g(\Delta)$ line this is seen mathematically from:

$$P(t) = \mathcal{N}d \int \frac{T^*}{\sqrt{2\pi}} \exp(-{T^*}^2 \Delta^2/2) \left[U(t,\Delta) - iV(t,\Delta) \right] d\Delta =$$
$$= Nd \sin \Theta_0 e^{-t^2/2T^{*2}}.$$

For a typical room temperature gas a visible transition has a width $1/(2\pi T^*) = 1.5\,\mathrm{GHz}$, so that the time constant of this *dephasing* mechanism is $T^* = 10^{-10}\mathrm{s}$.

FID and photon echo



Photon echo

In order to understand this effect – discovered and demonstrated in 1966 as an analogue of the spin echo in NMR – we consider the situation where it shows up most notably.
A sharp pulse of area $\pi/2$ brings all atoms into the state $U(0,\Delta) = 0$, $V(0,\Delta) = -1$, $W(0,\Delta) = 0$, i.e., the Bloch vector becomes horizontal for all atoms, which is an equal-weight superposition of the ground and excited states. In terms of the b amplitudes: $b_1 = 1/\sqrt{2}, b_2 = -i/\sqrt{2}$
After the excitation is stopped, FID begins and during T^* it smears out the polarization, as atoms with different Δ move around with different angular velocities.

Photon echo II

Now after a time interval $t' \gg T^*$, but still within T_2, a sharp π pulse is applied to the system, that rotates all Bloch vectors around the U axis by π, resulting in $V(t' + \epsilon, \Delta) = -V(t' - \epsilon, \Delta)$, leaving the other components unchanged. This puts the slowly-precessing vectors ahead of the more rapid ones. Now FID continues with the same angular velocities Δ for the individual atoms as before. As a consequence, the Bloch vectors begin to gather, and to rephase again, so that at $2t'$ they build up again the macroscopic polarization. This shows up in the emission of an echo pulse of area $\pi/2$. The process can be followed, of course, mathematically by the sequence of the appropriate transformations of the b_i amplitudes, or the components of $\mathbf{U}$.

Coherent transients with propagation: Introduction

The propagation equation for the slowly-varying amplitude becomes after multiplying by $d/\hbar$

$$\frac{\partial \Omega_r}{\partial z} + \frac{1}{c}\frac{\partial \Omega_r}{\partial t} = \frac{k}{2\epsilon}\frac{\mathcal{N} d^2}{\hbar}\int g(\Delta) V(\Delta) d\Delta.$$

We shall assume here that the phase is constant, so we exclude the equation for ϕ.
On a time scale less than T_2, T_1, and for atoms in exact resonance: $\Delta = 0$, the values of the *Bloch angle*, Θ, and V and W are given by

$$\Theta(z,t) = \int_{-\infty}^{t} \Omega_r(t', z) dt',$$

$$W(\Theta, \Delta = 0) = \pm \cos\Theta, \qquad V(\Theta, \Delta = 0) = \pm \sin\Theta.$$

Coherent propagation – continued

From now on the initial time instant is set to $-\infty$ (for convenience), and according to the usual convention the zero value of Θ is chosen to depend on the initial state:

$$\Theta(-\infty) = 0, \quad W(-\infty) = -1 \text{ attenuator,}$$
$$\Theta(-\infty) = 0, \quad W(-\infty) = 1 \text{ amplifier,}$$

as the medium initially in the ground state will absorb the radiation, while an inverted medium will amplify it. Here we assume that initially the value of W is homogeneous along the pencil-shaped sample.

