HOMEWORK NO. 1 – September 15, 2017

Exercise 1. If a, b, c, d are distinct objects, determine which of the five sets $\{a, b, c\}$, $\{b, c, a, b\}$, $\{c, a, c, b\}$, $\{b, c, b, a\}$, $\{a, b, c, d\}$ are equal.

Exercise 2. Let $A = \{a, b, c\}$, $B = \{a, b\}$, $C = \{a, b, d\}$, $D = \{a\}$ and $E = \{b, c\}$, where a, b, c are distinct objects. State whether each of the following statements is true or false:

(a) $B \subset A$,   (b) $E \neq C$,   (c) $D \not\subset B$,   (d) $D \subset A$,   (e) $A = B$

Exercise 3. Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{4, 5, 6, 7, 8, 9\}$, $C = \{2, 4, 6, 8\}$, $D = \{4, 5\}$, $E = \{5, 6\}$, $F = \{4, 6\}$, and X a set which satisfies the following conditions: $X \subset A$, $X \subset B$ and $X \not\subset C$. Determine which of the sets A, B, C, D, E, F can equal X.

Exercise 4. Which of the following sets is the empty set?

(a) $\{x \colon x$ is an odd integer and $x^2 = 4\}$
(b) $\{x \colon x$ is an integer and $x + 8 = 8\}$
(c) $\{x \colon x$ is a positive integer and $x < 1\}$

Exercise 5. Let $A = \{a, b, c, d\}$, $B = \{b, d, f, h\}$, $C = \{c, d, e, f\}$. Find

(a) $A \cap B$, $A \cap C$, $B \cap C$,
(b) $A \cup B$, $A \cup C$, $B \cup C$,
(c) $A \setminus B$, $B \setminus A$, $A \setminus C$, $C \setminus A$, $C \setminus B$, $B \setminus C$.

Exercise 6. Let R be the set of real numbers, $A = \{x \in R \colon 1 \leq x \leq 3\}$ and $B = \{x \in R \colon 2 \leq x \leq 4\}$. Find

(a) $A \cup B$,   (b) $A \cap B$,   (c) $(R \setminus A) \cap B$,   (d) $(R \setminus B) \cap A$,   (e) $(R \setminus A) \cap (R \setminus B)$,
    (f) $(R \setminus B) \cap (R \setminus A)$,   (g) $(R \setminus A) \cup (R \setminus B)$,   (h) $B \cup [A \cap (R \setminus B)]$,
    (i) $[(R \setminus A) \cap B] \cup [(R \setminus B) \cap A]$.

Exercise 7. Let Z be the set of all integers, $A = \{x \in Z \colon x$ is a multiple of 10$\}$, and $B = \{x \in Z \colon x$ is a multiple of 15$\}$. What is $A \cap B$? Can you generalize this result?

Exercise 8. Let $A = \{1, 2, 3, 8, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{3, 6, 9\}$. Determine $A \setminus B$, $C \setminus A$, $(A \setminus B) \cap C$, $(B \cup C) \setminus (A \setminus C)$, $\mathcal{P}(C)$, $\mathcal{P}(A \setminus B)$.

Exercise 9. Sketch the following sets: $(-2; 3] \setminus [1; 4]$, $(-2; 3] \cup [1; 4]$, $(-2; 3] \cap [1; 4]$.

Exercise 10. Sketch the following sets: $(2, 6] \setminus (1; 4)$, $(2, 6] \cap (1; 4)$, $(2, 6] \cup (1; 4)$.

Exercise 11. Let $A = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$. Determine whether each of the following statements is true or false:

(a) $\emptyset \in A$
(c) $\{\emptyset\} \in A$
(e) $\{\{\emptyset\}\} \in A$
(g) $\{\emptyset, \{\emptyset\}\} \in A$
(b) $\emptyset \subseteq A$
(d) $\{\emptyset\} \subseteq A$
(f) $\{\{\emptyset\}\} \subseteq A$
(h) $\{\emptyset, \{\emptyset\}\} \subseteq A$

Exercise 12. Let $A = \mathcal{P}(\{a, b\})$ and $B = \mathcal{P}(\{b, c\})$. Determine the elements of the following sets:

$$A \cup B, \quad A \cap B, \quad A \setminus B, \quad B \setminus A, \quad A \triangle B.$$

Exercise 13. Let $U = \{a, b, c, d, e\}$ be the universal set, $A = \{a, b, c, d\}$, $B = \{d, e\}$ and $C = \{a, b, e\}$. Determine the elements of the following sets:

$$A \cup B, \quad A \cap B, \quad \overline{B}, \quad A \setminus B, \quad A \triangle B, \quad (A \triangle \overline{C}) \setminus \overline{B}, \quad \mathcal{P}(B).$$

Exercise 14. Give the elements of the set $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$.

Exercise 15. Are there any sets A, B, C, such that $A \subseteq B \in C$ and $A \in B \subseteq C$?