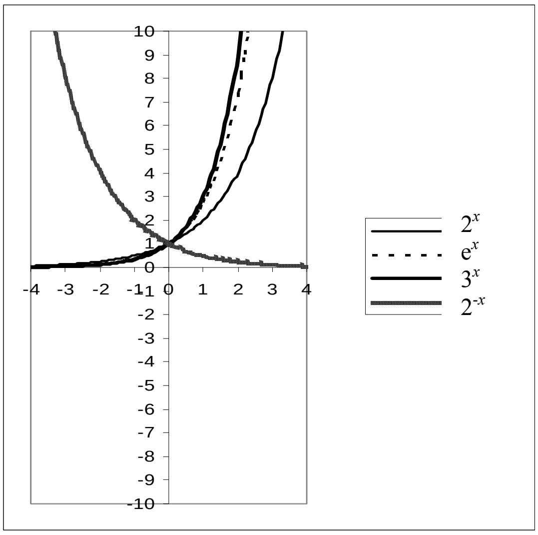
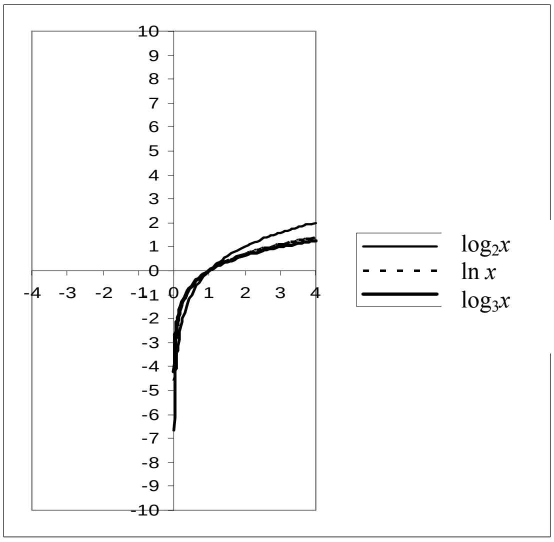
G3. Mathematical basics: exponential és logarithmic functions and applications: transformations

Exponential function: $y=a^x$, $a>0$ Logarithmic function: $y=log_a x$, $a>0 \Rightarrow a^y=x$





3.1. Operations with exponents. Calculate the following!

$2^x \cdot 2^y=$ $2^x/2^y=$ $(2^x)^y=$ $2^{-x}=$

$3^2 \cdot 3^y=$ $3^3/3^4=$ $(3^2)^3=$ $3^{-1}=$ $3^{-2}=$ $\left(\frac{1}{3}\right)^{-2}=$

3.1. Operations with logarithms.

3.1.1. Calculate the following logarithms:

$log_2 2=$	$log_{10}1= lg1=$	$ln\,1=$
$log_2 4=$	$lg10=$	$ln\,e=$
$log_2 8=$	$lg100=$	$ln\,e^2=$
$log_2 1=$	$lg1000=$	$ln\,(1/e)=$
$log_2(1/2)=$	$lg\,0{,}1=$	$log_3 9=$
$log_2(1/4)=$	$lg\,0{,}01=$	$log_3 27=$
$log_2(-2)=$	$lg\,(1/100)=$	$log_9 3=$
$log_{½}2=$	$lg\,0.0001=$	$log_4 16=$
$log_{½}4=$	$lg\,2=$	$log_{16} 4=$
$log_{½}1=$	$lg\,3=$	$log_{16} 1=$

