Quantum theory of light-matter interaction: Fundamentals

Lecture 11

Mechanical effects of light on atoms: laser cooling and trapping

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BEFEKTETÉS A JÖVŐBE

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Radiation pressure, basic quantum effects

Classical terms

Kepler: the tail of a comet always points away from the Sun. Maxwell: the momentum density of the electromagnetic radiation is given by $\mathbf{g} = \epsilon_0 \mathbf{E} \times \mathbf{B}$. Experimental observation of the radiation pressure: at the beginning of the 20th century[1,2].

[3]

Basic quantum effects

Matter: quantized energy levels, possibility of light absorption and emission. Light: quantized, the magnitude of the momentum of a photon is given by $\hbar k$. This momentum is gained/lost by the atom during an absorption/emission process (conservation of momentum).

[1] P. Lebedev, *Ann. Phys.*, **6**, 433 (1901).

[2] E. E. Nichols and G. F. Hull, *Phys. Rev.*, **17**, 26 (1903).

[3] Q1: Does this wheel rotate indeed because of radiation pressure?

Experimental applications

Diffraction of atoms

Electromagnetic standing waves mean a grating for material particle waves: the quantum mechanical wave function of the atomic center of mass is strongly modified by the presence of light.

Trapping

In contrast to to the Penning and the Paul traps, neutral atoms can also be trapped by light induced forces.

Cooling

Laser beams with appropriately chosen parameters can lead to atomic samples with narrow velocity distribution. These atoms are ideal for precise quantum mechanical experiments.

Semiclassical model

Although the notion of photon provides an easily interpretable physical picture, the basic effects can already be seen when we consider classical fields. This approach technically simplifies the model, but still allows the understanding of important applications.

Atomic Hamiltonian

The center-of-mass coordinate and momentum $(\mathbf{r}, \mathbf{p})$ are treated as quantum mechanical operators: $[r_i, p_j] = i\hbar\delta_{ij}$. The atomic Hamiltonian reads[4]:

$$H_a = \frac{p^2}{2m} + \frac{1}{2}\hbar\omega_0\sigma_z,$$

where near-resonant interaction is assumed (only two energy levels are taken into account from the internal degrees of freedom).

[4] Q2: What is the Hilbert-space on which this Hamiltonian acts?

Interaction

We assume dipole interaction $K = -DE(\mathbf{r}, t)$, where the field is written as $E(\mathbf{r}, t) = \mathcal{E}(\mathbf{r}) \cos(\omega t + \phi(\mathbf{r}))$. Using RWA, we have

$$K = \frac{1}{2}\hbar\Omega_r(\mathbf{r}) \left(e^{-i\phi(\mathbf{r})} |2\rangle\langle 1| \right) + \text{h.c.}$$

Light force

Analogously to Newton's law, and using the Ehrenfest theorem, the expectation value of the light force operator reads:

$$\langle \mathbf{F}(t) \rangle = \left\langle \frac{d\mathbf{p}}{dt} \right\rangle = \frac{i}{\hbar} \langle [H, \mathbf{p}] \rangle = \frac{i}{\hbar} \langle [K, \mathbf{p}] \rangle ,$$

where $H = H_a + K$. Note that although H_a commutes with $\mathbf{p}$[5], the phase $\phi(\mathbf{r})$ in K has rapid position dependence, and even the the Rabi frequency can depend on $\mathbf{r}$.

[5] Q3: How can we verify it?

Expectation value of the light force operator

By evaluating the commutator on the previous slide, we obtain:

$$\mathrm{Tr}\left[\mathbf{F}(\mathbf{r},t)\rho(\mathbf{r},t)\right] = -\frac{1}{2}\hbar\Omega_r(\mathbf{r})\left(U(\mathbf{r},t)\alpha(\mathbf{r},t) + V(\mathbf{r},t)\beta(\mathbf{r},t)\right),$$

where $\alpha = \nabla\Omega_r/\Omega_r$, $\beta = \nabla\phi$. The generalized Bloch-vector components are given by

$$\begin{pmatrix} U(\mathbf{r},t) \\ V(\mathbf{r},t) \\ W(\mathbf{r},t) \end{pmatrix} = \begin{pmatrix} 2\mathrm{Re}\left[\rho_{21}(\mathbf{r},t)\exp(-i\phi(\mathbf{r}))\right] \\ 2\mathrm{Im}\left[\rho_{21}(\mathbf{r},t)\exp(-i\phi(\mathbf{r}))\right] \\ \rho_{22}(\mathbf{r},t) - \rho_{11}(\mathbf{r},t) \end{pmatrix}.$$

The first term in $\mathrm{Tr}\left[\mathbf{F}\rho\right]$ above is nonzero only if the field amplitude is not constant, and it is proportional to the real part of ρ_{21}: it is related to dispersive effects (index of refraction). On the other hand, the second term takes absorption and amplification of light into account.