Definition of the pulse area

As we have seen, a key concept in coherent resonant optics is the area of the pulse, which now depends on the spatial coordinate:

$$\mathcal{A}(z) := \Theta(z, t = \infty) = \int_{-\infty}^{\infty} \Omega_r(t', z)dt'.$$

$\mathcal{A}(z)$ obeys a simple differential equation. We will show that:

$$\frac{d\mathcal{A}(z)}{dz} = \pm\frac{\alpha_B}{2}\sin\mathcal{A}(z)$$

with an appropriate constant α_B, which is the small signal absorption (-) (or gain +) coefficient for the inhomogeneously broadened line. This is the so called *area theorem* obtained first by S. McCall and E. Hahn in 1967 for coherent pulse propagation.

Derivation of the area theorem I

In order to prove the area theorem, integrate the propagation equation with respect to t, and note that the time integral of the term $\frac{\partial \Omega_r}{\partial t}$ vanishes due to $\Omega_r(z, t = \pm\infty) = 0$.

$$\int_{-\infty}^{\infty} \frac{\partial \Omega_r(z,t)}{\partial z} dt = \frac{d\mathcal{A}(z)}{dz} = \frac{k}{2\epsilon_0} \frac{\mathcal{N} d^2}{\hbar} \int_{-\infty}^{\infty} dt d\Delta \, g(\Delta) V(z,t,\Delta).$$

We shall substitute here V from the negative of the imaginary part of the time integral of the Bloch equation for $\dot{U} - i\dot{V}$ giving:

$$V(z,t,\Delta) = \operatorname{Im}\left[i e^{-i\Delta t} \int_{-\infty}^{t} dt' \Omega_r(z,t') W(z,t',\Delta) e^{i\Delta t'} \right].$$

Derivation of the area theorem II

As everything except for $ie^{i\Delta(t'-t)}$ is real here, we can write

$$\frac{d\mathcal{A}(z)}{dz} = \mathrm{Im}\left[i\frac{k}{2\epsilon_0}\frac{\mathcal{N}d^2}{\hbar}\int_{-\infty}^{\infty} dt \int d\Delta\, g(\Delta)\int_{-\infty}^{t} dt'\Omega_r(z,t')W(z,t',\Delta)e^{i\Delta(t'-t)}\right]$$

Interchange the integration with respect to t and t':

$$\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' = \int_{-\infty}^{\infty} dt' \int_{t'}^{\infty} dt =$$

substitute $s = t - t'$ giving $\int_{t'}^{\infty} dt e^{i\Delta(t'-t)} = \int_{0}^{\infty} e^{-i\Delta s} ds = \pi\delta(\Delta) - i\,\mathrm{Pr}\,\frac{1}{\Delta}$.

Derivation of the area theorem III

Performing the integration over Δ by the Dirac delta $\delta(\Delta)$

$$\frac{d\mathcal{A}(z)}{dz} = g(0)\frac{k}{2\epsilon_0}\frac{\mathcal{N}d^2\pi}{\hbar}\int\limits_{-\infty}^{\infty} dt'\Omega_r(z,t')W(z,t',\Delta=0)$$

$$= \pm\frac{\alpha_B}{2}\int\limits_{-\infty}^{\infty} dt'\dot{\Theta}(z,t)\cos\Theta(z,t) = \mp\frac{\alpha_B}{2}\sin\Theta(z,\infty) = \pm\frac{\alpha_B}{2}\sin\mathcal{A}(z),$$

where

$$\alpha_B = \pi g(0)\frac{k}{\epsilon_0}\frac{\mathcal{N}d^2}{\hbar},$$

$$\frac{d\mathcal{A}(z)}{dz} = \mp\frac{\alpha_B}{2}\sin\mathcal{A}(z)$$

and the sign depends on the initial condition, - for the attenuator and + for the amplifier