3.1.2. Expand:

$log_2(a+b)=$ $log_2(a \cdot b)=$ $log_2(a/b)=$ $log_2(a)^b=$

3.1.3. Transform the following to a simpler form!

$log_2(4)+ log_2(8)=$ $ln(x)+ ln(y)=$ $ln(x) – ln(y)=$

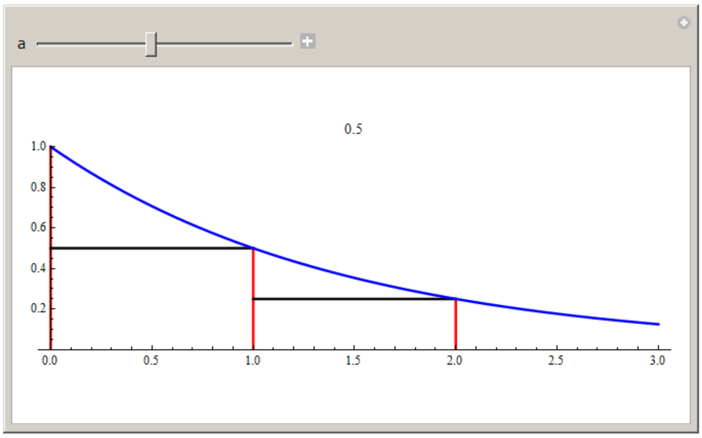
$0.5log_2(4)+2log_2(3)=$ $½[ln(x)+ ln(y)]=$

3.2. Plotting exponential and logarithmic functions. Sketch the following exponential functions and their inverse (logarithm) and give their domains and ranges! Give the values assigned to –1, 0, 1 (if sensible).

a) $y = 2^x$ b) $y = 3^x$ c) $y = e^x$ e=2.71828… d) $y = \left(\frac{1}{2}\right)^x$, e) $y = \left(\frac{1}{3}\right)^x$

f) y = - 2^x g) y = 1 - 2^x

3.3. Half-life of the exponential function. Half-life is the amount of time, during which the starting value decreases to its half.



For 0<a<1 the half-life (T1/2) is: $a^{x+T_{1/2}} = \frac{1}{2}a^x$, from which $a^{T_{1/2}} = \frac{1}{2}$, thus $T_{1/2} = -\frac{1}{\log_2 a}$

During a decay of a radioisotope, the number of isotopes is given by a decreasing exponential function:

$$N(t) = N(0)e^{-\lambda t}$$

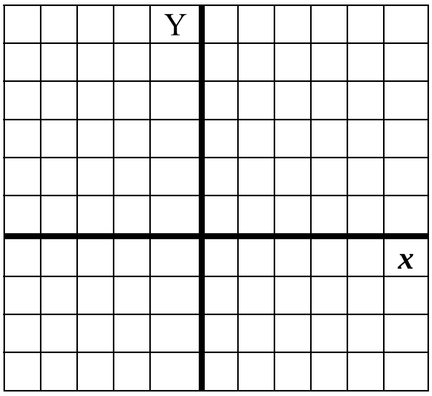
where *N(t)* is the number of isotopes at time *t*, *N(0)* is the value of *N* at time *t* = 0 (starting value), and λ is a constant depending on the core, which is called **decay constant**.

In this case the half-life is:

$\frac{N(0)}{2} = N(0)e^{-\lambda T_{1/2}}$, more simply $\frac{1}{2} = e^{-\lambda T_{1/2}}$, from which $\ln(\frac{1}{2}) = \ln(e^{-\lambda T_{1/2}}) = -\lambda T_{1/2}$, thus $\ln 2 = \lambda T_{1/2}$

The half-life is $T_{1/2} = \frac{\ln 2}{\lambda}$

3.3.1. Give the half-life of the function $y = \left(\frac{1}{2}\right)^x$! Plot it!



3.3.2. Give the half-life, if the decay of the radioisotope is given by the equation N(t)=3 e^{-5t} !

……………………..

……………………..

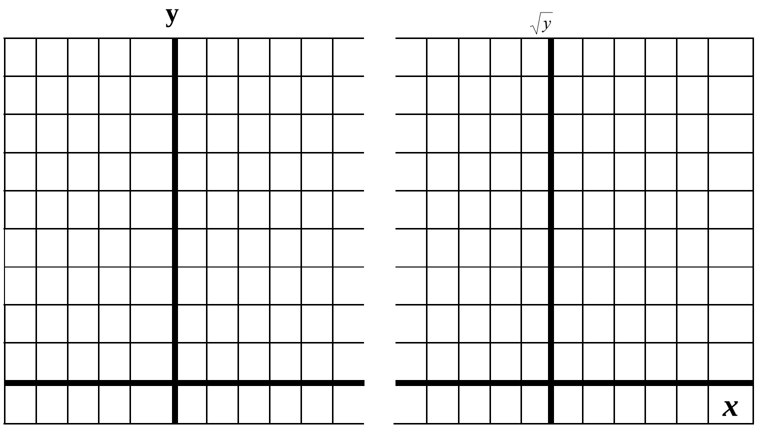
3.3.3. For a radioisotope, 62.3% of it decays during 8 days. Give the half-life!

