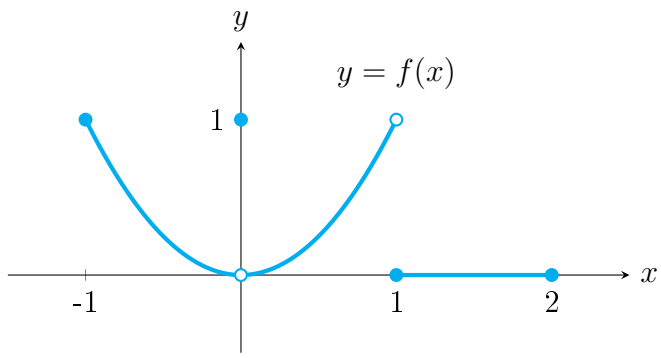
Homework No. 3 – October 1, 2017

Exercise 1. Using the graph, determine the following limits.



(a) $\lim_{x\to -1^+} f(x)$

(b) $\lim_{x\to 0^-} f(x)$

(c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 1} f(x)$

(e) $\lim_{x\to 2^-} f(x)$

(f) $\lim_{x\to 2^+} f(x)$

Exercise 2.

Let $f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & x > 2. \end{cases}$

(a) Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$.

(b) Does $\lim_{x\to 2} f(x)$ exist? If so, what is it? If not, why not?

(c) Find $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$.

(d) Does $\lim_{x\to 4} f(x)$ exist? If so, what is it? If not, why not?

Exercise 3. Let $f(x) = (x^2 - 9)/(x + 3)$. Make a table of the values of f at the points $x = -3.1,\ -3.01,\ -3.001$, and so on as far as your calculator can go. The estimate $\lim_{x\to 3} f(x)$. What estimate do you arrive at if you evaluate f at $x = -2.9,\ -2.99,\ -2.999, \ldots$ instead?

Exercise 4. Find the following limits.

(a) $\lim_{x\to -7}(2x + 5)$

(b) $\lim_{x\to 12}(10 - 3x)$

(c) $\lim_{x\to 2}(-x^2 + 5x - 2)$

(d) $\lim_{x\to -2}(x^3 - 2x^2 + 4x + 8)$

(e) $\lim_{t\to 6} 8(t - 5)(t - 7)$

(f) $\lim_{s\to 2/3} 3s(2s - 1)$

(g) $\lim_{x\to 2} \frac{x+3}{x+6}$

(h) $\lim_{x\to 5} \frac{4}{x-7}$

(i) $\lim_{y\to -5} \frac{y^2}{5-y}$

(j) $\lim_{y\to 2} \frac{y+2}{y^2+5y+6}$

(k) $\lim_{x\to -1} 3(2x-1)^2$

(l) $\lim_{x\to -4} (x+3)^{1984}$

(m) $\lim_{t\to -3} (5-t)^{4/3}$

(n) $\lim_{z\to 0} (2z-8)^{1/3}$

(o) $\lim_{h\to 0} \frac{3}{\sqrt{3h+1}+1}$

(p) $\lim_{h\to 0} \frac{5}{\sqrt{5h+4}+2}$

(q) $\lim_{h\to 0} \frac{\sqrt{3h+1}-1}{h}$

(r) $\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h}$

Exercise 5. Find the following limits.

(a) $\lim_{x\to 5} \frac{x-5}{x^2-25}$

(b) $\lim_{x\to -3} \frac{x+3}{x^2+4x+3}$

(c) $\lim_{x\to -5} \frac{x^2+3x-10}{x+5}$

(d) $\lim_{x\to 2} \frac{x^2-7x+10}{x-2}$

(e) $\lim_{t\to 1} \frac{t^2+t-2}{t^2-1}$

(f) $\lim_{t\to -1} \frac{t^2+3t+2}{t^2-t-2}$

(g) $\lim_{x\to -2} \frac{-2x-4}{x^3-2x^2}$

(h) $\lim_{y\to 0} \frac{5y^3+8y^2}{3y^4-16y^2}$

(i) $\lim_{u\to 1} \frac{u^4-1}{u^3-1}$

(j) $\lim_{v\to 2} \frac{v^3-8}{v^4-16}$

(k) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

(l) $\lim_{x\to 4} \frac{4x-x^2}{2-\sqrt{x}}$

(m) $\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2}$

(n) $\lim_{x\to -1} \frac{\sqrt{x^2+8}-3}{x+1}$

(o) $\lim_{x\to 2} \frac{\sqrt{x^2+12}-4}{x-2}$

(p) $\lim_{x\to -2} \frac{x+2}{\sqrt{x^2+5}-3}$

Exercise 6. Find the following limits.

(a) $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

(b) $\lim_{x\to 0} \frac{(x+h)^2-x^2}{h}$

Exercise 7. Find the following limits.