A steady-state solution for the Bloch-vector

Let us consider an atom initially at rest in the origin ($\mathbf{r} = 0$). The position (operator) dependent equations for the density matrix (or the Bloch vector) can be solved in this case. Assuming an excited-to-ground-state decay rate Γ, the resulting steady-state (st) solution can be used to calculate field-force operator expectation value $\mathcal{F}(\mathbf{r}) = \mathrm{Tr}\left[\mathbf{F}(\mathbf{r})\rho_{st}(\mathbf{r})\right]$. We obtain:

$$\mathcal{F}(\mathbf{r}) = \frac{1}{2}\hbar\Omega_r \left(V_{st}\beta + U_{st}\alpha\right) = \mathcal{F}_{diss}(\mathbf{r}) + \mathcal{F}_{react}(\mathbf{r}),$$

where Ω_r, α and β are to be evaluated at $\mathbf{r} = 0$. For a monochromatic plane wave, the "dissipative" term has a usual Lorentzian line shape, while, due to the lack of the intensity gradient, $\mathcal{F}_{react} = 0$. In a more general (steady-state) case, the expression for the "reactive" force reads:

$$\mathcal{F}_{react} = \frac{\hbar\Delta}{4}\left(\frac{\nabla{\Omega_r}^2}{\Delta^2 + (\Gamma/2)^2 + {\Omega_r}^2/2}\right), \qquad \text{(Freact)}$$

where $\Delta = \omega_0 - \omega$ is the detuning.

Analysis and the possibility of trapping

It can be verified by direct differentiation that $\mathcal{F}_{react}$ given by Eq. (Freact) corresponds to the following "optical potential":

$$\mathcal{V}_{opt} = -\frac{\hbar\Delta}{2} \ln\left(1 + \frac{{\Omega_r}^2/2}{\Delta^2 + (\Gamma/2)^2}\right).$$

Since the sign of the detuning determines whether this potential is attractive or repulsive, it can be seen[6] that atoms are "high-field seeking" for red-detuned ($\Delta > 0$) excitation, while they prefer positions with intensity minima for the blue-detuned case.

This effect offers the possibility of trapping neutral atoms. E.g., in magneto-optical traps (MOT), counter-propagating laser beams produce a standing wave pattern that can keep a cloud of cold atoms at a certain 3D position. Note that laser fields have a double role in this case: besides providing a trapping optical potential using the Zeeman-shifted atomic levels, they also decrease the speed of the atoms, i.e., provide a cooling mechanism (see the next slide).

[6] Q4: What steps have to be followed?

Pointlike atom in motion

An atom propagating along the axis of a monochromatic wave with a velocity of **v** experiences a field of

$$E(\mathbf{r}, t) = \mathcal{E} \cos(\omega t - \mathbf{kv}t),$$

For small enough velocities, we find

$$\mathcal{F}_{diss}(v) = \mathcal{F}_{diss}(0) - \mu v,$$

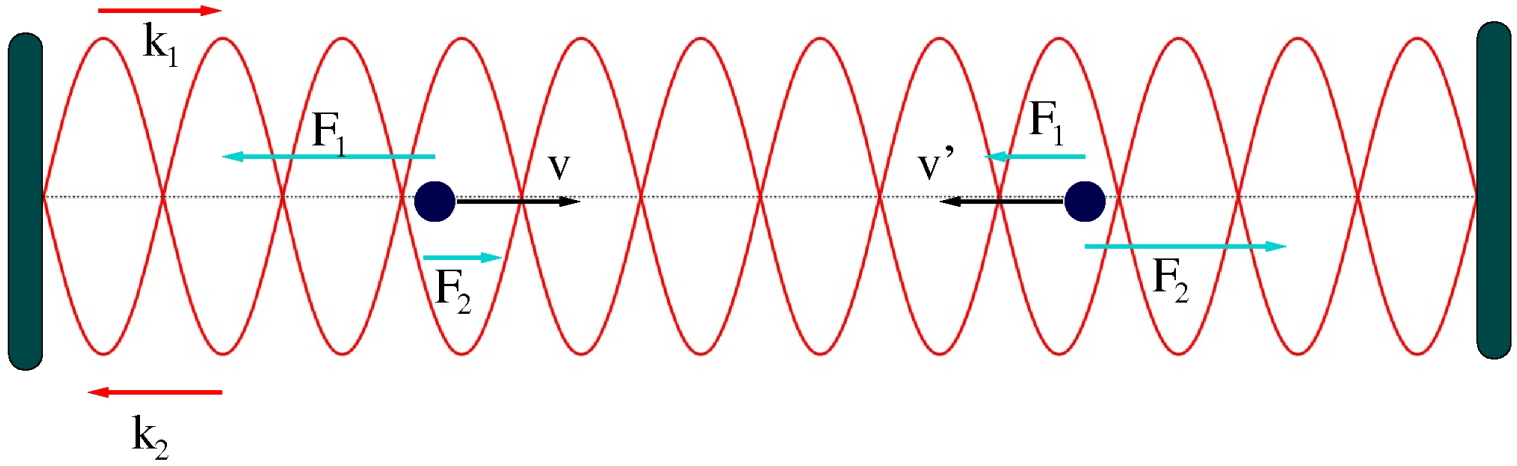
where the "friction coefficient" μ depends on the detuning:

$$\mu(\Delta) = \hbar k^2 \left(\frac{s}{(1+s)^2} \right) \left(\frac{\Delta\Gamma}{\Delta^2 + (\Gamma/2)^2} \right).$$

Here s is the saturation parameter. For a given value of s and positive Δ (red-detuned laser), there is indeed a friction force ($\mu > 0$).

Counterpropagating waves

When the fields are weak, their contribution can simply be added. Physically, both red-detuned laser modes exert friction forces on the atom, forces that have opposite direction and different magnitude. Using appropriate detuning, the atom can lose velocity, independently from the direction of its movement (see the figure below).



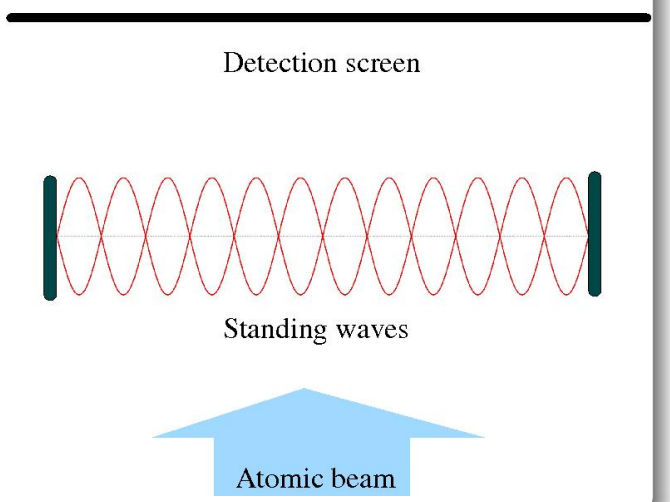
The minimal temperature that can be achieved via Doppler cooling is reached when the heating due to spontaneous emission is balanced by the cooling from the friction force. This leads to: $k_B T_{lim} \approx \frac{\hbar\Gamma}{2}$. For Sodium, T_{lim} is of the order of 240 μK.

Near-resonant Kapitza-Dirac-effect

Historically, Kapitza and Dirac predicted[7] that an *electron* beam could be diffracted by a standing light field as a result of stimulated Compton scattering. They concluded, however, that in 1933 there was no light source that could be used to perform the related experiment.

Atoms, molecules and probably viruses

Soon after the advent of lasers the effect was detected, and it was also realized that neutral atoms can also get diffracted by standing waves. Nowadays it is possible to experimentally demonstrate that even large molecules (e.g., C_{60}) show quantum mechanical interference having passed a standing wave grating[8].

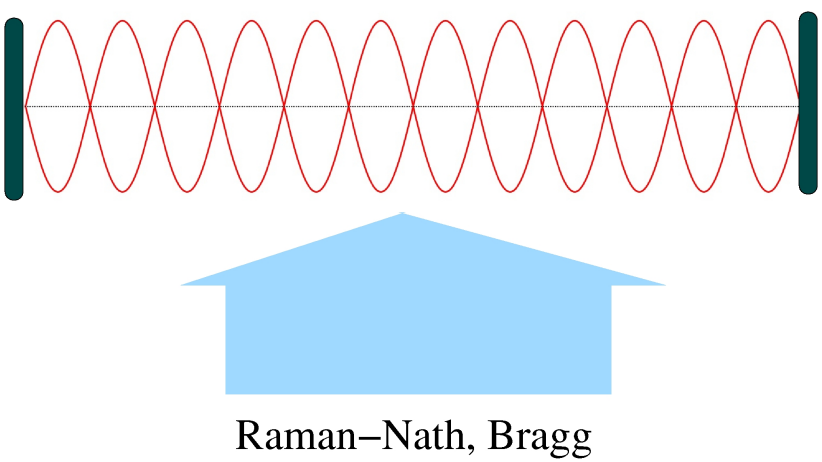


[7] P. L. Kapitza and P. A. M. Dirac, *Proc Cambridge Phil. Soc.* **29**, 297 (1933).