3.4. Linearising curved relationships.

Sometimes we are unable to fit a regressional line to our data, but for example a parabola can be fitted. We can fit lines more easily, so we transform the data, and plot the sqare root of them on the y-axis instead of the original data:

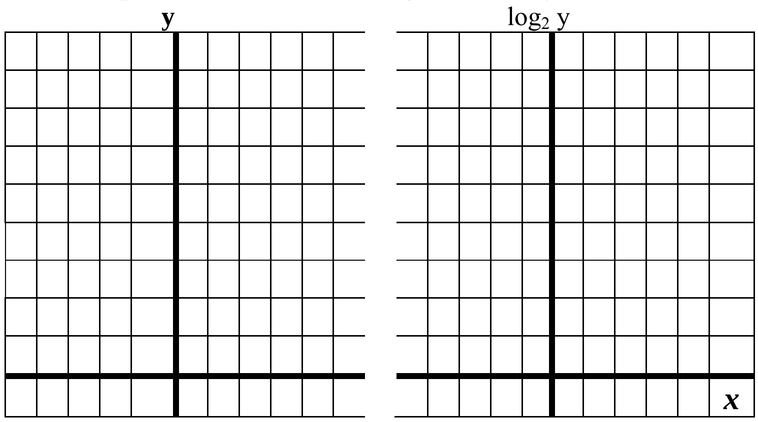
3.3.1.

x	y	$Y=\sqrt{y}$
1	1	1
2	4	2
3	9	3



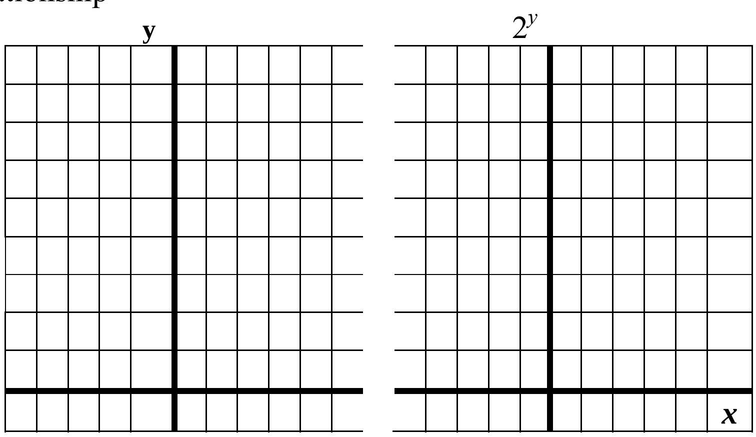
3.3.2. Transformation of an exponential relationship.Lets calculate the logarithm of y!

x	$y=2^x$	$\log_2 y$
-1	1/2	-1
0	1	0
1	2	1
2	4	2
3	8	3



3.3.3. Transformation of a logarithmic relationship

x	y	2^y
0.5	-1	1/2
1	0	1
4	2	4
8	3	8



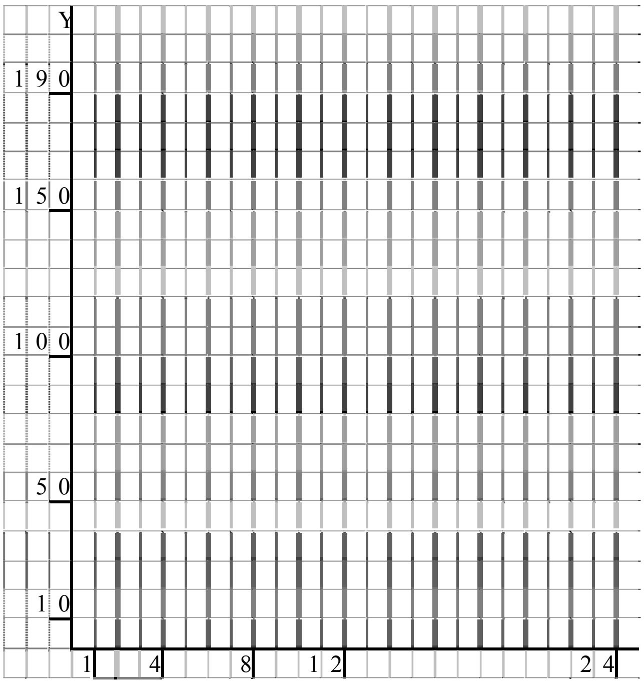
3.3.4. Transform the function $y=4\ 2^{3x}$, so that its graph becomes a line!