The constant in the area theorem

α_B is just the *linear steady state* absorption coefficient in the Boguer-Lambert-Beer law for an inhomogeneously broadened attenuator, where for small, but non-negligible $1/T_2$ the conditions $W = -1$, $\dot{U} = \dot{V} = 0$ gives the stationary solution from the Bloch equations: $V(\Delta) = -\Omega_r T_2(1 + \Delta^2 T_2^2)^{-1}$.
Then in the right hand side of the propagation equation we get:

$$\int g(\Delta)V(\Delta)d\Delta = -\Omega_r \int g(\Delta)T_2(1 + \Delta^2 T_2^2)^{-1} d\Delta \approx -\pi g(0)\Omega_r,$$

because the inhomogeneous line $g(\Delta)$ is usually much broader than the homogeneous one determined by the factor $(1 + \Delta^2 T_2^2)^{-1}$. Then the normalized function $\frac{1}{T_2 \pi} \frac{1}{1/T_2^2 + \Delta^2}$ can be approximated in this respect by a Dirac $\delta(\Delta)$.
So the linear and steady state limit $\frac{\partial \Omega_r}{\partial t} = 0$ for an attenuator gives

$$\frac{\partial \Omega_r}{\partial z} = -\pi g(0) \frac{k}{2\epsilon_0} \frac{\mathcal{N} d^2}{\hbar} \Omega_r \Longrightarrow \Omega_r(z) = \Omega_r(0) e^{-\frac{\alpha_B}{2} z},$$

with the same α_B as in the area theorem.

Solution of the area theorem

The solution of

$$\frac{d\mathcal{A}(z)}{dz} = \mp\frac{\alpha_B}{2}\sin\mathcal{A}(z)$$

$$\mathcal{A}(z) = 2\arctan\{e^{\mp\frac{\alpha_B}{2}z}\tan\left(A_0/2\right)\},$$

where $\mathcal{A}_0$ is the pulse area at the entry face of the medium.

In the case of the attenuator ($-$ sign), we see that there is a critical area $\mathcal{A}_0 = \pi$, under which $\mathcal{A}(z)$ diminishes during the propagation, while for initial pulse areas $\pi < \mathcal{A}_0 < 3\pi$, the limit for large z is 2π. More generally, for initial areas $(2k-1)\pi < \mathcal{A}_0 < (2k+1)\pi, k = 1, 2\ldots$, the limit is $2k\pi$, an even multiple of π, depending on the value of $\mathcal{A}_0$.

For the amplifier, ($+$ sign), a small area pulse is amplified so that the limit is π, and more generally stable areas are $(2k+1)\pi$, with $k = 0, 1\ldots$

2π secanthyperbolic pulse and self-induced transparency

The coherent M-B equations have exact analytic solutions, even in the presence of inhomogeneous broadening. The simplest one for $W(z, t = -\infty) = -1$ is the "2π secanthyperbolic pulse" (of area 2π, hence its name):

$$\Omega_r = \frac{dE_0(z,t)}{\hbar} = \frac{2}{\tau}\text{sech}\frac{t - z/v}{\tau}.$$

The pulse duration, τ is arbitrary, but must obey $\tau \ll T_1, T_2$ to remain within the coherent regime. Its propagation speed is given by

$$v = c(1 + \alpha_B c\tau)^{-1}.$$

This group velocity can be slower than the speed of light by orders of magnitude if $\alpha_B c\tau \gg 1$. In fact, $v \sim 10^{-3}c$ has been demonstrated experimentally, the first example of "slow light".
McCall and Hahn called the effect corresponding to the transmission of the 2π sech pulse as *self-induced transparency (SIT)*.

Solitons

Any pulse of initial area $\pi < \mathcal{A}_0 < 2\pi$ can be shown to become a 2π sech pulse, after a sufficiently long propagation.
That is why the 2π sech pulse is the so called *one-soliton solution* of the inhomogeneously broadened Maxwell-Bloch equations, which became an important prototype of the so-called *integrable nonlinear systems*, and the discovery of McCall and Hahn contributed a lot to the *mathematical theory* of integrable systems and solitons, as explicit but complicated expressions for stable n-soliton solutions with n travelling *peaks* could be found theoretically.

McCall and Hahn performed detailed experiments on coherent pulse propagation in ruby at low temperatures (to supress homogeneous relaxation) confirming their theoretical findings, and their work was followed by several other studies.