(a) $\lim_{x\to 0^+} \frac{1}{3x}$

(b) $\lim_{x\to 0^-} \frac{5}{2x}$

(c) $\lim_{x\to 2^-} \frac{3}{x-2}$

(d) $\lim_{x\to 3^+} \frac{1}{x-3}$

(e) $\lim_{x\to -8^+} \frac{2x}{x+8}$

(f) $\lim_{x\to -5^-} \frac{3x}{2x+10}$

(g) $\lim_{x\to 7} \frac{4}{(x-7)^2}$

(h) $\lim_{x\to 0} \frac{-1}{x^2(x+1)}$

(i) $\lim_{x\to 0^+} \frac{2}{3x^{1/3}}$

(j) $\lim_{x\to 0^-} \frac{2}{3x^{1/3}}$

(k) $\lim_{x\to -0.5^-} \sqrt{\frac{x+2}{x+1}}$

(l) $\lim_{x\to 1^+} \sqrt{\frac{x-1}{x+2}}$

(m) $\lim_{x\to -2^+} \left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right)$

(n) $\lim_{x\to 1^-} \left(\frac{1}{x+1}\right)\left(\frac{x+6}{x}\right)\left(\frac{3-x}{7}\right)$

(o) $\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$

(p) $\lim_{h\to 0^-} \frac{\sqrt{6}-\sqrt{5h^2+11h+6}}{h}$

(q) $\lim_{x\to -2^+} (x+3)\frac{|x+2|}{x+2}$

(r) $\lim_{x\to -2^-} (x+3)\frac{|x+2|}{x+2}$

(s) $\lim_{x\to 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$

(t) $\lim_{x\to 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

Exercise 8. Find the following limits.

(a) $\lim \frac{1}{x^2-4}$ as $x \to 2^+$, $x \to 2^-$, $x \to -2^+$, $x \to -2^-$;

(b) $\lim \frac{x}{x^2-1}$ as $x \to 1^+$, $x \to 1^-$, $x \to -1^+$, $x \to -1^-$;

(c) $\lim \left(\frac{x^2}{2} - \frac{1}{x}\right)$ as $x \to 0^+$, $x \to 0^-$, $x \to -1$, $x \to \sqrt[3]{2}$;

(d) $\lim \frac{x^2-1}{2x+4}$ as $x \to -2^+$, $x \to -2^-$, $x \to 1^+$, $x \to 0^-$;

(e) $\lim \frac{x^2-3x+2}{x^3-2x^2}$ as $x \to 0^+$, $x \to 2^+$, $x \to 2^-$, $x \to 2$, $x \to 0$;

(f) $\lim \frac{x^2-3x+2}{x^3-4x}$ as $x \to 2^+$, $x \to -2^+$, $x \to 0^-$, $x \to 1^+$, $x \to 0$.

Exercise 9. Find the following limits.

(a) $\lim_{x\to\infty} \frac{2x+3}{5x+7}$

(b) $\lim_{x\to\infty} \frac{2x^2+3}{5x^2+7}$

(c) $\lim_{x\to-\infty} \frac{x^2-4x+8}{3x^3}$

(d) $\lim_{x\to\infty} \frac{1}{x^2-7x+1}$

(e) $\lim_{x\to-\infty} \frac{x^2-7x}{x+1}$

(f) $\lim_{x\to\infty} \frac{x^4+x^3}{12x^3+128}$

Exercise 10. Find the limits of each function as $x \to \infty$ and $x \to \infty$.

(a) $f(x) = \frac{2}{x} - 3$

(b) $g(x) = \pi - \frac{2}{x^2}$

(c) $f(x) = \frac{1}{2+(1/x)}$

(d) $g(x) = \frac{1}{8-(5/x^2)}$

(e) $h(x) = \frac{-5+(7/x)}{3-(1/x^2)}$

(f) $h(x) = \frac{3-(2/x)}{4+(\sqrt{2}/x^2)}$

(g) $f(x) = \frac{2x+3}{5x+7}$

(h) $f(x) = \frac{2x^3+7}{x^3-x^2+x+7}$

(i) $g(x) = \frac{x+1}{x^2+3}$

(j) $g(x) = \frac{3x+7}{x^2-2}$

(k) $h(x) = \frac{7x^3}{x^3-3x^2+6x}$

(l) $h(x) = \frac{1}{x^3-4x+1}$

(m) $f(x) = \frac{10x^5+x^4+31}{x^6}$

(n) $g(x) = \frac{9x^4+x}{2x^4+5x^2-x+6}$

(o) $h(x) = \frac{-2x^3-2x+3}{3x^3+3x^2-5x}$

(p) $f(x) = \dfrac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$

(q) $g(x) = \dfrac{2\sqrt{x} + x^{-1}}{3x - 7}$

(r) $h(x) = \dfrac{2 + \sqrt{x}}{2 - \sqrt{x}}$

(s) $f(x) = \dfrac{\sqrt[3]{x} - \sqrt[5]{x}}{\sqrt[3]{x} + \sqrt[5]{x}}$

(t) $f(x) = \dfrac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$

(u) $h(x) = \dfrac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + \sqrt{x}}$

(v) $g(x) = \dfrac{\sqrt[3]{x} - 5x + 3}{2x + x^{2/3} - 4}$