3.3.5. The next dadaset measures the effect (blood level) of 20 mg-s of a drug as a function of time. The first coloumn gives the time, the second one gives the effect, and the third one gives the logarithm of the effect.

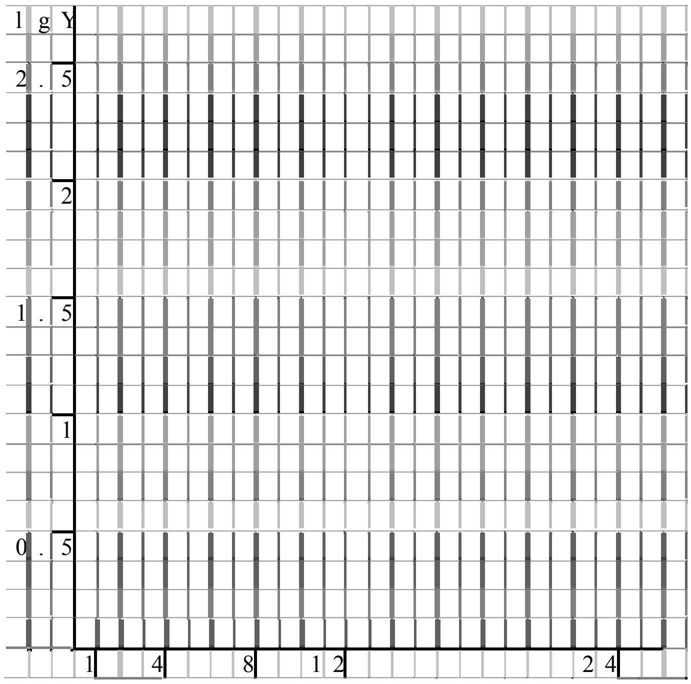
Draw a scatter plot of the data, with time as the independent variable (horizontal axis)!

a) The original measurments are on the y-axis in the first chart
b) The logarithm of the measurements are on the y-axis in the second chart

x: idő (óra)	y	$log_{10}\,y$
1	184.33	2.27
4	87.63	1.94
8	33.05	1.52
12	9.30	.97
24	2.80	.45



a)



b)

What do you think of the connection based on the plot? Is it positive or negative?

....

Is the connection approximately linear?

....

The equation of the regression based on the second plot is:
$log_{10}\,y = 2.206 - 0.079 \cdot x$.
Give the equation of the fitted exponential curve in the first plot: y=……………………………………

Type the dataset into SPSS and check your results! Give the equation of the line!

3.3.5. Give the equation of the connections between the variables based on the plots below! Give the connections both for the linear and logarithmic scales!

http://www.clinchem.org/cgi/reprint/56/9/1413
Hendrik Neubert* Jeremy Gale and David Muirhead: Online High-Flow Peptide Immunoaffinity Enrichment and Nanoflow LC-MS/MS: Assay Development for Total Salivary Pepsin/Pepsinogen. Clinical Chemistry 56:9 1413–1423 (2010)