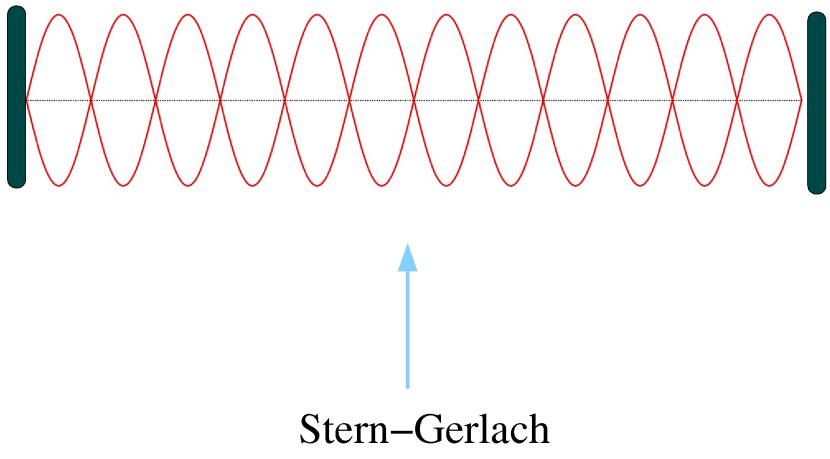
[8] S. Gerlich et al. *Nat. Commun.* **2**, 263 (2011).

Different regimes in atomic diffraction

For the Raman-Nath and Bragg regimes, the width of the incoming atomic beam is large compared to the period of the standing wave pattern, while the atoms probe the field locally for the Stern-Gerlach regime. The difference between the first two cases is whether the kinetic term can be neglected in the Hamiltonian (R-N) or not (B).



Raman–Nath, Bragg



Stern–Gerlach

Model Hamiltonian

Generally, it suffices to treat quantum mechanically only the momentum (and position) component that is parallel with the wave vector of the standing waves. Omitting the subscript, in a frame rotating at the laser frequency ω, we have:

$$H = \frac{p^2}{2m} + \hbar\Delta|2\rangle\langle 2| + \frac{\hbar}{2}\Omega_r(x)f(t)\left(|2\rangle\langle 1| + |1\rangle\langle 2|\right), \qquad \text{(Ham)}$$

where $\Omega_r(x) = \Omega_r \cos(kx)$, and $f(t) = \Theta(t) - \Theta(t+T_{int})$ is a simple model for the interaction time.

Raman-Nath regime

In the following we focus on the parameter range, where the kinetic (first) term in Eq. (Ham) can be ignored (although x and p are still considered as operators). This is the Raman-Nath approximation. It is convenient to use momentum representation in this case, and recall the translation property $\exp(ip_0x/\hbar)|p\rangle = |p+p_0\rangle$.

Details

Let us look for solutions corresponding to a fixed momentum eigenvalue, p. That is, we assume that $|\Psi(p,t)\rangle = u(p,t)|p\rangle|2\rangle + v(p,t)|p\rangle|1\rangle$. The time-dependent Schrödinger equation reads

$$
\begin{aligned}
i\frac{\partial u(p,t)}{\partial t} &= \frac{\Omega_r}{4}\left[v(p+\hbar k,t)+v(p-\hbar k,t)\right]+\Delta u(p,t),\\
i\frac{\partial v(p,t)}{\partial t} &= \frac{\Omega_r}{4}\left[u(p+\hbar k,t)+u(p-\hbar k,t)\right].
\end{aligned}
\tag{Sch}
$$

Let us restrict ourselves to the resonant case ($\Delta = 0$) and choose the following initial conditions: $|\Psi(t=0)\rangle = v(0,0)|p=0\rangle|1\rangle$, i.e., ground state with zero transversal momentum at $t=0$. It is easily seen that only momenta with integer multiples of $\hbar k$ occur in the solution: $u(p,t) = \sum u_m(-m\hbar k,t)$, $v(p,t) = \sum v_m(-m\hbar k,t)$.