Problems related to self-induced transparency

Calculate the FWHM of the 2π *sech pulse*

Show that the Bloch angle corresponding to the above solution is
$\Theta(z, t) = 4 \arctan\left[\exp(t - t_0)/\tau\right]$.

Find the corresponding expressions for V *and* W.

Consider the sharp line limit of the problem where $g(\Delta) = \delta(\Delta)$*, and introduce the new variables* $\xi = x$ *and* $\tau = t - x/c$*. Show that the M-B system with propagation can be transformed into the form:*

$$\frac{\partial^2 \Theta}{\partial \xi \partial \tau} = -\gamma \sin \Theta$$

known as the sine-Gordon equation

M-B system for the amplifier

We consider now in some more detail the coherent propagation in an initially inverted medium=a coherent amplifier.

The Maxwell-Bloch system with $T_1 = T_2 = \infty$, without inhomogeneous broadening:

$$\dot{U} - i\dot{V} = -i\Delta(U - iV) - i\Omega_r W,$$
$$\dot{W} = -\Omega_r V,$$
$$\frac{\partial \Omega_r}{\partial z} + \frac{1}{c}\frac{\partial \Omega_r}{\partial t} = \frac{k}{2\epsilon_0}\frac{\mathcal{N} d^2}{\hbar} V.$$

Initial conditions for an inverted medium $U = V = 0$, $W = 1$, and no field present: $\Omega_r = 0$, correspond to an unstable equilibrium state. Quantum fluctuations that are neglected in this semiclassical picture will start the spontaneous emission of the atoms. Quantum field theoretical considerations show that this brings about fluctuations of the dipole moment amplitude such that $\sqrt{U^2 + V^2} \sim N^{-1/2}$, where N is the total number of atoms in the sample.

Initial condition for the atoms

For simplicity, we have already omitted the equation for phase modulation, and in accordance with this, we set $U \equiv 0$, without loss of generality. Then $V = \sin\Theta, W = \cos\Theta$ gives $\dot{\Theta} = \Omega_r$.
The effect of quantum fluctuations can be simulated by a small initial "tipping" angle $\Theta_0 \sim N^{-1/2} \ll 1$, and the system starts from $V = \sin\Theta_0 \approx \Theta_0$ and $W = \cos\Theta_0 \lesssim 1$
In contrast to the attenuator, no explicit analytic solutions are known for this initial condition. We can however learn the physics by assuming a spatially homogeneous solution, i.e., setting $\frac{\partial \Omega_r}{\partial z} = 0$.
This leads to the pendulum-like equation:

$$\ddot{\Theta} = \beta \sin\Theta, \qquad \text{with } \beta = \frac{\omega_0}{2\epsilon_0}\frac{\mathcal{N} d^2}{\hbar},$$

where Θ is measured from the upper unstable position, and ω_0 is the transition frequency.

Energy release from the sample

Explicit periodic solution of this pendulum equation with $\Theta(0) = \Theta_0$ in terms of elliptic integrals are well known. The periodic solution lasting forever is however due to the spatially homogeneous ansatz, meaning also that the energy of the field and that of the atoms are exchanged periodically.
A physically more relevant model is if we include the effect of the release of the field at the boundary of the sample, so the equation for the time derivative of the field strength Ω_r is to be completed by a term $-\kappa\Omega_r = -\kappa\dot{\Theta}$, describing these losses. It is reasonable to put the damping constant to be equal to $\kappa = c/L$, the rate at which a photon of velocity c leaves the sample of length L. The equation is now

$$\ddot{\Theta} + \kappa\dot{\Theta} = \beta \sin\Theta.$$

Solution of the overdamped equation

In the strongly overdamped situation: $\ddot{\Theta} \ll \kappa\dot{\Theta}$, the solutions of $\kappa\dot{\Theta} = \beta \sin\Theta$ for the angle and the field strength are