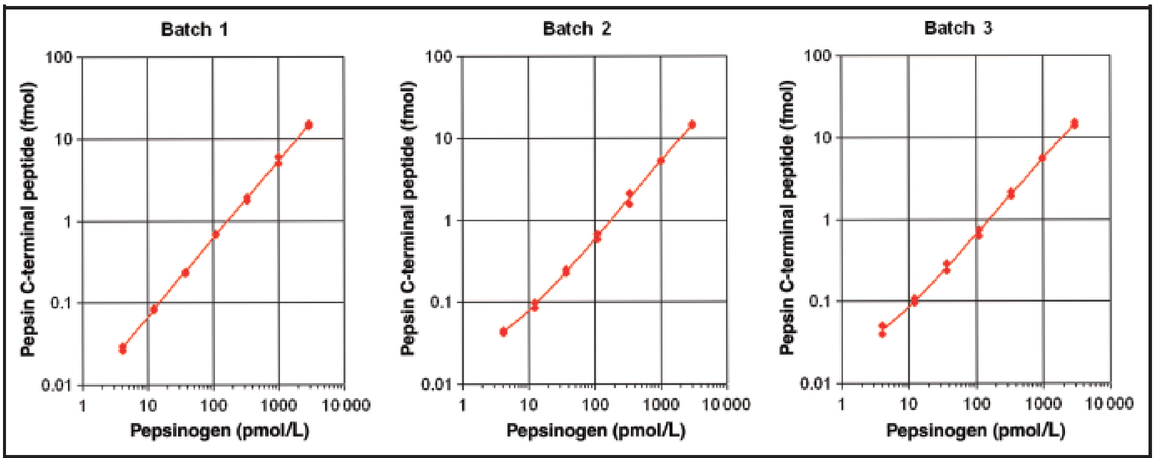


Fig. 5. Three calibration lines obtained during assay validation ranging from 4.08 to 2980 pmol/L pepsinogen in human saliva.
The calibration was modeled against a weighting factor of $1/y^2$.

Batch 1.
Equation on the logarithmic scale
Equation on the linear scale

Batch 2.
Equation on the logarithmic scale
Equation on the linear scale

Batch 3.
Equation on the logarithmic scale
Equation on the linear scale

3.3.6. Give the equation of the line both for linear and logarithmic scales!

Clinical Chemistry 55:11 2049–2052 (2009). Plasma Proprotein Convertase Subtilisin/ Kexin Type 9: A Marker of LDL Apolipoprotein B-100 Catabolism?

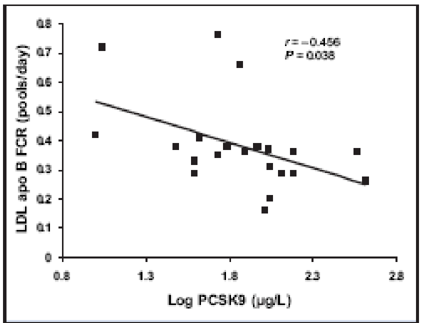


Fig. 1. Correlation between the LDL apo B-100 FCR and plasma PCSK9 concentration in 21 males.

The data can be found in the file clinchem2049.sav. Run the regression in SPSS!

Individual plasma PCSK9, lathosterol concentrations and LDL-apoB-100 kinetic parameters

Subject no	PCSK9 (μg/L)	Lathosterol (μmol/L)	LD-apoB-100 FCR (pools/day)	LDL-apoB-100 PR (mg/kg/day)
1	11	7.32	0.72	10.7
2	107	4.63	0.37	6.16
3	54	3.7	0.35	6.27
4	39	3.62	0.29	5.24
5	10	5.87	0.42	7.68
6	54	7.85	0.76	16.44
7	73	2.82	0.66	10.07
8	109	8.52	0.2	3.57
9	39	8.06	0.33	4.46
10	102	14.17	0.16	3.4
11	94.3	8.94	0.38	8.91
12	110.7	9.77	0.31	9.11
13	61	4.87	0.38	9.26
14	403.1	7.94	0.26	5.14
15	151.8	7.57	0.29	8.82
16	41.7	6.87	0.41	6.46
17	130	10.62	0.29	9.43
18	366.6	6.02	0.36	9.57
19	150	8.62	0.36	8.58
20	30.2	3.78	0.38	12.95
21	78	9.88	0.36	8.45

FCR: fractional catabolic rate

PR: production rate

Logarithmic scale (base 10)

Analyse/regression/Curve estimation/ PCSK9 ->Independent, lgLDLapoBPR -> Dependent, Models: linear, ☑ Display ANOVA table

Equation: ………………………….

r: ……………………….

r^2: ……………………….

Logarithmic scale (base *e*)

Analyse/regression/Curve estimation/ PCSK9 ->Independent, lnLDLapoBPR -> Dependent, Models: linear, ☑ Display ANOVA table

Equation: ………………………….

r: ……………………….

r^2: ……………………….

Linear scale:

Analyse/regression/Curve estimation/ PCSK9 ->Independent, LDLapoBPR -> Dependent, Models: logarithmic, ☑ Display ANOVA table

Equation: ………………………….

r: ……………………….

r^2: ……………………….