Time evolution in the Raman-Nath regime

Using the series expansion, the Schrödinger equation (Sch) reads:

$$i\frac{du_m}{dt} = \frac{\Omega_r}{4}\left[u_{m-1} + u_{m+1}\right] \text{ (m odd)}, \quad i\frac{dv_m}{dt} = \frac{\Omega_r}{4}\left[v_{m-1} + v_{m+1}\right] \text{ (m even)}.$$

The solutions of these equations are known to be in the form of mth-order Bessel functions of the first kind. Thus the probability $P_m = |u_m|^2$ or $|v_m|^2$ of the atom having transverse momentum $m\hbar k$ is given by

$$P_m(t) = {J_m}^2\left(\frac{\Omega_r t}{2}\right).$$

Discussion

Physically, the order of the populated transverse momentum states, m, cannot increase infinitely without violating the conditions that define the Raman-Nath approximation. As the transversal kinetic energy becomes comparable to, or even larger than the interaction energy, the Bragg-regime is reached. For more details see Ref. [1].

Stern-Gerlach regime

In contrast to the previous cases, the Stern-Gerlach regime uses an atomic beam that is well localized in space compared to the optical potential. That is, the atom sees a local potential rather than the whole cosine standing wave. At resonance, the local Hamiltonian without the kinetic term has the following form:

$$H_{SG} = \frac{\hbar}{2}\Omega_r \cos kx \left(|2\rangle\langle 1| + |1\rangle\langle 2|\right).$$

Local diagonalization of the Hamiltonian

The eigenvalues and eigenvectors of the Hamiltonian above are given by:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle \pm |2\rangle\right), \quad E_{\pm}(x) = \pm\frac{1}{2}\Omega_r \cos(kx).$$

These are the position-dependent dressed states.

Equations of motion

Let us consider the following initial conditions: at $t = 0$ the atom is in its ground state $|1\rangle$ and localized around the position x_0. That is, $|\Psi(t = 0)\rangle = |x_0\rangle|1\rangle = 1/\sqrt{2}|x_0\rangle(|-\rangle + |+\rangle)$. The local forces acting on the states $|x_0\rangle|-\rangle$ and $|x_0\rangle|+\rangle$ can be calculated by obtaining the negative gradient ($-d/dx$ in 1D) of the corresponding energy eigenvalues. Using these results, we obtain that the expectation values of the positions obey pendulum-like equations:

$$\frac{d^2}{dt^2}\langle x_\pm\rangle = \pm\frac{\hbar k}{2m}\Omega_r \sin(k\langle x_\pm\rangle).$$

Stern-Gerlach analogy

These equations of motion show that a localized atomic wavepacket initially at rest is split into two parts corresponding to the different internal states and oscillate within the potential well. This is in complete analogy with the Stern-Gerlach effect for spin-1/2 particles in a magnetic field gradient.

Outlook

Recall that our discussion remained on the semiclassical level. Important phenomena could be understood using this approach, but let us keep in mind that complete quantum mechanical treatment would require field quantization as well. We also neglected the back-action of the atoms on the electromagnetic field (atomic emission and absorbtion did not modify the intensity of the field), which can be a severe approximation. Especially, for cavity quantum electrodynamics, when emitted photons cannot immediately escape, the coupled dynamics of the field and the atoms have to be solved consistently. Important cooling mechanism were also not part of this lecture, see [2] for more details.

Questions

1. Does the wheel shown on page 3 rotate indeed because of radiation pressure?
2. What is the Hilbert-space on which the atomic Hamiltonian H_a acts?
3. Why does the transverse momentum component p commute with H_a?
4. Show that atoms are "high-field seeking" for red-detuned ($\Delta > 0$) excitation.
5. What is the basic principle of Doppler cooling?
6. What kind of detuning is needed for Doppler cooling?

Questions (continued)

7. What is the Raman-Nath approximation?
8. What are the regimes that are usually considered for near-resonant Kapitza-Dirac effect?
9. Can the wave function of a large molecule interfere with itself?
10. What is the analogy between a narrow atomic beam that is diffracted by standing waves and the Stern-Gerlach experiment?

References

General, review texts

1. P. Meystre and M. Sargent, *Elements of Quantum Optics*, Spinger (Berlin, Heidelberg) (2007).
2. C. Cohen-Tannoudji, Atomic Motion in Laser Light, in J. Dalibard, J.-M. Raimond and J. Zinn-Justin, eds., *Fundamental Systems in Quantum Optics*, North-Holland (Amsterdam) (1992).

Original papers mentioned

- P. Lebedev, *Ann. Phys.*, **6**, 433 (1901).
- E. E. Nichols and G. F. Hull, *Phys. Rev.*, **17**, 26 (1903).
- P. L. Kapitza and P. A. M. Dirac, *Proc Cambridge Phil. Soc.* **29**, 297 (1933).
- S. Gerlich et al. *Nat. Commun.* **2**, 263 (2011).