$$\Theta = 2 \arctan[\exp(\beta t/\kappa) \tan(\Theta_0/2)],$$

$$\Omega_r = \dot{\Theta} = \frac{d}{\hbar} E_0 = \frac{1}{T_R} \operatorname{sech} \frac{t - t_d}{T_R}, \qquad \text{(SRAD)}$$

where the time constant of the process is

$$T_R := \frac{\kappa}{\beta} = 2\epsilon_0 \frac{\hbar c}{d^2 \omega_0 \mathcal{N} L}.$$

while the so called delay time is

$$t_d = T_R \ln(\Theta_0/2).$$

This is a secanthyperbolic pulse again, but this time of area π.

Problem: Check the validity of the above solution and prove the relation for the delay time t_d with Θ_0.

Time constant of collective emission

The effect when a system of many atoms emits spontaneously and coherently is called *superradiation*, in general, while the specific case of the pencil-shaped elongated system we considered above, is sometimes called *superfluorescence*.
This is *NOT* amplified spontaneous emission:
superfluorescence$\neq$ASE $\equiv$ *superluminescence* which is another incoherent process.
Compare the result for T_R with $\tau_0 = 3\pi\varepsilon_0 \frac{\hbar c^3}{d^2\omega_0^3}$, the spontaneous decay time of a single atom. Noting that $\omega_0/c = 2\pi/\lambda$

$$T_R = \tau_0 \frac{8\pi}{3} \frac{1}{\mathcal{N}\lambda^2 L}.$$

We see that T_R is smaller than τ_0 by the factor of $\sim \mathcal{N}\lambda^2 L/8$, where $\mathcal{N}\lambda^2 L$ is the number of atoms in the volume $\lambda^2 L$.

Superradiance

This means that

1. the collective spontaneous emission of atoms is *faster* than single-atom emission.
2. as by (SRAD) the amplitude E_0 is proportional to $1/T_R \sim \mathcal{N}\lambda^2 L$, the intensity is proportional to the *square* of the number of atoms in the volume $\lambda^2 L$.
3. the emitted pulse is delayed, and has a sharp peak, instead of the decaying exponential of a single-atom emission.

These effects have been first discussed by R. Dicke, who in 1954 developed a *microscopic* theory of cooperative spontaneous emission of N atoms, and called the effect *superradiance*. The three characteristics enumerated above appeared in his theory quite differently from the way we obtained them. Since then a very large number of theoretical, as well as experimental work investigated superradiance.

Coherent amplification and others

A related effect is *coherent amplification*, when the system is initially in the inverted state, and in addition an incident pulse falls in that is to be amplified when passing through the sample. The important features of this and many other details on coherent transients can be learnt from the book [1].

In addition, there are a number of important effects discovered more recently in connection with coherent transients in three-level systems etc., and we can only enumerate a few:

- Lasing without inversion
- Electromagnetically induced transparency
- Slowing down and stopping light in Bose-Einstein condensates

For more details on these we can only refer to the original literature.

Questions

1. What is the physical meaning of the relaxation times T_1 and T_2?
2. What can we say about the length of the Bloch vector without relaxation?
3. What do we mean by "free induction decay"?
4. What mechanisms appear in photon echo?
5. What kind of pulses appear in photon echo experiments?
6. What is the definition of pulse area?

Questions (continued)

7. What is the physical meaning of pulse area theorem?
8. What do we call "2π secanthyperbolic pulse"?
9. What kind of solitons appear in inhomogeneously broadened Maxwell-Bloch equations?
10. What is the pendulum equation?
11. What are the characteristics of superradiance?
12. What is the difference between amplified spontaneous emission and superradiance?

References

1. M. G. Benedict et. al., *Superradiance*, IOP (Bristol) (1996).
2. P. Meystre and M. Sargent, *Elements of Quantum Optics*, Spinger (Berlin, Heidelberg) (2007).